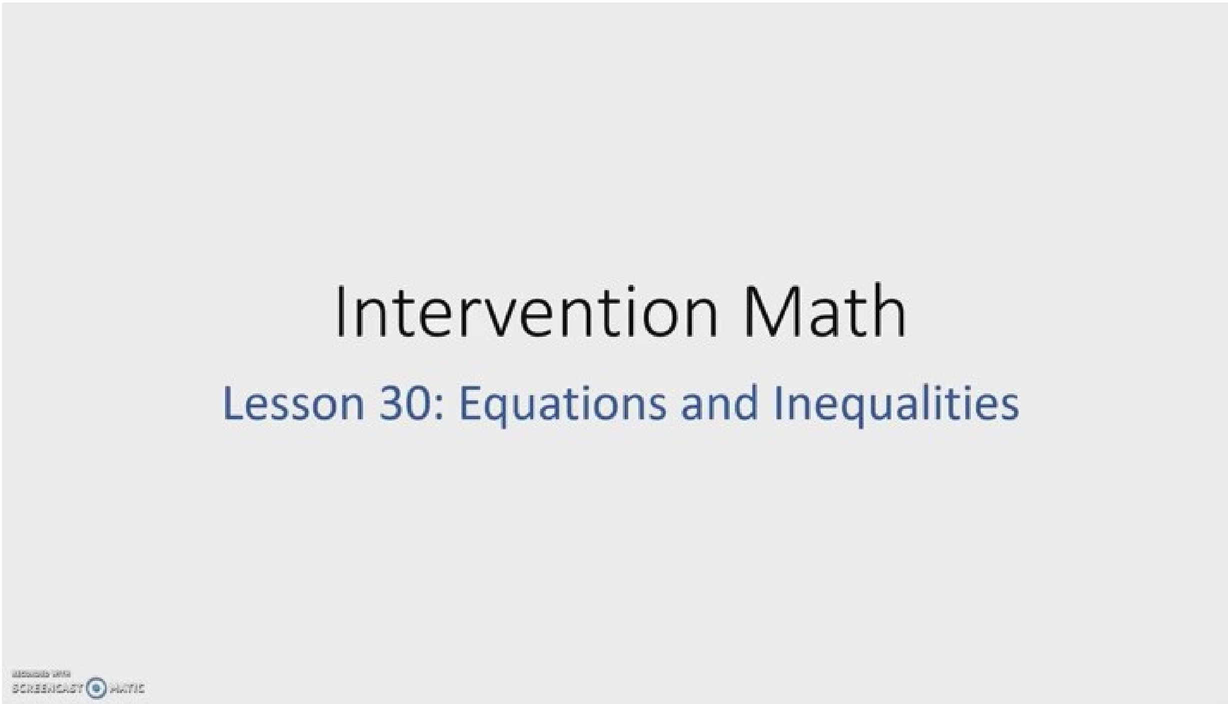
PDF

EQUATIONS AND INEQUALITIES

This unit begins with examining the meaning of variables and algebraic expressions followed by solving one-step equations and one-step inequalities. This topic will be extended to include solving various types of basic equations and inequalities that involve fractions and decimals. The understanding of inequality solutions will be reinforced by interpreting solutions through graphs on the number line and through listing integers that prove true in the solutions.



Introduction to Variables and Expressions

Did you know?

Most countries in the world use the Celsius scale to measure temperature. Two critical temperatures on the Celsius scale are 0° freezing and 100° boiling. On the other hand, in the United States we use the Fahrenheit scale most of the time. The same two critical temperatures on the Fahrenheit scale measure 32° freezing and 212° boiling. It is possible to convert between temperature scales by using algebra. If the Celsius temperature is multiplied by $\frac{9}{5}$ and then added to 32, the Fahrenheit temperature can be determined.

32, the Fahrenheit temperature can be determined.

Algebra ca... already kn... ve could write

$\frac{9}{5} \cdot$ **Cels**...

In arithmet...

$\frac{9}{5} \cdot \square +$ ...

where $\square$ re... der.

In algebra w... ace holders used are called **variables**. Variables are letters such as *x*, *n*, or *a* that are used to represent the unknown value. (You may use any letter as a variable; these were just a few examples.)

Practice Worksheet: Writing Algebraic Expressions
Answer Key (Password Protected)

Practice Worksheet: Evaluate Expressions
Answer Key (Password Protected)

When choosing a variable to represent an unknown value, make sure not to use the letter o because it could be mistaken for the number zero.

Solving One-Step Equations

A mathematical sentence such as $374 + x = 795$ is called an **equation** because it contains an equal sign.

The **solution** to an equation is the value of the variable that results in a true statement. The process of finding this solution is called **solving the equation** which uses opposite operations in order to isolate the variable. When we say opposite operations, we mean the opposite of adding is subtracting, the opposite of multiplying is dividing, and vice versa.

At this point it should be mentioned that an equation must be balanced. This means that whatever is done on one side of the equation must also be done on the other side of the equal sign.

Let's take a look at a few examples of solving equations.

Example 1: Solve $x + 4 = 17$

-Since we want *x* by itself, we have to perform the opposite of $+4$ on both sides of the equal sign, opposite of $+4$ is -4.

-Performing opposite operations can be done different ways; below you will see that this can be done vertically or horizontally; you

you will see that this can be done vertically or horizontally, you
make the cl

$x+4=$

$-4 \quad -$

$x+0=$

$x=$

-check the s ... the solution 13.

Check: 13

$17=17$

Practice Worksheet: Basic Equations
Answer Key (Password Protected)

Since this is true, **13** is the correct solution. ✓

Example 2: Solve n =7

Equations: Addition and Subtraction

Keep Equations in Balance

Equations are balanced on the equals sign, so to keep the sides in balance, whatever math operation is completed on one side of the equation must also be completed on the other side of the equation.

Example 1: Solve $x+3.6=8.5$ for x.

We need to **isolate** x on the left side of the equals sign; that is, we must undo the "+ 3.6" by subtracting 3.6. To keep the equation in balance, we must also subtract 3.6 from the right side.

$$\begin{array}{rr} x + \cancel{3.6} & = 8.5 \\ -\cancel{3.6} & -3.6 \\ \hline x & = 4.9 \end{array}$$

Subtract 3.6 from BOTH sides.

*Subtracting 3.6 on the left side of the equation cancels out the "+ 3.6" which leaves just x on the left side. This is called "isolating x".

For this problem $x=4.9$.

Check: Substitute 4.9 in for x in the original equation and simplify.

$x + 3.6 = 8.5$

[redacted]

Exa[redacted]

We [redacted]
mus[redacted]ce,
we m[redacted]

*Ad[redacted]
"-16" which leaves just m on the left side.

Equations: Multiplication and Division

Keep Equations in Balance

Equations are balanced on the equals sign, so to keep the sides in balance, whatever math operation is completed on one side of the equation must also be completed on the other side of the equation.

Example 1: Solve $9x = -54$ for x.

We need to **isolate** x on the left side of the equals sign; that is, we must undo the "9 times x" by dividing by 9. To keep the equation in balance, we must also divide by 9 on the right side.

$$9x = -54$$

$$\frac{\cancel{9}x}{\cancel{9}} = \frac{-54}{9} \qquad \text{Divide BOTH sides by 9.}$$

$$x = -6$$

*Dividing by 9 on the left side of the equation cancels out the "multiplied by 9" which leaves just x on the left side.

For this problem $x = -6$.

Check: Substitute –6 in for x in the original equation and simplify.

[redacted]

$y = 256$

Practice Worksheet: More Basic Equations

Answer Key (Password Protected)

*Multiplying by 8 on the left side of the equation cancels out the "divided by 8" which leaves just y on the left side.

Inequalities

An **inequality** is a math sentence that expresses quantities that are **greater than**, **less than**, **greater than or equal**, or **less than or equal** to a given number or mathematical expression.

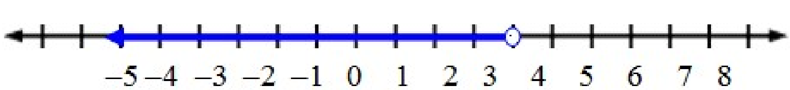
Inequality Symbols				
Symbol	$<$	$>$	$\leq$	$\geq$
Expression	less than	greater than	less than or equal	greater than or equal

Alexia may spend less than \$5 for lunch. How much money may Alexia spend for lunch?

She can spend any amount less than \$5 for lunch. To show the amounts she can spend, we'll look at a graph on the number line.

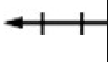
Let a stand for the amount she may spend and write an inequality: $a < 5$

Graph the inequality.



Use an **ope**[redacted]

She may sp[redacted]3 or
$2.50



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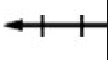

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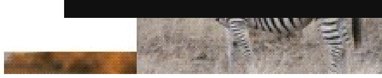

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Inequalities – Addition and Subtraction

Inequalities are math sentences that make a statement using inequality signs.

Example 1: Find all numbers that, when three is added, the result is less than eight.

Look for key phrases in the problem that can be translated to math notation.

Find all numbers that, when three is added, the result is less than eight

Practice Worksheet: Inequalities - Add and Subtract

Answer Key (Password Protected)

x $+3$ <8

Inequalities – Multiplication and Division

Inequalities are math sentences that make a statement using inequality signs.

Example 1: Find all numbers that, when multiplied by four, the result is less than 12.

Look for key phrases in the problem that can be translated to math notation.

Find all numbers that, when multiplied by four, the result is less than 12.

$$x \qquad \times 4 \qquad < 12$$

*The expression $x \times 4$ can be written as $4 \times x$. The expression $4 \times x$ can be simplified to $4x$. In mathematics, it is understood that $4x$ means $4 \times x$ or $x \times 4$.

The inequality is $4x < 12$.

Keep both sides of the inequality in balance.

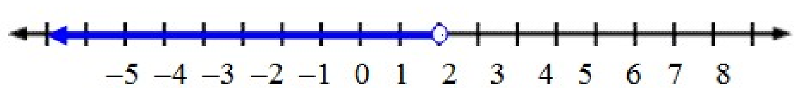
$$4x < 12$$

$$\frac{4x}{4} < \frac{12}{4}$$ Undo "$\times$ 4" by dividing by 4 on both sides.

$$x < 3$$

The solution is "all numbers less than 3".

A graph may be used to represent all solutions.

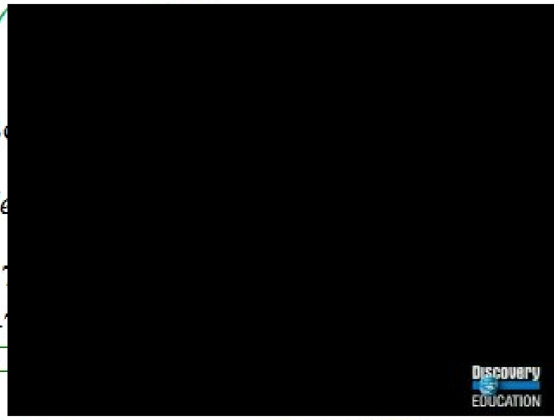


Check: Pick a number less than 3 to see if that number gives a **true** statement. We'll pick zero. Substitute zero in for x in the original inequality.

$4x < 12$ Substitute 0 for x

$4(0) < 12$ Simplify

$0 < 12$ ✓



Example 2: S

Keep both side

–7

–7 by -7

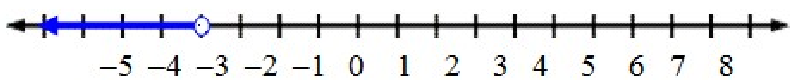
$x < -3$

*Reverse the inequality symbol when multiplying or dividing by a negative number.

Practice Worksheet: Inequalities – Multiply and Divide
Answer Key (Password Protected)

The solution is "all numbers less than –3".

A graph may be used to represent all solutions.



*To demonstrate that the inequality sign must be reversed, we'll test a point that is greater than –3 to show that it tests **false**. We'll test zero since it is greater than –3.

$$-7x > 21$$
$$-7(0) > 21$$
$$0 > 21$$ ✗ False

Now, we'll test a number that is less than –3 to show that it tests **true**, and that the solution is indeed "all numbers less than 3". We'll test –5.

$$-7x > 21$$
$$-7(-5) > 21$$
$$35 > 21$$ ✓ True

Rule: When **multiplying or dividing** an inequality by a **negative number**, **reverse** the direction of the **inequality sign**.

Example 3: Find all numbers that, when divided by eight, the result is less than 4.

Look for key phrases in the problem that can be translated to math notation.

Find all numbers that, when divided by eight, the result is less than 4.

$$\downarrow \qquad\qquad \downarrow \qquad\qquad \downarrow$$

$$x \qquad\qquad \div 8 \qquad\qquad < 4$$

*The expression $x \div 8$ can be written as $\frac{x}{8}$.

The inequality is $\frac{x}{8} < 4$.

Keep both sides of the inequality in balance.

$$\frac{x}{8} < 4$$

$$(\cancel{8})\frac{x}{\cancel{8}} < 4(8) \qquad \text{Undo "} \div 8 \text{" by multiplying by 8 on both sides.}$$

$$x < 32$$

The solution is "all numbers less than 32".

Check: Pick a number less than 32 to see if that number gives a **true** statement. We'll pick zero. Substitute zero in for x in the original inequality.

$$\frac{x}{8} < 4 \qquad \text{Substitute 0 for } x$$

$$\frac{0}{8} < 4 \qquad \text{Simplify}$$

$$0 < 4 \checkmark \qquad \text{True}$$

Example 4: Solve $15 < \frac{x}{12}$ for x.

Keep both sides of the inequality in balance.

$$15 < \frac{x}{12}$$

$$(12)15 < \frac{x}{\cancel{12}}(\cancel{12}) \qquad \text{Undo "} \div 12 \text{" by multiplying by 12 on both sides.}$$

$180 < x$	*Move x to the left side.
$x > 180$	** Switch the sides of the inequality and reverse the inequality sign.

*The inequality $180 < x$ means 180 is less than all values of x OR just simply x is any value greater than 180.

**When moving the unknown (x) from the right side to the left side, it is necessary to reverse the direction of the inequality sign to keep the meaning of the inequality the same.

The solution is “all numbers greater than 180”.

Check: Pick a number greater than 180 to see if that number gives a **true** statement. We’ll pick 240 since it is easily divisible by 12. Substitute 240 in for x in the original inequality.

$15 < \frac{x}{12}$	Substitute 240 for x
$15 < \frac{240}{12}$	Simplify
$15 < 20$ ✓	True