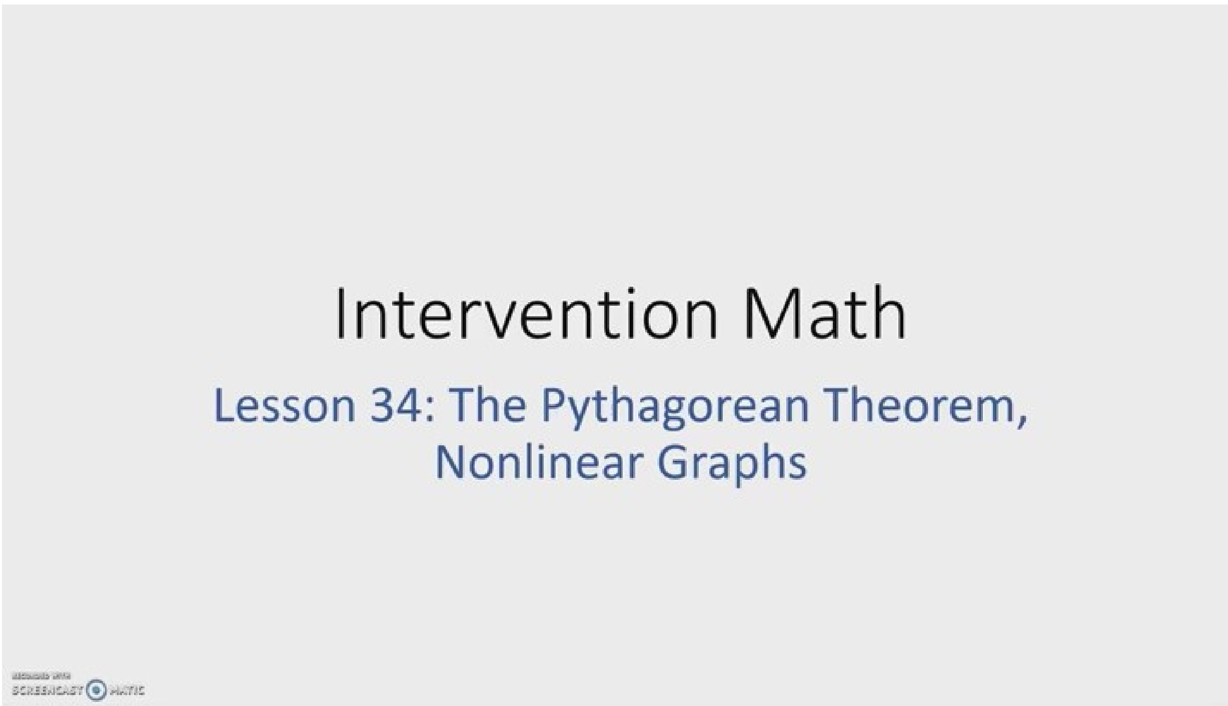
PDF

THE PYTHAGOREAN THEOREM
NONLINEAR GRAPHS

This unit begins with the Pythagorean Theorem and how it applies when calculating the lengths of the legs and the hypotenuse of a right triangle. The Pythagorean Theorem is a very useful formula to know when determining various right-angle measurements. The second part of this unit is about nonlinear functions and their graphs. The graphs of the equations will be nonlinear (not a straight line). Some examples are graphs of quadratic functions and graphs of inverse relationships.



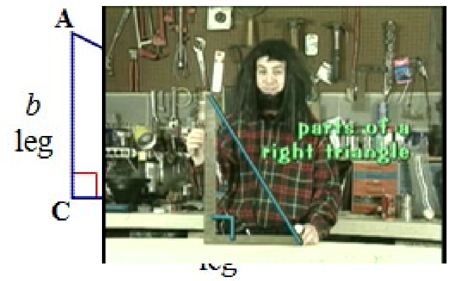
The Pythagorean Theorem

The Pythagorean Theorem is used to find the lengths of the sides of right triangles.

The Pythagorean Theorem

$$a^2 + b^2 = c^2$$

a and b are the lengths of the legs and c is the length of the hypotenuse (the side opposite the right angle).



Practice Worksheet: Pythagorean Theorem

Answer Key (Password Protected)

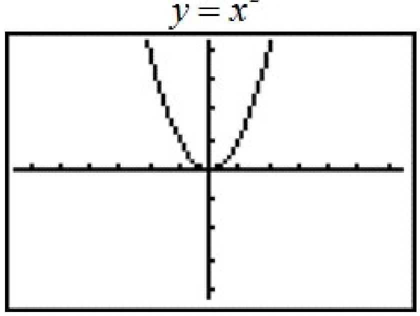
use the Pythagorean Theorem in finding the missing measures, follow the examples below.

Graphing Quadratic Functions

> **Quadratic Function**
>
> A quadratic function is a function of the form $y = ax^2 + bx + c$ where a, b, and c are real numbers and $a \neq 0$.

The graph of a quadratic function is a curve known as a **parabola** and is shown below.

$y = x^2$



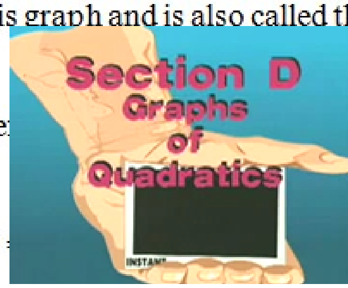
- The lowest point on this parabola is the **minimum value** of the function.
- The point (0, 0) on this graph and is called the **vertex** of the parabola.
- In a parabola there is a vertical line called the **axis of symmetry** drawn through the vertex, that reflects the parabola across the line of $x = h$ or in other words splits the parabola into two equal parts. In the case below the axis of symmetry would be the y-axis or $x = 0$.

If the coefficient of x-squared is a negative number, the parabola opens downward, and therefore has a maximum value at the vertex.

- The equation of this gr ... $(-1)x^2$. The coefficient of x-square ...

- The highest point on t ... **m value** of the function.

- The point (0, 0) on this graph and is also called the **vertex** of the parabola.



A table of values is very use ... ph a quadratic equation.

Example 1: Graph y ...

a) Make a table of values using positive and negative x-values.

Practice Worksheet: Graphing Quadratic Equations

Answer Key (Password Protected)

x	$x^2 + 2$	y

Inverse Variation

An inverse variation is a function that is defined by an equation in the following form: $xy = k$ where k is a nonzero real-number constant.

*There are many situations in which one quantity varies indirectly as another:

- as the rate increases, the time decreases when traveling a set distance.

- the volume of a gas in a container decreases as the pressure increases and the temperature remains constant.

Consider the following expressions:

$xy = k$

$\frac{\not{x}y}{\not{x}} = \frac{k}{x}$ Divide both sides by x.

$y = \frac{k}{x}$

$y = k \cdot \frac{1}{x}$ Another [redacted]

Thus, y is directly proportio[redacted]erse of x, $\frac{1}{x}$.

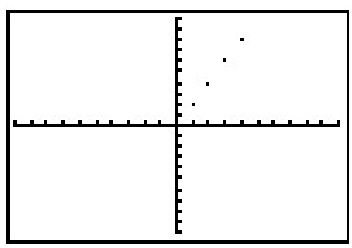
y* varies inversely to *x

Linear and Nonlinear Equations

Now that we have graphed equations, let's see if we can reverse the process to predict what the equation might look like if we only "see the graph or the table of values". In this section we will examine both a linear graph and a quadratic graph.

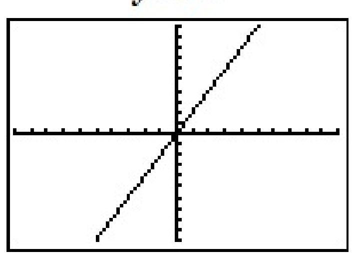
Example 1: Study the table of values and look for a relationship between the x-values and the y-values.

x	y
1	2
2	4
3	6
4	8



The y-value is equal to the x-value multiplied by two. We can write an equation for the relationship, $y = 2x$. The graphed points from the table suggest a linear equation. Since the graph is a straight line, we can assume the equation is **linear** or x is to the first degree.

$\boldsymbol{y = 2x}$



The equation for the table of values is $y = 2x$ and the graph is **linear**.

Example 2: Study the table and look for a relationship between the x-values and the y-values.

Practice Worksheet: Graphing Linear Equations
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Simple and Compound Interest

People save money for various reasons. One reason for opening a savings account is to earn *interest*.

Simple Interest

First, let's review the simple interest formula. To determine simple interest, multiply the amount of money in the account by the interest rate (a percent established by the bank). The result is the amount of interest that would be received in one year.

If the money is in a savings account for longer than a year, then multiply by the number of years.

When a period of time (determined by the bank) is less than a year, the time is written as a fractional part of a year. (For example, six months would be written as 6/12 or 1/2 of a year.)

Simple Interest Formula $I = p\,r\,t$

Interest (I) = principal (p) × rate (r) × time (t)
$ % (in years)
(as decimal)

Example 1: Mary is investing $500.00 at an interest rate of 6.5% annually. How much money will she have after 3 years?

$p = \$500 \quad r = 6.5\% = 0.065 \quad t = 3 \text{ years}$

$$I = prt$$
$$I = 500 \times .065 \times 3$$
$$I = 97.50$$

Mary will earn $97.50 interest in three years.

*Note: Mary will have a total of $500 + $97.50 = $597.50 total after saving for three years.

Compound Interest

Another type of interest is compound interest. With this type of interest, the value of a savings account grows much faster. Compounding can take place with different periods.

...with different periods.

Practice Worksheet: Simple Interest

Answer Key (Password Protected)

Some examples of compounding are:

Practice Worksheet: Compound Interest

Answer Key (Password Protected)

- once a day (daily, 365 times a year)
- once a month (monthly, 12 times a year)
- once a quarter (quarterly, 4 times a year)
- once a year (yearly, 1 time a year).

Graph Paper (PDF)

Compound Interest Formula

$$\text{Total Amount } (A) = P\left(1+\frac{r}{n}\right)^{nt}$$

P = principal (\$)

r = annual rate (% as a decimal)

n = the number of times interest is compounded per year

t = time in years

The formula looks complicated. Let's take a closer look and simplify the process.

Example 2: John is investing \$500.00 at an interest rate of 6.5% that is compounded monthly. How much money will he have after 3 years?

*Note: This problem is very similar to the previous problem, but now we will apply compound interest.

We have one new part: compounded monthly. This is the number of periods per year. This means the simple interest could be calculated every month (12 times per year) and added on to the amount in savings each month. Instead of actually calculating the interest 12 times, we apply the compound interest formula to predict how much money will be saved in three years under these terms.

$p = \$500 \quad r = 6.5\% = 0.065 \quad t = 3 \text{ years} \quad n = 12 \text{ (compounded monthly)}$

First, let's substitute the given information one step at a time.

$$A = P\left(1+\frac{r}{n}\right)^{nt}$$

$$\left(\quad n \right)$$

$$A = 500\left(1+\frac{r}{n}\right)^{nt} \qquad p = 500$$

$$A = 500\left(1+\frac{0.065}{n}\right)^{nt} \qquad r = 6.5\% = 0.065$$

$$A = 500\left(1+\frac{0.065}{n}\right)^{n(3)} \qquad t = 3$$

$$A = 500\left(1+\frac{0.065}{12}\right)^{(12)(3)} \qquad n = 12$$

*Note: The 12 substitutes in two places.

Now, let's solve the problem one step at a time.

$$A = 500\left(1+\frac{0.065}{12}\right)^{(12)(3)}$$

$$A = 500\left(1+\frac{0.065}{12}\right)^{36} \qquad (12)(3) = 36$$

$$A = 500(1+0.005417)^{36} \qquad \frac{0.065}{12} \approx 0.005417$$

$$A = 500(1.005417)^{36} \qquad 1+0.005417 = 1.005417$$

$$A = 500(1.214686) \qquad (1.005417)^{36} = 1.214686$$

$$A = 607.34 \qquad \text{Multiply}$$

Total Amount = 607.34

John's investment will earn him $607.34.
*Note: Answers will vary a few cents due to rounding.