COMPLEX NUMBERS

In this unit you will learn about complex numbers and how to perform mathematical operations with complex numbers such as adding, subtracting, multiplying and dividing.

Definition of Complex Numbers

Complex numbers are in the form of $a+bi$ where a and b are real numbers and i is called the imaginary unit. The number i is, by definition, a number such that:

$$i^2 = -1$$

That is, $i = \sqrt{-1}$

> ### Definition of Equality
>
> For two complex numbers, $a+bi$ and $c+di$,
>
> $a+bi = c+di$ if and only if $a = c$ and $b = d$.

Example #1: Solve for x and y using the definition of equality.

$x-3i = 2-yi$

$x = 2 \quad -3i = -yi$

$3 = y$

Example #2: Solve for x and y using the definition of equality.

$5x+7i = 10-14yi$

$5x = 10 \qquad 7i = -14yi$

$x = 2 \qquad y = \frac{-1}{2}$

Addition and Subtraction of Complex Numbers

Definition of Addition and Subtraction

$$(a+bi)+(c+di)=(a+c)+(b+d)i$$

and

$$(a+bi)-(c+di)=(a-c)+(b-d)i$$

*Note: Make sure you add or subtract the real parts with real parts and the imaginary parts with imaginary parts.

Example #1: Add $(2-3i)+(7+i)$

$(2+7)+(-3+1)i$

$9+(-2)i$

$9-2i$

Example #2: Subtract $(9-5i)-(6-2i)$

$(9-6)+(-5-(-2))i$

$3+(-3)i$

$3-3i$

Multiplying Complex Numbers

When multiplying complex numbers, it is easier to just say FOIL the complex numbers as opposed to giving the formal definition.

*One thing to remember at this point is that $i^2 = -1$

Example #1: Multiply $(4+2i)(2-5i)$

$8+(-20i)+4i+(-10i^2)$	FOIL
$8-20i+4i+(-10\cdot -1)$	simplify
$8-16i+10$	combine like terms
$18-16i$	

Dividing Complex Numbers

The division of complex numbers will use what is called a **conjugate** if there are two terms in the denominator.

Definition of Conjugate

For any complex number $a+bi$,

the conjugate is the complex number $a-bi$.

*Notice that the only difference between a given complex number and it's conjugate is the sign.

Example #1: Determine the conjugate of the following:

a.) $4-7i$ $\quad$ $4+7i$

b.) $-2+6i$ $\quad$ $-2-6i$

c.) $-3-4i$ $\quad$ $-3+4i$

Example #2: Divide and express in standard form: $\dfrac{1}{2-3i}$

1.) multiply the numerator and denominator by the conjugate of $2-3i$

$$\frac{1}{2-3i}\cdot\left(\frac{2+3i}{2+3i}\right)$$

2.) distribute numerators and FOIL the denominators

$$\frac{2+3i}{4+6i-6i-9i^2}$$

3.) convert i^2 to -1 and simplify

$$\frac{2+3i}{4-9(-1)}=\frac{2+3i}{4+9}=\frac{2+3i}{13}$$

Complex Numbers and Radicals

Principal Square Root of Negative Real Numbers

$$\sqrt{-x} = \sqrt{-1} \cdot \sqrt{x} = i\sqrt{x}$$

$$\sqrt{-9} = \sqrt{-1} \cdot \sqrt{9} = i \cdot 3 = 3i$$

*When writing the square root of a negative number, write i before the radical if the radicand is not a perfect square:

$$\sqrt{-2} = i\sqrt{2}$$

This will prevent you from accidentally writing i under the radical sign.

When multiplying negative radicals first write each radical in standard form, and then multiply and simplify.

Example #1: $\sqrt{-9} \cdot \sqrt{-4}$

$= 3i \cdot 2i$

$= 6i^2$

$= 6(-1)$

$= -6$

Example #2: Write each in standard form.

a.) $\sqrt{-16}$ b.) $7 + \sqrt{-3}$ c.) $\dfrac{1}{2 - \sqrt{-4}}$

a.) $\sqrt{-16} = \sqrt{-1} \cdot \sqrt{16} = i \cdot 4 = 4i$

b.) $7 + \sqrt{-3} = 7 + i\sqrt{3}$

c.) $\frac{1}{2-\sqrt{-4}} = \frac{1}{2-2i}\left(\frac{2+2i}{2+2i}\right)$

$$= \frac{2+2i}{4+4i-4i-4i^2}$$

$$= \frac{2+2i}{4-4i^2}$$

$$= \frac{2+2i}{8}$$

$$= \frac{2(1+i)}{8}$$

$$= \frac{1+i}{4}$$