LINEAR SYSTEMS

In this unit you will solve linear equations based on real world applications. You will also solve systems of equations using substitution and elimination.

Linear Equations

An **algebraic equation** is a mathematical statement that relates two expressions involving at least one variable.

Sample equations:

$$3x + 4 = 13 \qquad \frac{7}{x+2} = \frac{4}{x-3} \qquad 2x^2 + 4x - 6 = 5 \qquad \sqrt{x-5} = x+8$$

The **replacement set**, or **domain** for a variable, is the set of numbers that are given to replace for the variable.

The **solution set** are the elements in the domain that makes the equation true.

To **solve** an equation means to find the solution set for the equation and is done so by using the following properties of equality to isolate the variable.

Properties of Equalities

When a, b, and c are real numbers:

1.) If $a = b$, then $a + c = b + c$	**Addition Property**
2.) If $a = b$, then $a - c = b - c$	**Subtraction Property**
3.) If $a = b$, then $ac = bc$	**Multiplication Property**
4.) If $a = b$, then $\frac{a}{c} = \frac{b}{c}$	**Division Property**
5.) If $a = b$, then either may replace the other in any statement without changing the statement.	**Substitution Property**

Example #1: Solve $7x - 10 = 4x + 5$

a.) add 10 to both sides

$7x = 4x + 15$

b.) subtract $4x$ from both sides

$3x = 15$

c.) divide both sides by 3

$x = 5$

If the equation you are solving contains fractions, you can eliminate the fraction by multiplying each term by the LCD. This is an easier way to solve the equation.

Example #2: $5 - \frac{3x-4}{5} = \frac{7-2x}{2}$

a.) the LCD in this case is 10, so multiply all terms by 10

$$10(5) - 10\left(\frac{3x-4}{5}\right) = 10\left(\frac{7-2x}{2}\right)$$

b.) simplify all fractions

$50 - 2(3x - 4) = 5(7 - 2x)$

c.) distribute the (-2) and the 5

$50 - 6x + 8 = 35 - 10x$

d.) combine like terms on the left

$58 - 6x = 35 - 10x$

e.) add $10x$ to both sides

$58 + 4x = 35$

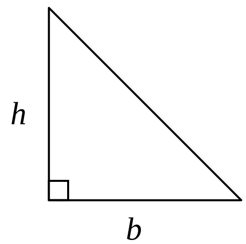
f.) subtract 58 from both sides

$4x = -23$

g.) divide both sides by 4

$$x = \frac{-23}{4} \text{ or } -23/4$$

Sometimes you will be asked to solve equations that contain more than one variable. For example, if *b* and *h* are the base and height of a triangle, the area of the triangle is given by:



$$A = \frac{1}{2}bh$$

Depending on the situation, you may be asked to solve this for *b* or for *h*. In either case, simply consider either *b* or *h* (depending on which you are solving for) as a constant and the other as the variable.

Example #3: Solve $A = \frac{1}{2}bh$ for *h*.

a.) multiply both sides by 2 to eliminate the fraction

$$2A = 2\left(\frac{1}{2}bh\right)$$

$$2A = bh$$

b.) isolate *h* by dividing both sides by *b*

$$\frac{2A}{b} = \frac{bh}{b}$$

c.) simplify

$$\frac{2A}{b} = h$$

Sometimes you may have to distribute or factor before solving an equation that contains all variables.

Example #4: Solve for Z in terms of the other variables

$A = Z + Zrt$

a.) think of A, r, and t as constants

b.) since there are 2 Z's (the variable you want to solve for), you will have to factor out the Z from the right hand side of the equation to produce only 1 Z.

$A = Z(1 + rt)$

c.) divide both sides by $(1 + rt)$ to solve for Z

$$\frac{A}{(1+rt)} = \frac{Z(1+rt)}{(1+rt)}$$

d.) simplify

$$\frac{A}{(1+rt)} = Z$$

Word Problems

*There are many practical problems that can be solved algebraically – and there is no one method that will work for all. However, there are strategies that you can use that will help you organize your thoughts.

Word Problem Strategies

1.) Read the problem carefully – more than once if necessary – and determine what the problem is asking you to find.

2.) Decide what information you know the least about and name it with a variable.

3.) If necessary draw figures to represent the information.

4.) Form an equation based on the information given.

5.) Solve the equation.

6.) Reread the problem to make sure your solution 1) makes sense, and 2) answers the question.

Number problems

Example #1: Find three consecutive even integers such that the sum of the first two exceeds the last by 6.

1.) the problem is asking you to find three even integers

2.) x = first integer

$x + 2$ = second integer

$x + 4$ = third integer

*Note: Understand that even integers are 2 units apart (2, 4, 6, 8, …) as are odd integers (1, 3, 5, 7…). That is how you determine x is the first, $x + 2$ is the second and so forth.

3.) no figure is necessary for this problem

4.) set up the equation

$$\underbrace{x + (x + 2)} = (x + 4) + 6$$

This sum of the first two is bigger than (x + 4) by 6. That is why you add 6 to (x + 4) to make it equal.

5.) solve the equation

$$2x + 2 = x + 10$$

$x = 8$, $x + 2 = 10$ and $x + 4 = 12$

Solution: The three even integers are 8, 10, and 12.

Check: $8 + 10 = 12 + 6$
$18 = 18$

Geometry

Example #2: Draw a diagram – If one side of a triangle is one-fourth the perimeter, the second side is 7 feet, and the third side is two-fifths the perimeter, what is the perimeter and the lengths of the sides.

1.) the problem is asking you to find the lengths of the sides of the triangle along with the perimeter (remember perimeter of a triangle is adding all the sides).

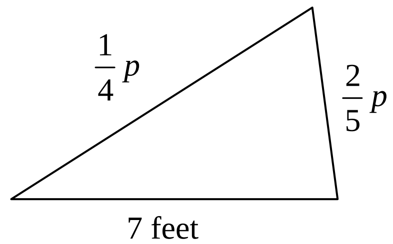
2.) p = perimeter

$\frac{1}{4}p$ = length of one side

$\frac{2}{5}p$ = length of another side

7 feet = length of last side

3.) draw a figure and label it



4.) set up an equation

$$p = \frac{1}{4}p + \frac{2}{5}p + 7$$

5.) solve the equation, remember to multiply everything by the LCD to eliminate the fractions.

$$20(p) = 20\left(\frac{1}{4}p\right) + 20\left(\frac{2}{5}p\right) + 20(7)$$

$20p = 5p + 8p + 140$
$20p = 13\text{p} + 140$

$7p = 140$

$p = 20$ the perimeter is 20 feet

$\frac{1}{4}(20)$ = length of one side

5 feet = length of one side

$\frac{2}{5}(20)$ = length of another side

8 feet = length of another side

Solution: The perimeter of the triangle is 20 feet and the lengths of the sides are 5 feet, 8 feet, and 7 feet.

Check: $(1/4)(20) + 7 + (2/5)(20) = 20$
$5 + 7 + 8 = 20$
$20 = 20$

Mixture Problems

A variety of applications can be classified as mixture problems and although they come from different areas, they are mathematically treated the same.

Example #3: How many liters of a mixture containing 80% alcohol should be added to 5 liters of a 20% solution to produce a 30% solution?

1.) the problem is asking you to find the amount of an 80% solution needed to mix with 5 liters of a 20% solution to produce a new solution.

2.) identify the unknown

x = amount of 80% solution

3.) no figure is necessary

4.) set up an equation

amount of 80% + 5 liters of 20% = total amount of 30%

-there are x liters of 80%

-there are 5 liters of 20%

-there are $(x + 5)$ liters of the 30% (the mixture)

$.80x + .20(5) = .30(x + 5)$

5.) solve the equation

$.80x + 1 = .30x + 1.50$

$.50x = .50$

$x = 1$

Solution: 1 liter of 80% solution will be added to 5 liters of a 20% solution to produce the 30% solution.

Check: $.80(1) + .20(5) = .30(1 + 5)$

$$.80 + 1 = 1.80$$

$$1.80 = 1.80$$

Systems of Equations

A system of equations is a set of equations in the same variable and can be solved a number of ways. For this unit we will discuss solving systems algebraically using substitution and elimination.

Recall that there are 3 possible solutions to a system of equations: 1) one solution (x, y), 2) no solution, and 3) many solutions.

Solving by Substitution

To solve a system using substitution

1.) solve one equation for a variable
2.) substitute the result into the other equation
3.) solve
4.) substitute the result into either equation to find the second variable.

Example #1: Solve $x + 3y = 7$ by substitution
$3x - 2y = -12$

Hint: When using substitution, it is easier if you solve for a variable having a coefficient of 1.

1.) Solve the first equation for x

$x + 3y = 7$
$x = 7 - 3y$

2.) substitute $7 - 3y$ into the second equation for x and solve for y

$3x - 2y = -12$

$3(7 - 3y) - 2y = -12$

$21 - 9y - 2y = -12$

$-11y = -33$

$y = 3$

3.) substitute 3 for y in either equation to find x

$x = 7 - 3y$

$x = 7 - 3(3)$

$x = 7 - 9$

$x = -2$

Solution: (–2, 3) Remember, the solution to a system will be an ordered pair.

Elimination

Another way to solve systems is by eliminating a variable by adding or subtracting the equations from each other. This can only be done when the coefficients of a variable are the same.

Example #2: Solve by elimination $3x - 2y = 12$

$$6x - 5y = -9$$

a.) In this system you will notice that there is no variable with the same coefficient, so, we will have to alter one of the equations to produce like coefficients. We can multiply the first equation by (2) and then subtract the two equations, or, we can multiply it by (–2) and add the two. Let’s multiply by (–2) and add.

$$-2(3x - 2y = 12) \qquad -6x + 4y = -24$$
$$6x - 5y = -9 \qquad 6x - 5y = -9$$

b.) add the two equations to produce

$-y = -33$

$y = 33$

c.) substitute y with 33 to solve for x in either equation

$3x - 2(33) = 12$

$3x - 66 = 12$

$3x = 78$

$x = 26$

Solution: (26, 33)

Applications of Systems

Solving real-world problems can be done using systems of equations. Follow the example below.

Example #1: Roco Casting has one bronze alloy in stock that is 90% copper and another bronze alloy that is 70% copper. How many pounds of each should be melted together to obtain 320 pounds of a 75% copper alloy?

a.) let x = number of pounds of 90%

let y = number of pounds of 70%

b.) write two equations based on the information

$x + y = 320$
$.90x + .70y = .75(320)$

c.) solve using substitution or elimination, we will use elimination

$.9(x + y = 320)$ $.9x + .9y = 288$
$.9x + .7y = 240$

subtract the two equations to produce:

$.2y = 48$

$y = 240$

d.) substitute 240 for y in the first equation to solve for x

$x + 240 = 320$

$x = 80$

Solution: 80 pounds of 90% alloy should be melted with 240 pounds of 70% alloy to produce 320 pounds of 75% alloy.

Check: $.90(80) + .70(240) = .75(320)$
$72 + 168 = 240$
$240 = 240$