LAW OF SINES AND LAW OF COSINES

In this unit you will learn how to solve triangles that are **not** right triangles by using the Law of Sines and the Law of Cosines.

Law of Sines

Law of Cosines

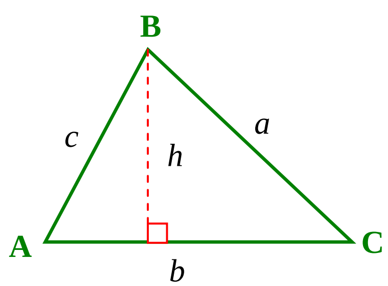
Law of Sines

Area

The area of a triangle can be found using the Law of Sines if you know side-angle-side information. The area can also be found if the measure of any two angles is known along with the length of any side.

Given the triangle ABC with height h and sides with lengths a b, and c, the area of the triangle is given by: $A = \frac{1}{2}bh$. A different area formula can be found using $\sin A = \frac{h}{c}$ or $h = c \sin A$. By combining these equations, a new formula for area is derived:

$$\text{Area} = \frac{1}{2}bh = \frac{1}{2}bc\sin A$$



Two other formulas for the area of the triangle are:

$$\text{Area} = \frac{1}{2}ac\sin B \text{ and } \frac{1}{2}ab\sin C$$

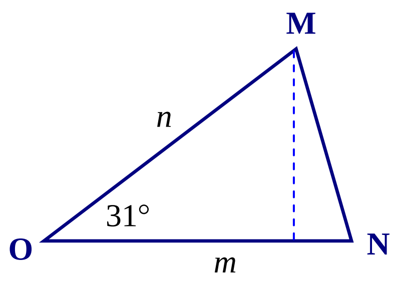
Since all formulas represent the area of the same triangle we can say:

$$\frac{1}{2}bc\sin A = \frac{1}{2}ac\sin B = \frac{1}{2}ab\sin C$$

And the Law of Sines is derived by dividing each by $\frac{1}{2}abc$ to obtain:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Example #1: Find the area of $\triangle MNO$ if m = 12, n = 13 and $m\angle 0 = 31°$.



a.) area = $\dfrac{1}{2} mn \sin O$

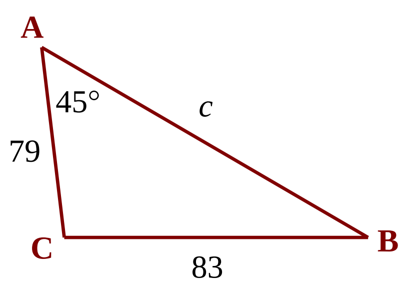
b.) area = $\dfrac{1}{2}(12(13)(\sin 31°)$

$\approx$ 40 square units

Solving a Triangle

To use the Law of Sines to solve a triangle, the measures of sides opposite given angles must be known. Follow the example below.

Example #2: Solve *ABC*



a.) you know the measure of angle $A(45°)$, side $a(83)$ and side $b(79)$ so you can find angle B

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\frac{\sin 45}{83} = \frac{\sin B}{79}$$ multiply both sides by 79

$$79\left(\frac{\sin 45}{83}\right) = 79\left(\frac{\sin B}{79}\right)$$

$$\frac{79(\sin 45)}{83} = \sin B$$

$$.6730 = \sin B$$ use $\boxed{2nd}$ $\boxed{\sin}$ to find B

$$42° \approx B$$

b.) we know that $\angle A + \angle B + \angle C = 180°$ (definition of a triangle), so we can find the measure of angle C

$$45° + 42° + \angle C = 180°$$

$$87° + \angle C = 180°$$

$$\angle C = 93°$$

c.) use the Law of Sines to find side c

$$\frac{\sin A}{a} = \frac{\sin C}{c}$$

$$\frac{\sin 45}{83} = \frac{\sin 93}{c}$$ (cross multiply)

$$c \sin 45 = 83 \sin 93$$ (divide by $\sin 45$)

$$\frac{c \sin 45}{\sin 45} = \frac{83 \sin 93}{\sin 45}$$

$$c \approx 117.2$$

Law of Cosines

Solving a Triangle

Sometimes it is not possible to use the Law of Sines to solve a triangle. Therefore we have the Law of Cosines that can be used in the following situations:

a.) to find the third side of a triangle if two sides and the included angle are known

b.) to find the measure of an angle of a triangle when all 3 sides are known

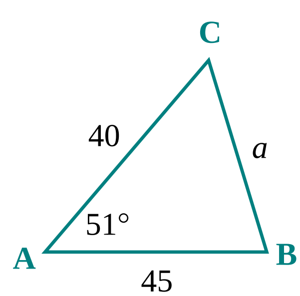
Law of Cosines

If $\triangle ABC$ is any triangle with *a*, *b*, and *c* representing the measures of sides opposite angles *A*, *B*, and *C*, then:

$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$b^2 = a^2 + c^2 - 2ac\cos B$$
$$c^2 = a^2 + b^2 - 2ab\cos C$$

Example #1: Solve $\triangle ABC$. Round the lengths and angle measures to the nearest tenth.

a.) we know the measure of two sides and the included angle, so we can use the Law of Cosines to find side "*a*"



$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$a^2 = 40^2 + 45^2 - 2(40)(45)(\cos 51)$$
$$a^2 = 1600 + 2025 - 3600\cos 51$$
$$a^2 = 1600 + 2025 - 2265.55$$
$$a^2 = 3625 - 2265.55$$
$$a^2 = 1359.45$$
$$a = 36.9$$

b.) we can now use the Law of Sines to find a second angle

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\frac{\sin 51}{36.9} = \frac{\sin B}{40}$$ multiply both sides by 40

$$40\left(\frac{\sin 51}{36.9}\right) = 40\left(\frac{\sin B}{40}\right)$$

$$.8424 = \sin B$$ use $\boxed{2nd}$ $\boxed{\sin}$ to find B

$$57.4° = \angle B$$

c.) determine the third angle

$$\angle A + \angle B + \angle C = 180$$

$$51 + 57.4 + C = 180$$

$$71.6° = \angle C$$