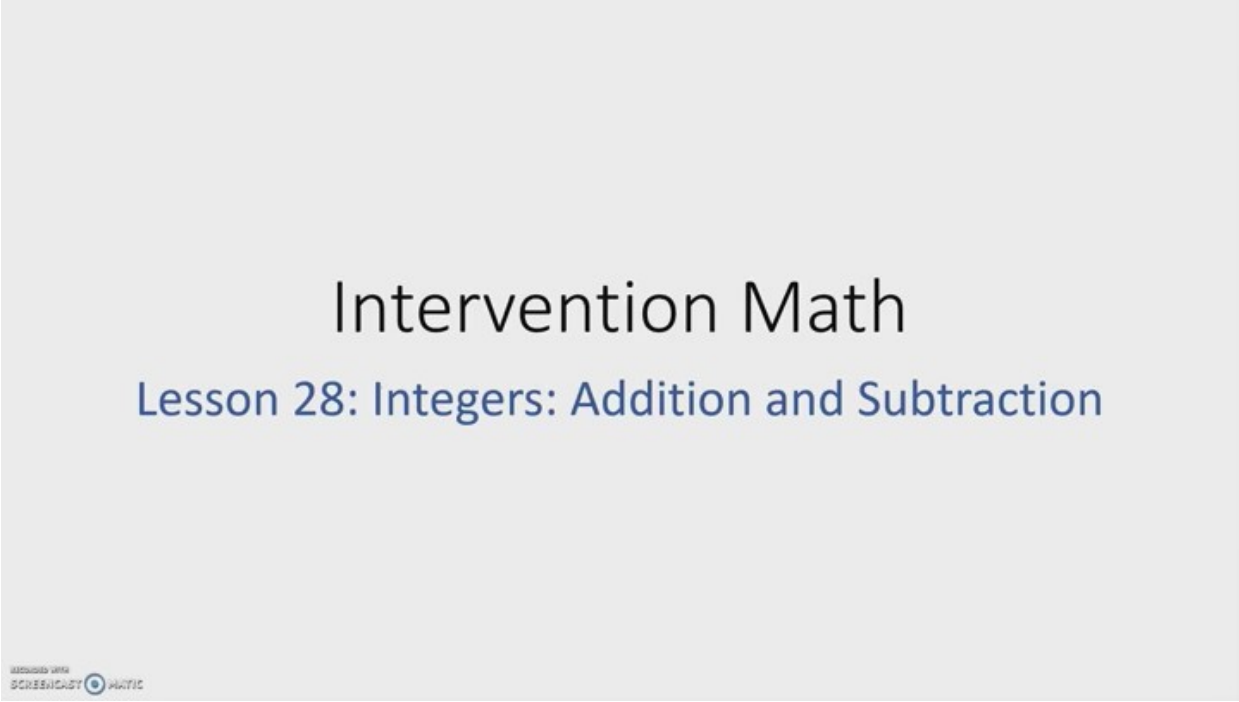
PDF

INTEGERS: ADDITION AND SUBTRACTION

This unit is about number theory, integers, and the use of signed numbers. The integers are a subgroup of the Real Numbers. Integers have signs, but do not have any decimals or fractions. Integers will be discussed as a subgroup of the set of real numbers. The order of operations will be reviewed as a reminder of the rules that must be followed in mathematics when evaluating expressions. A close look at comparing integers and finding the absolute value of an integer will aid in taking a closer look at the introduction of adding and subtracting integers. In the final part of this unit, the rules that apply to adding and subtracting integers will be applied to adding and subtracting fractions and decimals with signs.



Real Numbers

Real numbers are the numbers that can be represented on the number line. They include the whole numbers, their opposites, and all the other numbers in between them.

The real numbers are a union of the rational and irrational numbers.

Rational Numbers - Rational numbers are numbers that can be written as a quotient of two integers.

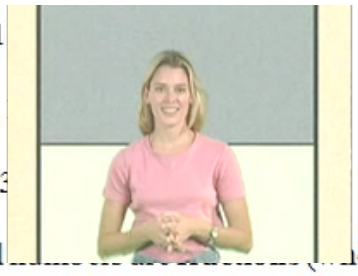
The natural numbers, whole numbers, and integers are subsets of the rational numbers. Note: A member of any of these sets can be expressed as a

quotient of two integers.

Natural Numbers: {1

Whole Numbers: {0,

Integers: {...–5, –4, –3



Also included in the rational ere the numerator and denominators are integers) and **repeating and terminating decimals**. Fractions may not have zero

[redacted]

Irrational Numbers – Irrati hat **cannot** be expressed as the ratio of two tional numbers are (a) square roots of non-perfe c) decimals that do not develop into a repeating

(a) $\sqrt{94}$

(b) 3.14159265358979323846264338327...

Order of Operations

In order to find the numerical value (**evaluate**) of any combination of numbers and operations (**expression**) correctly, mathematicians have established the order of operations which tells us which operations to do first in any mathematical problem.

P (parentheses)

E (exponents – powers)

M (multiply) } **work left to right**

D (divide) }

A (add) } **work left to right**

S (subtract) }

This "saying" may be used to help remember the order of operations.

Please
Excuse
My **D**ear
Aunt **S**ally

*Note: Multiplication and division are at the same level, meaning multiplication does NOT take priority over division. Work these two operations as they occur, left to right. The same is true about addition and subtraction. Work the two operations as they occur, left to right.

Thus, if "multiplication and division" or "addition and subtraction" are the only two operations in the expression, work the problem from left to right!

Example 1: Evaluate $6\times4+2$.

$6\times4+2$ Multiply 6×4

$24+2$ Add $24+2$

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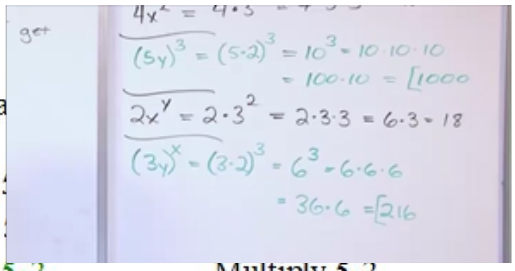


Example 2: Eva

$4(6+3)-$

$4(9)-$

$36-5\cdot2$ Multiply $5\cdot2$

$36-10$ Subtract

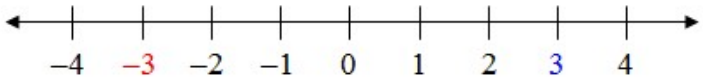
26

Practice Worksheet: Order of Operations

Answer Key (Password Protected)

Comparing Integers

The set of whole numbers consists of 0, 1, 2, 3 … and can be represented on a number line. We can match each whole number with another number that is the same distance from 0 but on the opposite side of 0.



If you take a look at the number line above, 3 or (positive 3) and –3 (negative 3) are on opposite sides of 0 but the same distance from 0. These numbers are called opposites and make up the set of **integers.** Integers are the set of positive and negative whole numbers.

Examples: Name the integer that is suggested by each situation.

a) The temperature is 5° below 0.

–5 Below 0 suggests a **negative** integer.

b) Emily's lemonade stand made a $24 profit.

24 A profit suggests a **positive** integer.

To compare integers, we will use the symbol "<" which means *less than* or the symbol ">" which means *greater than*. If you remember from the previous unit, these symbols are called **inequality** signs. An inequality can either be true or false. For example, the sentence $12 > 8$ is true and the sentence $6 > 9$ is false.

On a number line the numbers increase as you move from left to right. For any two numbers, the number that is farther to the right is the larger number and the number farther to the left is the lesser number. Let's take a look at comparing some integers using the number line below.

Write a true sentence u .

a) 3 □ 9 ⇒

Since 3 is to the e, 3 is less than 9.

Practice Worksheet: Represent Integers
Answer Key (Password Protected)

b) –5 □ 11 ⇒ –5 < 11

Practice Worksheet: Compare Integers
Answer Key (Password Protected)

Since –5 is to the left of 11 on the number line, –5 is less than 11.

Absolute Value

Absolute value of an integer is the distance the integer is from zero. The distance is measure in positive units.

Absolute value is represented by two vertical bars around the number.

| | |

Example 1: The absolute value of –5 is 5 because –5 is five units from 0.

$$|-5| = 5$$

5 units →

{…–8, –7, –6, –5, –4, –3, –2, –1, 0, 1, 2, 3, 4, 5, 6, 7, 8…}

Example 2: The absolute value of 5 is 5 because 5 is 5 units from 0.

$$|5| = 5$$

← 5 units

{…–8, –7, –6, –5, –4, –3, –2, –1, 0, 1, 2, 3, 4, 5, 6, 7, 8…}

Introduction to Adding Integers

{… –8, –7, –6, –5, –4, –3, –2, –1, 0, 1, 2, 3, 4, 5, 6, 7, 8 …}

Integers are used to show positive and negative quantities. When adding integers

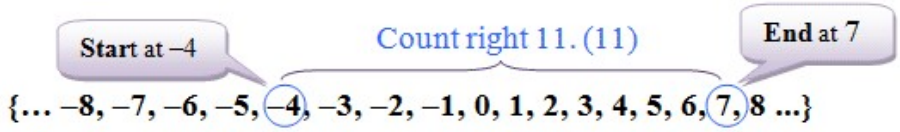
Introduction to Subtracting Integers

{… –8, –7, –6, –5, –4, –3, –2, –1, 0, 1, 2, 3, 4, 5, 6, 7, 8 …}

Integers are used to show positive and negative quantities. For subtracting integers, let's look at the ***difference*** between two integers.

Example 1: Solve: 7 – (–4)

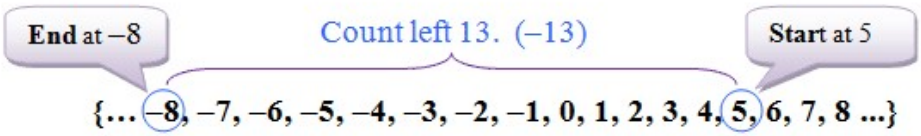
Locate the 7 and the –4. What is the *difference* between the two numbers? Counting right from –4 to 7, the difference is 11.



Solution: 7 – (–4) = 11

Example 2: Solve: –8 – 5

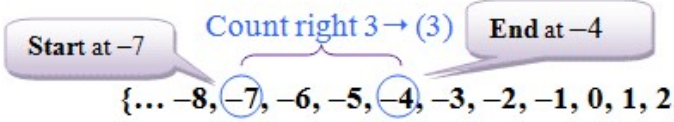
Locate the –8 and the 5. What is the *difference* between the two numbers? Counting left from 5 to –8, the difference is –13.



Solution: –8 – 5 = –13

Example 3: Solve: –4 – (–7)

Locate the –4 and the –7. What is the *difference* between the two numbers? Counting right from –7 to –4, the difference is 3.



Solution: –4 – (–7) = 3

Counting out the answer on a number line is a good way to understand the types of

Adding and Subtracting Integers

When adding and subtracting integers, it will be very helpful to understand the following rules:

Adding Integers

Same sign: If the numbers have the same sign, add the numbers and keep the sign given.

Example 1:	$(-5) + (-9) = -14$	Both integers are negative, so the answer is negative.
Example 2:	$6 + 15 = 21$	Both integers are positive, so the answer is positive.

Different signs: If the numbers have different signs, subtract the absolute value of the numbers and use the sign of the larger absolute value.

Example 3:	$(-9) + 3 = -6$	Subtract 9 and 3 to get 6, and then determine that –9 has the larger absolute value; so, you will use the negative sign.
Example 4:	$17 + (-4) = 13$	Subtract 17 and 4 to get 13, and then determine that 17 has the larger absolute value; so, you will use the positive sign, or in this case no sign at all represents positive.

Subtracting Integers

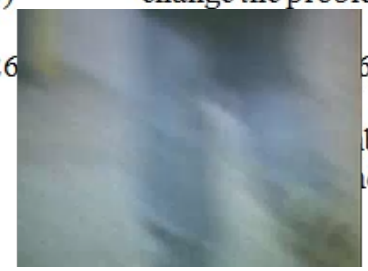
Change the problem to addition and add the opposite of the second number. Then go to addition rules.

Example 5:	$15 - (-8) =$	
	$15 + (-8) =$	change the problem to addition
	$15 + (+8) =$	change (–8) to a (+8)
	23	Add the numbers because they now have the same sign.
Example 6:	$18 - 26 =$	

$18 + (26) =$ change the problem to addition

$18 + (-26$ 6)

-8 bers because they now have d use the sign of the larger



Practice Worksheet: Add and Subtract Integers
Answer Key (Password Protected)

Practice Worksheet: Add and Subtract Integers and Absolute Value of Integers
Answer Key (Password Protected)

Practice Worksheet: Compare Integers and Absolute Value of Integers
Answer Key (Password Protected)

Fractions and Decimals with Signs

Fractions and decimals may have signs also. The rules for determining the signs of the answers to computations with integers also apply to computations with fractions and decimals. Let's take a look at a few examples and see how to apply the rules.

Example 1: Find the sum of $-2\ 3/4 + -5\ 7/8$.

> *Addition Rule*: When the signs are the same (both negative), ADD and use the same sign.

$$\begin{array}{r} -2\frac{3}{4}=2\frac{6}{8} \\ +\ -5\frac{7}{8}=5\frac{7}{8} \\ \hline -7\frac{13}{8}=-8\frac{5}{8} \end{array}$$

The sum of $-2\ 3/4 + -5\ 7/8$ is $-8\ 5/8$.

Practice Worksheet: Add and Subtract Decimal Numbers with Signs

Answer Key (Password Protected)

Example 2: Find the sum of 5 2/3 + –7 1/9.

> *Addition Rule*: When the signs are different, SUBTRACT and take the sign of the number farther from zero on the number line.

*Note: Since –7 1/9 is farther from zero than 5 2/3 on the number line, the sign of the answer is **negative**. Also, write the –7 1/9 above the 5 2/3 to subtract.

$$\begin{array}{r} \left(\frac{9}{9}+\frac{1}{9}\right) \\ -7\frac{1}{9}=\not{7}^{6}\frac{1}{9}=6\frac{10}{9} \\ +\quad 5\frac{2}{3}=\quad 5\frac{6}{9}=5\frac{6}{9} \\ \hline -1\frac{4}{9} \end{array}$$

The sum of 5 2/3 + –7 1/9 is –1 4/9.

Example 3: Find the difference of 2/3 – 7/8.

$$\begin{array}{r} \frac{2}{3}=\frac{16}{24} \\ -\frac{7}{8}=\frac{21}{24} \\ \hline \end{array}$$

> *Subtraction Rule*: Rewrite the problem to "add the opposite".

2/3 – 7/8 = 2/3 + –7/8 *The opposite of 7/8 is –7/8.

Since the signs are different, "subtract" and take the sign of the number farther from zero on the number line.

(2/3 + –7/**8**)

*To subtract, write –7/8 as the top number since 21/24 is larger than 16/24.

$$\begin{array}{rl} -\frac{7}{8}=\frac{21}{24} & \text{*Subtract } 21-16. \\ +\quad \frac{2}{3}=\frac{16}{24} & \text{*Take the sign of } -7/8. \\ \hline -\frac{5}{24} & \end{array}$$

The difference of 2/3 – 7/8 is –5/24.

Example 4: Find the difference of –2 3/10 – (–5 3/4).

Subtraction Rule: Rewrite the problem to "add the opposite".

–2 3/10 + (+5 3/4) *The opposite of –5 3/4 is + 5/3/4.

*To subtract, write 5 3/4 as the top number since 5 is larger than 2.

$$\begin{array}{r} 5\frac{3}{4} = 5\frac{15}{20} \\ +-\ 2\frac{3}{10} = 2\frac{6}{20} \\ \hline 3\frac{9}{20} \end{array}$$

*Subtract 15 – 6 (no borrowing is needed).

*Take the **sign of 5 3/4**.

The difference of –2 3/10 – (–5 3/4) is 3 9/20.