VECTORS

In this unit you will learn about vectors which are used to describe many physical quantities which specify both direction and magnitude. The measure of wind blowing at 25 mph northeast is based on direction and magnitude.

Addition of Vectors

Geometric Vectors

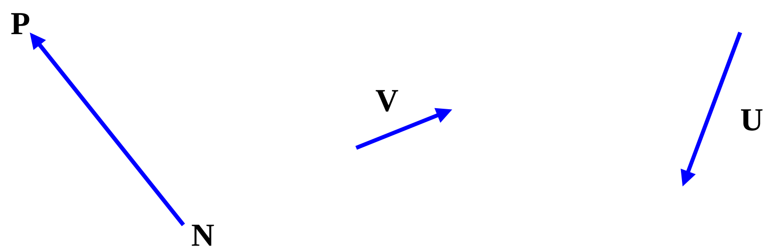
Addition of Vectors

Vectors

A **vector** is a segment that has direction. Vectors are used to represent motion or forces acting upon objects. Vector quantities have both magnitude (length) and direction such as forces, velocities, and acceleration.

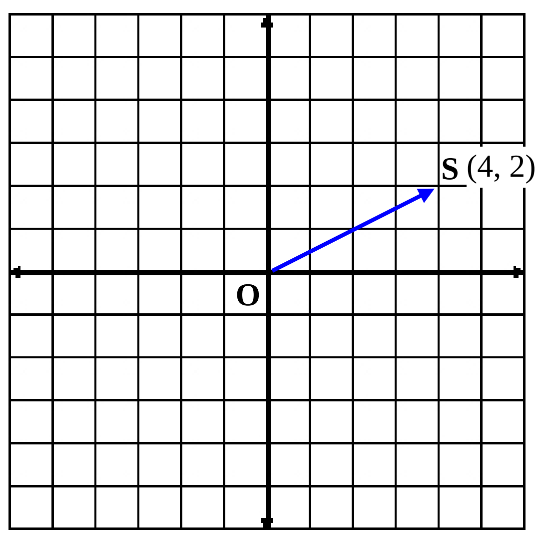
A quantity that can be described by magnitude only is known as a **scalar quantity** such as pressure and temperature.

An arrow is placed at the end of a vector to indicate the direction. In the illustration below, the vector from point N to point P can be written as $\overrightarrow{NP}$. "N" is called the initial point and "P" is called the terminal point. Vectors are frequently named using a letter such as **V** and **U** printed in bold face.



A vector placed in a rectangular coordinate plane is said to be in standard position when the initial point is at the origin.

In the diagram at the right, vector OS ($\overrightarrow{OS}$) is in standard position and can be represented by the ordered pair (4, 2).



If the coordinates of the endpoints of a vector in a coordinate system are given (x_a, y_a) and (x_b, y_b), it is possible to find the corresponding standard vector.

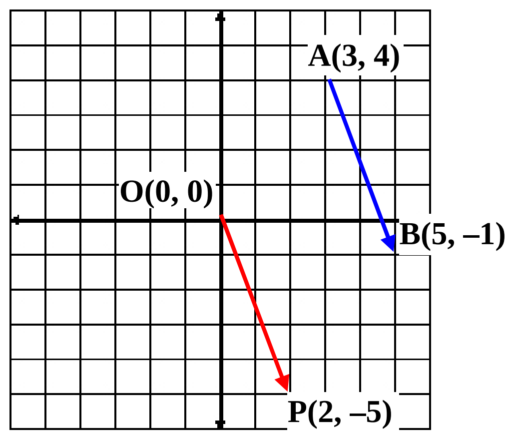
The initial point "O" of the standard vector $\overrightarrow{OP}$ is always (0, 0). Therefore, you only have to find the coordinates of the terminal point "P" given by:

$$(x_p, y_p) = (x_b - x_a, y_b - y_a)$$

Example #1: Given the vector $\overrightarrow{AB}$ with initial point A(3, 4) and the terminal point B(5, –1), find the standard vector $\overrightarrow{OP}$ of $\overrightarrow{AB}$.

$$(x_p, y_p) = (5-3, -1-4)$$
$$= (2, -5)$$

The ordered pair that represents the standard vector $\overrightarrow{OP}$ is (2, –5).



Magnitude of Vectors

An algebraic vector is an ordered pair of real numbers, and to avoid confusion between a point on the coordinate plane (*x*, *y*) we will denote an algebraic vector as <*x*, *y*>.

At the beginning of the unit we discussed that a vector has both magnitude and direction. The magnitude of a vector is its length and is found using the following definition.

The magnitude of a vector **V** = <*x*, *y*> is denoted by $|V|$ and is found by

$$|V| = \sqrt{x^2 + y^2}$$

Example #2: Find the magnitude of vector **V** = <2, –5>

$$|V| = \sqrt{2^2 + (-5)^2}$$

$$|V| = \sqrt{29}$$

Addition of Algebraic Vectors

Many applications of vectors involve vector addition and is performed by adding the corresponding components as indicated in the following definition:

If **u** = <*a*, *b*> and **v** = <*c*, *d*> then

u + **v** = <*a* + *c*, *b* + *d*>

Example #3: Let **u** = <4, –3> and **v** = <0, –5> find **u** + **v**.

u + **v** = <4 + 0, –3 + (–5)>

u + **v** = <4, –8>

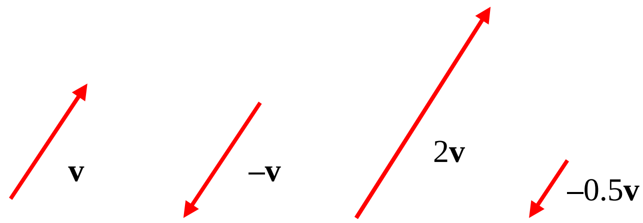
Scalar Multiplication

In the product "*a***V**", *a* is a real number called a scalar and the operation of "*a***V**" is called **scalar multiplication** and is explained in the following definition.

If **u** = <*a*, *b*> and *k* is a scalar, then

*k***u** = <*ka*, *kb*>

If *k* is positive, then *k***v** has the same direction as **v**. If *k* is negative, then *k***v** has the opposite direction as **v**.



Example #4: If **U** = <3, –2>, find –4**U.**

–4**U** = <(–4 · 3), (–4 · –2)>

–4**U** = <–12, 8>

Basic Properties of Vectors

Vector addition and scalar multiplication have algebraic properties just like real numbers. These properties allow us to manipulate symbols representing vectors just as we manipulate symbols for real numbers in algebra.

Addition Properties		**Scalar Multiplication**	
Commutative	**u** + **v** = **v** + **u**	**Associative**	**u(vw)= (uv)w**
Associative	**u** + (**v** + **w**) = (**u** + **v**) + **w**	**Distributive**	**u(***m* + *n***)** = **u***m* + **u***n*
Additive Identity	**u** + 0 = **u**	**Distributive**	*m***(u** + **v)** = *m***u** + *m***v**
Additive Inverse	**u** + (–**u**) = 0	**Multiplicative Identity**	(1)(**u**) = **u**

Geometric Vectors

A **geometric vector** is a directed line segment represented by an arrow. Two vectors are equal if they have the same magnitude and direction. Therefore, a vector may be translated from one location to another as long as the magnitude and direction do not change.

The sum of two vectors can be defined using the **tail-to-tip rule**. The initial point of **V** is placed at the terminal point of **U** and **U** + **V** is drawn from the tail end of **U** to the tip end of **V**. *Study figure 1 below*.

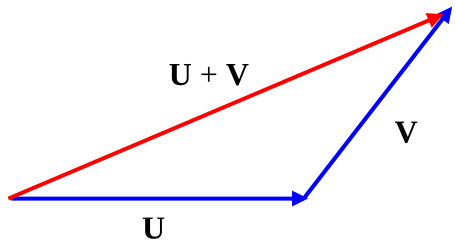


Fig. 1

The sum of two nonparallel vectors can be defined using the **parallelogram rule**. The sum of the two nonparallel vectors **U** and **V** is the diagonal of the parallelogram formed by using **U** and **V** as adjacent sides. *Study figure 2 below.*

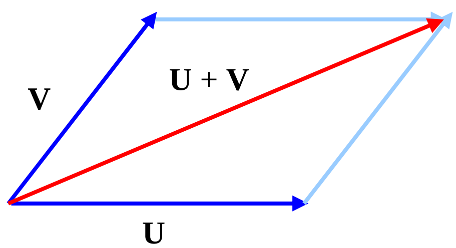


Fig. 2

*The vector **U** + **V** is called the resultant of the two vectors **U** and **V**.

Both of the above rules give the same result. The choice of which to use depends on the situation. To prove that the tail-to-tip and parallelogram rule give the same sum, let's compare figure 1 and figure 2 from above.

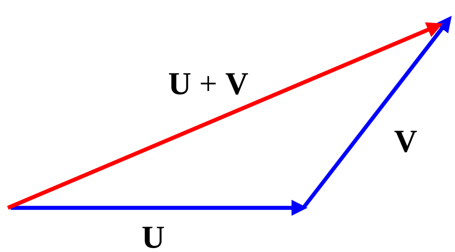


Fig. 1

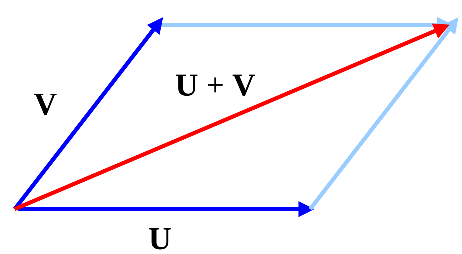
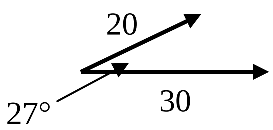


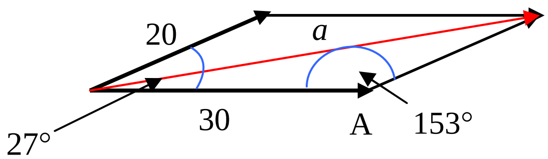
Fig. 2

Example #1: Two forces of 20 pounds and 30 pounds act on a point in the plane. If the angle between the directions of the two forces is $27°$, find the magnitude of the resultant force.

a.) The vectors representing the forces are shown below.



b.) Draw a parallelogram using the forces given. The magnitude of the resultant force is the length of "*a*", the diagonal in the parallelogram.



* The measure of angle A is $153°$ because adjacent angles in a parallelogram total $180°$ and since we know one angle is $27°$, we subtract that from 180 to produce $153°$.

c.) To find the length of "*a*", we can use the Law of Cosines because we know the length of two sides and the measure of the included angle.

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$a^2 = 20^2 + 30^2 - 2(20)(30)\cos 153°$$

$$a^2 = 400 + 900 - 1200\cos 153°$$

$$a^2 = 1300 - (-1069)$$

$$a^2 = 2369$$

$$a = 49$$

Thus, the magnitude of the resultant force is 49 pounds.