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EXPONENT PROPERTIES PART I

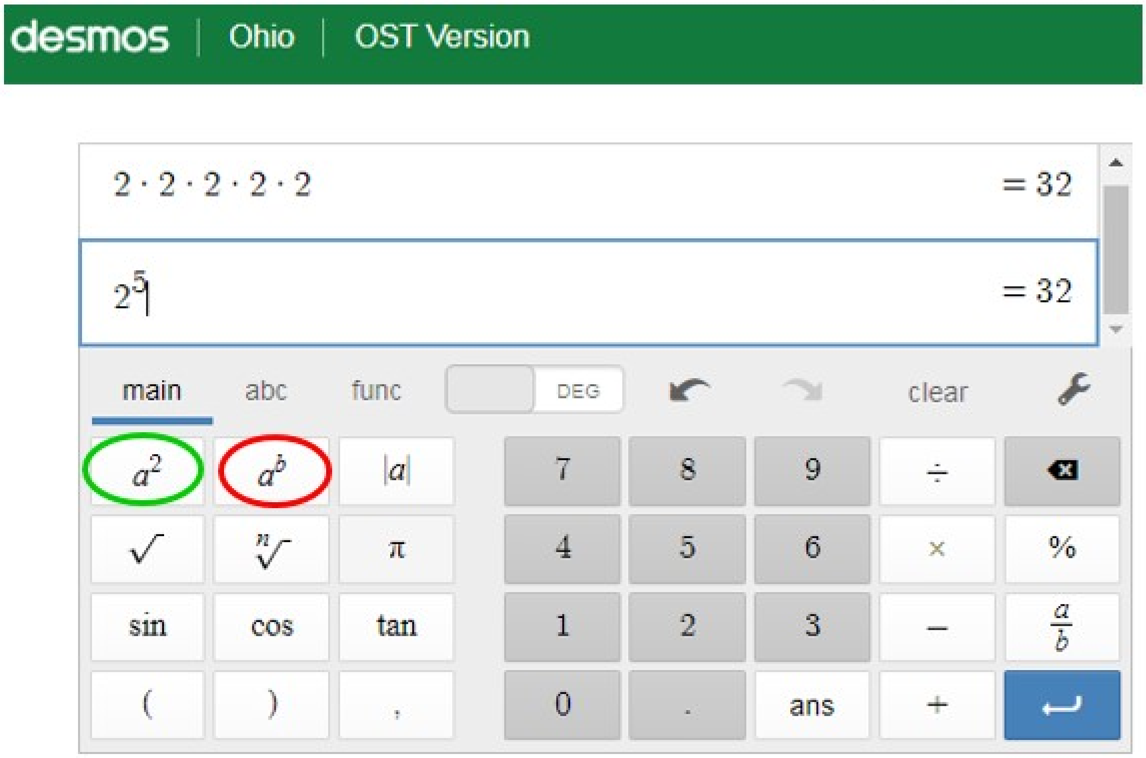
Unit Overview

In this unit, you will multiply and divide expressions involving exponents with a common base, take a power to a power, and take positive rational numbers to whole-number powers.

Product of Powers Property

To type an exponent on your Desmos Scientific Calculator:

- use the a^b button (circled below):
- to square a number, you can also use the a^2 button (circled below):



In the expression, 3^{100}, three is a product of itself 100 times. The number 3 represents the **base** and the 100 represents the **exponent**. The exponent tells how many times the base is used as a factor.

Example #1: $5^4 = 5\cdot5\cdot5\cdot5$

$$= 25\cdot5\cdot5$$

$$= 125\cdot5$$

$$= 625$$

Example #2: $3^3 = 3\cdot3\cdot3$

$$= 9\cdot3$$

$$= 27$$

Product of Powers

For all real numbers "a" and
all integers m and n

$$a^m \cdot a^n = a^{m+n}$$

The property above states that if you are multiplying like bases (in the case above "a"), then to simplify you will add the exponents.

Example #3: $2^3 \cdot 2^4 = 2^{3+4} = 2^7$

To prove this example, we will expand each term, count the bases and rewrite the term using one base and one exponent.

$$\overbrace{2\cdot2\cdot2}^{2^3}\ \overbrace{2\cdot2\cdot2\cdot2}^{2^4}$$

-count the number of bases and use this as your exponent

2^7 *This is the same answer we found when we used the product of powers rule.

The product of powers rule can be used to find the product of **monomials**. A monomial is an algebraic expression that is a constant (number), a variable, or a product of a constant and one or more variables. The constant (or numerical factor) is called the **coefficient**.

Examples of monomials: 3, m, $-5xy$

To simplify a product of monomials:

1) Multiply the coefficients.

2) Use the product of powers rule to simplify the variables.

Example #4: Simplify $(3x)(-20x^3)$.

$$(3x)(-20x^3)$$

$$(3\cdot -20)(x\cdot x^3)$$

$$-60x^4$$

Example #5: Simplify $(-2m^2n)(-mp^2)(4n^2p^2)$.

$$(-2m^2n)(-mp^2)(4n^2p^2)$$

$$(-2\cdot -1\cdot 4)(m^2\cdot m)(n\cdot n^2)(p^2\cdot p^2)$$

$$8m^3n^3p^4$$

Example #6: Simplify $(-20pq^3)\left(\frac{3}{5}pq^2\right)$.

$$(-20pq^3)\left(\frac{3}{5}pq^2\right)$$

$$(-20\cdot\frac{3}{5})(p\cdot p)(q^3\cdot q^2) \qquad \left\{\frac{-20}{1}\cdot\frac{3}{5}=-\frac{\cancel{20}^{\,4}}{1}\cdot\frac{3}{\cancel{5}_{\,1}}=-12\right\}$$

$$-12p^2q^5$$

Power Properties

Sometimes, in algebra, it is necessary to raise a power to a power as in the example $(x^3)^2$. In this case, you would use the exponent rule called "Power of a Power" Rule.

Power of a Power

For all real numbers "a" and all integers "m" and "n":

$$(a^m)^n = a^{m \times n}$$

As you can see by the rule stated above, if you have a power raised to another power, then you multiply the exponents.

Example #1: Simplify $(3^2)^4$.

$$(3^2)^4 = 3^{2 \times 4} = 3^8$$

Example #2: Simplify $(m^3)^5$.

$$(m^3)^5 = m^{3 \times 5} = m^{15}$$

Example #3: Simplify $(y^n)^3$.

$$(y^n)^3 = y^{n \times 3} = y^{3n}$$

Another property of exponents is used for any number of factors inside parentheses. This rule or property is called Power-of-a-Product.

Power-of-a-Product

For all numbers "a" and "b", and all integers "n"

$$(ab)^n = a^n b^n$$

Notice that in this property the exponent is taken to each factor. Let's try a few examples using this property.

Example #4: Simplify $(2x)^3$.

$$(2x)^3 = 2^3 x^3 = 8x^3$$

Example #5: Simplify $(5yz)^2$.

$$(5yz)^2 = 5^2 y^2 z^2 = 25y^2 z^2$$

The last two properties, Power-of-a-Power, and Powers-of-a-Product, can be used together as shown in the examples below.

Example #6: Simplify $(x^3 y^2)^4$.

$$(x^3 y^2)^4 = (x^3)^4 \cdot (y^2)^4 = x^{12} y^8$$

Example #7: Simplify $(3n^2 m^4)^3$.

$$(3n^2 m^4)^3 = 3^3 \cdot (n^2)^3 \cdot (m^4)^3 = 27n^6 m^{12}$$

Example #8: Simplify $\left(\frac{2}{3} x^5 y\right)^4$

$$\left(\frac{2}{3} x^5 y\right)^4 = \left(\frac{2}{3}\right)^4 \cdot (x^5)^4 \cdot y^4 = \frac{2^4}{3^4} x^{20} y^4 = \frac{16}{81} x^{20} y^4$$

Quotient (Division) Properties

When simplifying quotients, you can do so by first expressing the powers in terms of their factors. Take a look at the example below and see if you can derive a rule on how to simplify monomial quotients.

Example #1: $\frac{x^6}{x^4} = \frac{\cancel{x}\cdot\cancel{x}\cdot\cancel{x}\cdot\cancel{x}\cdot x\cdot x}{\cancel{x}\cdot\cancel{x}\cdot\cancel{x}\cdot\cancel{x}} = x^2$

Did you figure out a rule for dividing monomials? If you said that you can subtract the exponents, you are correct. Study the property below.

Quotient-of-Powers Property

For all nonzero real numbers *a* and all integers *m* and *n*,

$$\frac{a^m}{a^n} = a^{m-n}$$

Example #2: Use the property stated above to simplify the following monomials.

a.) $\frac{2^7}{2^4} = 2^{7-4} = 2^3 = 8$

b.) $\frac{a^x}{a} = a^{x-1}$

c.) $\frac{a^{x+y}}{a^z} = a^{x+y-z}$

d.) $\frac{a^{x+1}}{a} = a^{x+1-1} = a^x$

e.) $\frac{p^8q^5r^2}{p^3q^2r} = p^{8-3}q^{5-2}r^{2-1} = p^5q^3r^1 = p^5q^3r$

Note: When there are coefficients, you simply reduce the fraction!

f.) $\frac{-6t^7}{8t} = \frac{-3t^7}{4t} = \frac{-3t^{7-1}}{4} = \frac{-3t^6}{4} = -\frac{3}{4}t^6$

Exponent rules can also be used when scientific notation is involved:

g) $\frac{5\times10^3}{2\times10^2} = 2.5\times10^1$

Power-of-a-Fraction Property

For all real numbers x and y, where $y \neq 0$, and all integers n,

$$\left(\frac{x}{y}\right)^n = \frac{x^n}{y^n}$$

Notice in the property above that the exponent is given to the term in the numerator and the term in the denominator. Let's take a look at some examples involving this property.

Example #3: Simplify $\left(\frac{3}{2}\right)^3$.

$$\left(\frac{3}{2}\right)^3 = \frac{3^3}{2^3} = \frac{27}{8}$$

Example #4: Simplify $\left(\frac{x^3}{y^2}\right)^4$.

$$\left(\frac{x^3}{y^2}\right)^4 = \frac{(x^3)^4}{(y^2)^4} = \frac{x^{12}}{y^8}$$ (Multiply the exponents.)

Example #5: Simplify $\left(\frac{2w^5}{5y^4}\right)^3$

Notice the coefficients are actually raised to the power and not multiplied by the exponent as are the exponents.

$$\left(\frac{2w^5}{5y^4}\right)^3 = \frac{(2w^5)^3}{(5y^4)^3} = \frac{(2)^3(w^5)^3}{(5)^3(y^4)^3} = \frac{8w^{15}}{125y^{12}}$$

Example #6: Simplify $\left(\frac{t^3u^5}{\ldots}\right)^{3x}$

$$\left(\frac{t^3u^5}{v^{2x}}\right)^{3x} = \frac{(t^3u^5)^{3x}}{(v^{2x})^{3x}} = \frac{(t^3)^{3x}(u^5)^{3x}}{(v^{2x})^{3x}} = \frac{t^{9x}u^{15x}}{v^{6x^2}}$$

Example #7: Simplify Scientific Notation

$$\frac{(3\times10^4)^3}{(2\times10^2)^2} = \frac{27\times10^{12}}{4\times10^4} = 6.75\times10^8$$

Note: Make sure your final answer is always in scientific notation with the number between 1 and 10 times ten to a power.