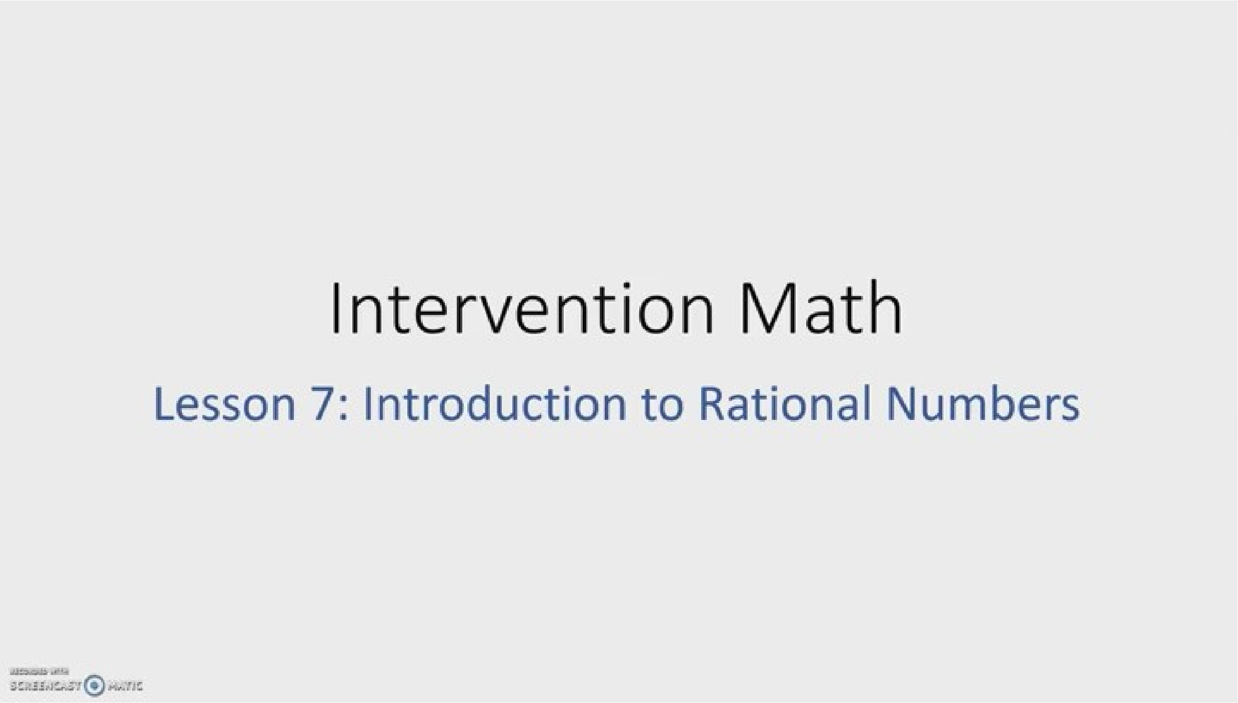
PDF

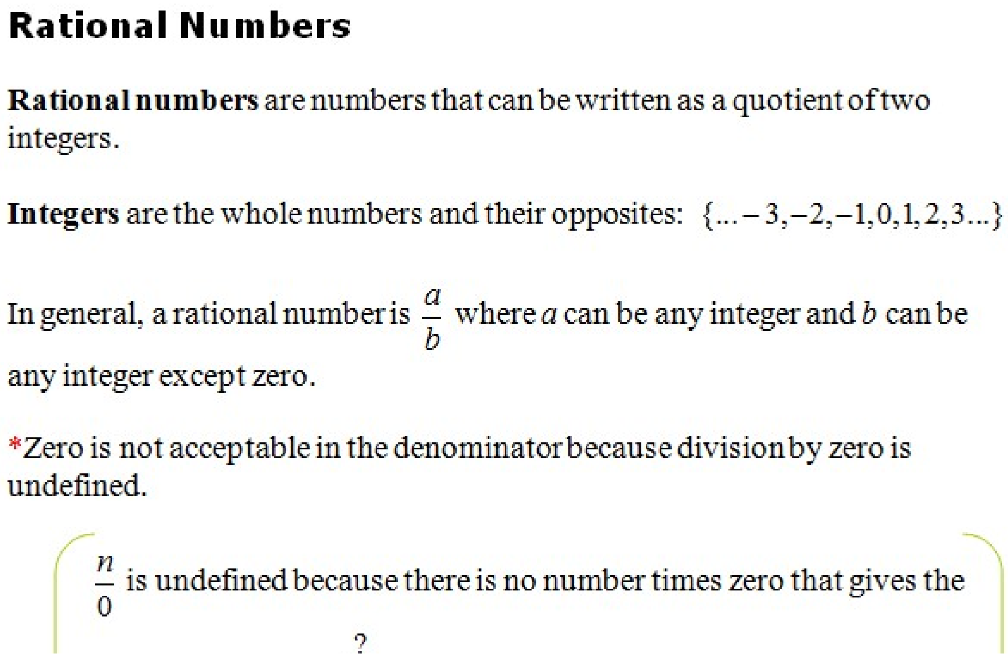
INTRODUCTION TO RATIONAL NUMBERS

This unit covers many facets of foundation topics that will be useful when working with rational numbers. Prime factors of a number are used to determine the Greatest Common Factor (GCF) and the Least Common Multiple (LCM). Finding equivalent fractions, simplifying fractions, and examining mixed numbers are basic concepts that are used in preparation for comparing and calculating with fractions.



Intervention Math

Lesson 7: Introduction to Rational Numbers



Rational Numbers

Rational numbers are numbers that can be written as a quotient of two integers.

Integers are the whole numbers and their opposites: $\{\ldots -3, -2, -1, 0, 1, 2, 3 \ldots\}$.

In general, a rational number is $\frac{a}{b}$ where a can be any integer and b can be any integer except zero.

*Zero is not acceptable in the denominator because division by zero is undefined.

$\frac{n}{0}$ is undefined because there is no number times zero that gives the

?

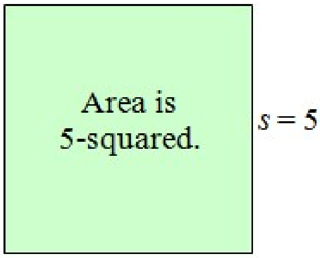
number. Think: $0\overline{)n}$ $(? \times 0 = n)$ No integer will make this statement

Squared and Cubed Numbers

Squared Numbers

Five to the second power, 5^2, equals 5×5 and can be read “5-squared”.

Think of the area of a square with a side that measures 5 units.



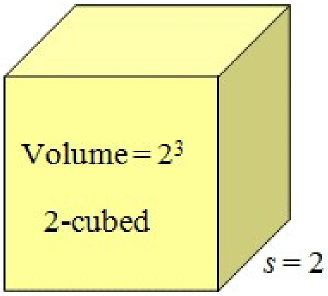
$$A = s^2$$
$$A = s \times s$$
$$A = 5 \times 5$$
$$A = 5^2 = \text{5-squared} = 25$$

Five to the second power, 5^2, equals 5×5 and can be read “5-squared”.

Cubed Numbers

Two to the third power, 2^3, equals $2 \times 2 \times 2$ and can be read “2-cubed”.

Think of the volume of a cube with a side that measures 2 units.



$$V = l \times w \times h$$
$$V = 2 \times 2 \times 2$$
$$V = 2^3 = \text{2-cubed} = 8$$

Two to the third power, 2^3, equals $2 \times 2 \times 2$ and can be read “2-cubed”.

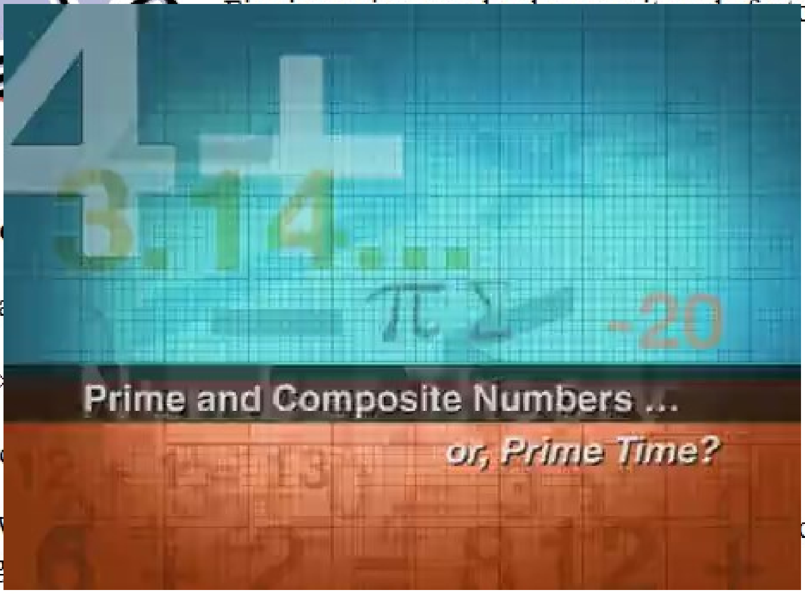
*Note: Other powers are simply read as follows:

Three to the sixth power, 3^6, equals $3 \times 3 \times 3 \times 3 \times 3 \times 3 = 729$.

Prime Factorization

A **prime** number has ***exactly*** two factors, which are one (1) and the number itself.

ors are



A comp

Eight is a

8

The fact

*Note: om least to g

*** One is *neither* prime nor composite. ***

One has does not fit the de r.

To find the prime umber as a product of prime n

Here are the prime

Example 1: Find the prime factorization of 40.

Practice Worksheet: Prime and Composite Numbers

Answer Key (Password-Protected)

Think of two factors of 40, and then continue on until all factors are prime factors.

Practice Worksheet: Prime Factorization

Answer Key (Password Protected)

$40 = 2 \times 20$

Greatest Common Factor

Finding GCF Using Prime Factorization

To find the **Greatest Common Factor (GCF)** of two numbers, apply prime factorization by finding *all* the common factors in each prime factorization, and then multiplying them.

Example 1: Find the GCF of 36 and 54.

Find the prime factorization of both numbers.

4

2 ×

Find the co ll the common prime facto orizations.

$54 = 2 \times 3 \times 3 \times 3$ $54 = 2 \times 3 \times 3 \times 3$

Practice Worksheet: List Factors

Answer Key (Password Protected)

There is a two (2), a three (3), and a second three (3) that is found in both prime factorizations.

Practice Worksheet: GCF

Answer Key (Password Protected)

The GCF is $2 \times 3 \times 3 = 18$.

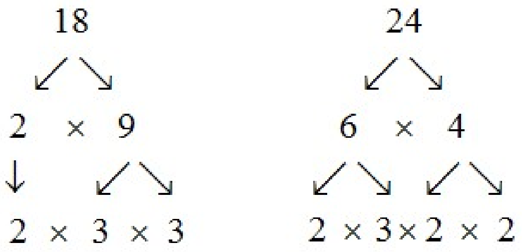
Least Common Multiple

Finding LCM Using Prime Factorization

To find the **Least Common Multiple (LCM)** of two numbers, apply prime factorization to find all the prime factors in each prime factorization, and then multiply the *highest occurrence of each **different*** factor.

Example 1: Find the LCM of 18 and 24 using prime factorization.

Step 1: Find the prime factorization of both numbers.



Step 2: Write each factorization using exponents.

$18 = 2 \times 3^2$ $24 = 2^3 \times 3$

Step 3: Find the LCM; that is, find the product of the highest occurrence of each prime factor.

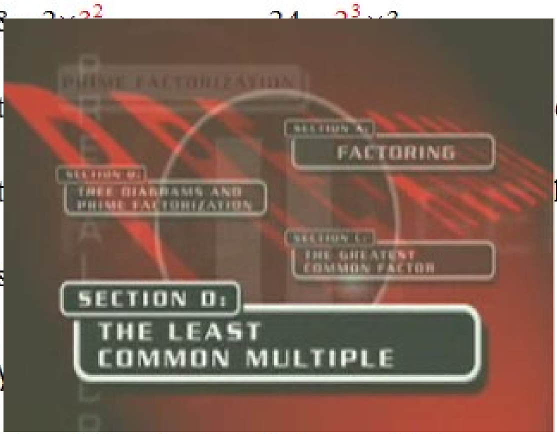
Look for the highest exponent of each factor.

$18 = 2\times3^2$ $24 = 2^3\times3$

The highest e 24....2^3.

The highest he 18....3^2.

The LCM is



Finding LCM By

The LCM may also be determined by listing multiples.

- Practice Worksheet: LCM
- Answer Key (Password Protected)

Find the least common multiple (LCM) by listing multiples:

1. List the multiples of both numbers.
2. Find the first multiple (the least) that is common to both sets of

Introduction to Fractions

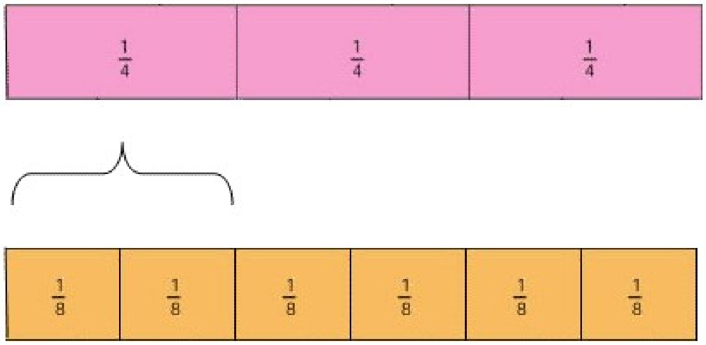
Equivalent Fractions

Fractions that represent the same amount are called **equivalent** fractions.

Look at the fraction strips below that represent the quantity $\frac{3}{4}$.

Three-fourths can also be expressed in eighths.

It takes two $\frac{1}{8}$'s to make one $\frac{1}{4}$. Therefore, three $\frac{1}{4}$'s = six $\frac{1}{8}$'s.



$$\frac{3}{4}=\frac{6}{8}$$

Equivalent fractions can be found by multiplying the numerator (top) and the denominator (bottom) of a fraction by the same number.

$$\frac{3\times2}{4\times2}=\frac{6}{8}$$

*Notice that this process is actually multiplying the original fraction by 2/2

ch 1, the value of 3/4 remains the same, it just has a different
pearance (6/8).

Practice Worksheet: Equivalent Fractions
Answer Key (Password Protected)

Practice Worksheet: Reduce Fractions
Answer Key (Password Protected)

$$\frac{3}{4}\times 1 \to \frac{3}{4}\times\frac{2}{2}=\frac{6}{8}$$

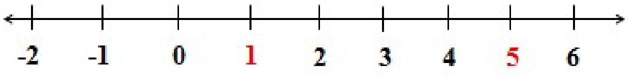
Comparing Fractions

To compare fractions, make sure the denominators are the same, and then compare the numerators.

Comparing Fractions with the Same Denominator

Example 1: Compare $\frac{5}{6}$ and $\frac{1}{6}$.

Compare the **numerators**: $5 > 1$



Since $5 > 1$, it follows that $\frac{5}{6} > \frac{1}{6}$.

Comparing Fractions with the Different Denominators

To compare fractions with *different denominators*, find equivalent fractions that have the same denominator.

Express each fraction with the **least common denominator (LCD)**. The LCD is the *least common multiple (LCM)* of the denominators.

To compare fractions with different denominators:

1. Find the least common denominator (LCD).
2. Find equivalent fractions by expressing each fraction with like denominators.
3. Compare the numerators.

Example 1: Compare $\frac{3}{4}$ and $\frac{4}{5}$.

Step 1: Find the least common denominator of 4 and 5.

$$4 = 2\times 2 = 2^2$$
$$5 = 5$$

Practice Worksheet: Compare Fractions

Answer Key (Password Protected)

$$\text{LCD} = 2^2 \times 5 = 4 \times 5 = 20$$

Practice Worksheet: Ordering Fractions

Answer Key (Password Protected)

Step 2: Set up equivalent fractions with the LCD.

Mixed Numbers

Mixed numbers are numbers that have a whole number and a fraction.

Examples of Mixed Numbers: $2\frac{2}{3}$ $7\frac{5}{8}$ $29\frac{3}{4}$

Improper fractions are fractions where the numerator is larger than the denominator.

Examples of Improper Fractions: $\frac{8}{3}$ $\frac{61}{8}$ $\frac{119}{4}$

When working with fractions, it is necessary to know how to convert mixed numbers to improper fractions and vice versa.

Changing Mixed Numbers to Improper Fractions

Example 1: Express $1\frac{5}{12}$ as an improper fraction.

$$1\frac{5}{12} = 1+\frac{5}{12} = \frac{12}{12}+\frac{5}{12} = \frac{12+5}{12} = \frac{17}{12}$$

*A quick way to find the improper fraction is to multiply the denominator by the whole number, and add on the numerator. Then, place that number over the denominator.

$$1\frac{5}{12} = \frac{12 \times 1 + 5}{12} = \frac{12+5}{12} = \frac{17}{12}$$

The improper fraction for 1 5/12 is 17/12.

Example 2: Express $2\frac{4}{9}$ as an improper fraction.

$$2\frac{4}{9} = \frac{9 \times 2 + 4}{9} = \frac{18+4}{9} = \frac{22}{9}$$

The improper fraction for 2 4/9 is 22/9.

Practice Worksheet: Change Improper Fractions to Mixed Numbers

Answer Key (Password Protected)

Changing Improper Fractions to Mixed Numbers

Practice Worksheet: Change Mixed Numbers to Improper Fractions

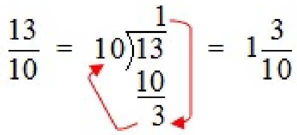
Answer Key (Password Protected)

Example 3: Express $\frac{13}{10}$ as a mixed fraction.

Think of $\frac{13}{10}$ as $\frac{10}{10} + \frac{3}{10}$, then as $1 + \frac{3}{10}$ because $\frac{10}{10} = 1$, then as $1\frac{3}{10}$.

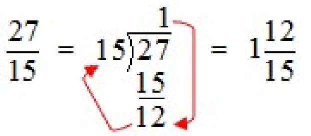
In this problem, the whole number is 1 $(\frac{10}{10})$ and the remaining part is $\frac{3}{10}$.

*A quick way to find the mixed number is to divide the numerator by the denominator and express the remainder as a fraction.



The mixed number for 13/10 is 1 3/10.

Example 4: Express $\frac{27}{15}$ as a mixed fraction.



The fraction must be simplified.

$$1\frac{12}{15} = 1\frac{12 \div 3}{15 \div 3} = 1\frac{4}{5}$$

The mixed number for 27/15 is 1 4/5.