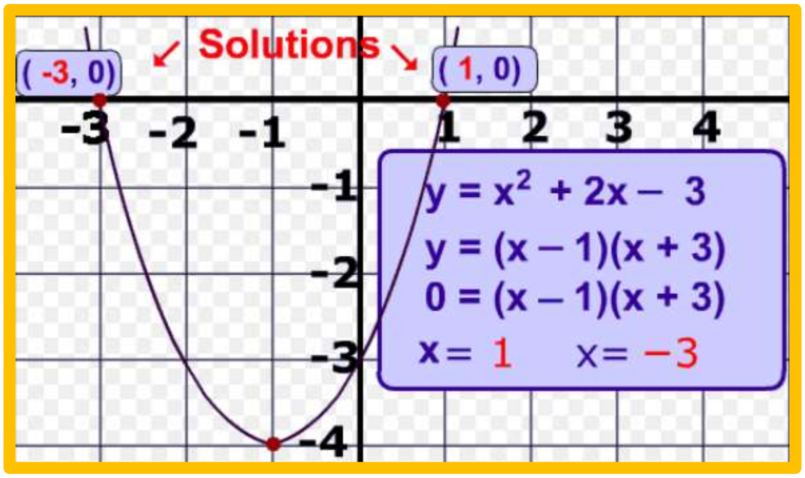
SOLVING QUADRATIC EQUATIONS BY GRAPHING



Unit Overview

In this unit, students will be able to:

- Solve quadratic equations through graphing
- Solve linear equations through graphing

Key Concepts

- *x*-intercept/root/zero/solution
- Number of solutions

In this unit, we will be using your TI-30XIIS calculator, as well as the Desmos online graphing calculator. Click on the words DESMOS to open up the graphing calculator.

Connection to Previous Units

- We will be incorporating the strategies to graph quadratic functions that we learned in Units 23 and 24.
- We will also be connecting to Units 3 and 4 where we solved equations.

Solving Quadratic Equations by Graphing

Quadratic equations, like quadratic functions, contain x^2 within the equation (sometimes after multiplying polynomials together).

A **solution to an equation** is any value that makes the equation true.

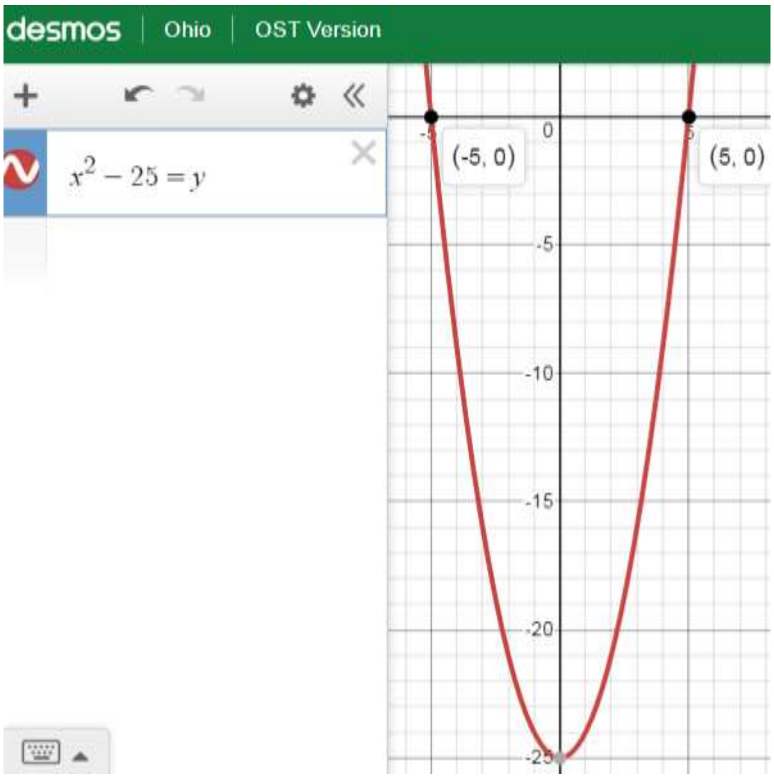
Quadratic equations have none, one or two solutions

Example A: Solve the equation, $x^2 - 25 = 0$.

You can likely determine one solution in your head, 5, because $5^2 = 25$.

Another solution is -5 → $(-5)^2 = 25$.

Let's take the original equation, replace 0 with *y*, and graph $x^2 - 25 = y$ on Desmos:



For the graph, we replaced 0 with *y*. The only points on the graph where the *y*-coordinates = 0 are on the *x*-axis.

What are the *x*-intercepts on the graph? (-5, 0) and (5, 0)

The solutions to the equation are the *x*-intercepts on the graph.

To solve a quadratic equation by graphing:

1st: get all the terms on one side of the equation and 0 on the other side

2nd: replace 0 with y

3rd: graph the function and identify the x-intercepts

Remember that from past units, x-intercepts are also known as roots, zeros, and solutions → when you put 0 in for y, you get the solutions for the equations.

Example B: Solve the equation, $x^2 - 5x = 6$

1st: get everything on one side:

$x^2 - 5x = 6$ or $x^2 - 5x = 6$

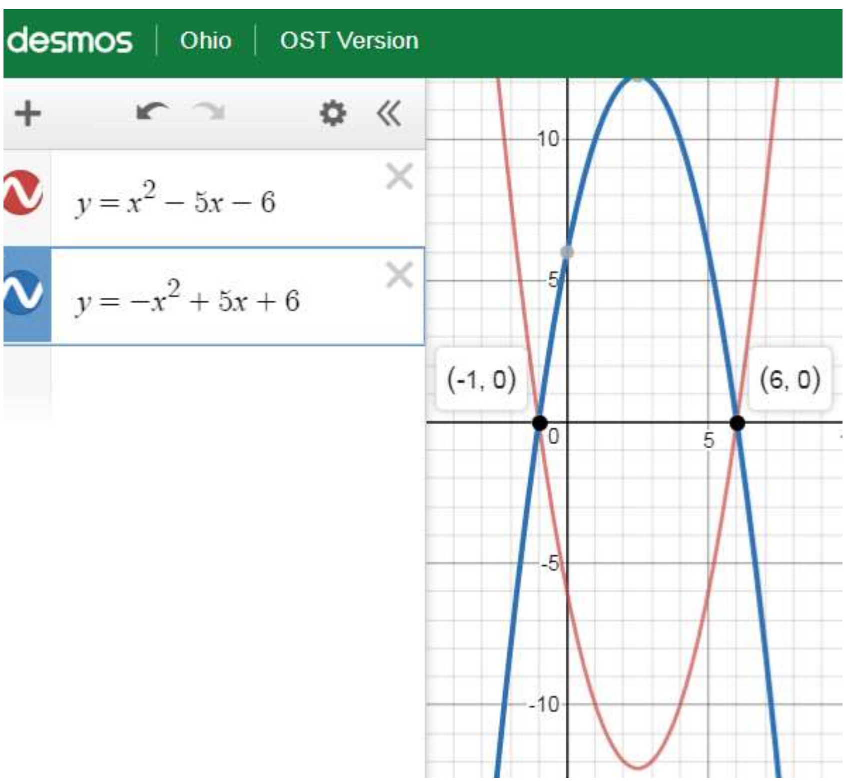
$-6 \quad -6 \qquad -x^2 + 5x \quad -x^2 + 5x$

$x^2 - 5x - 6 = 0$ $\quad 0 = -x^2 + 5x + 6$

2nd: replace 0 with y: $x^2 - 5x - 6 = y$ or $y = -x^2 + 5x + 6$

3rd: graph and identify the x-intercepts:

I will graph both equations above:



For both parabolas, the *x*-intercepts are (-1, 0) and (6, 0). Thus, the solutions for the original equation are *x* = -1 or 6

Check your answers your TI-30XIIS calculator by substituting those solutions in for *x*:

$x^2 - 5x = 6$ or $x^2 - 5x = 6$

$(-1)^2 - 5 \bullet (-1) = 6$ $(6)^2 - 5 \bullet (6) = 6$

$1 + 5 = 6$ $36 - 30 = 6$

Example C: Solve the equation, $(x - 8)^2 - 6 = 48$

1st: get everything on one side:

$(x - 8)^2 - 6 = 48$

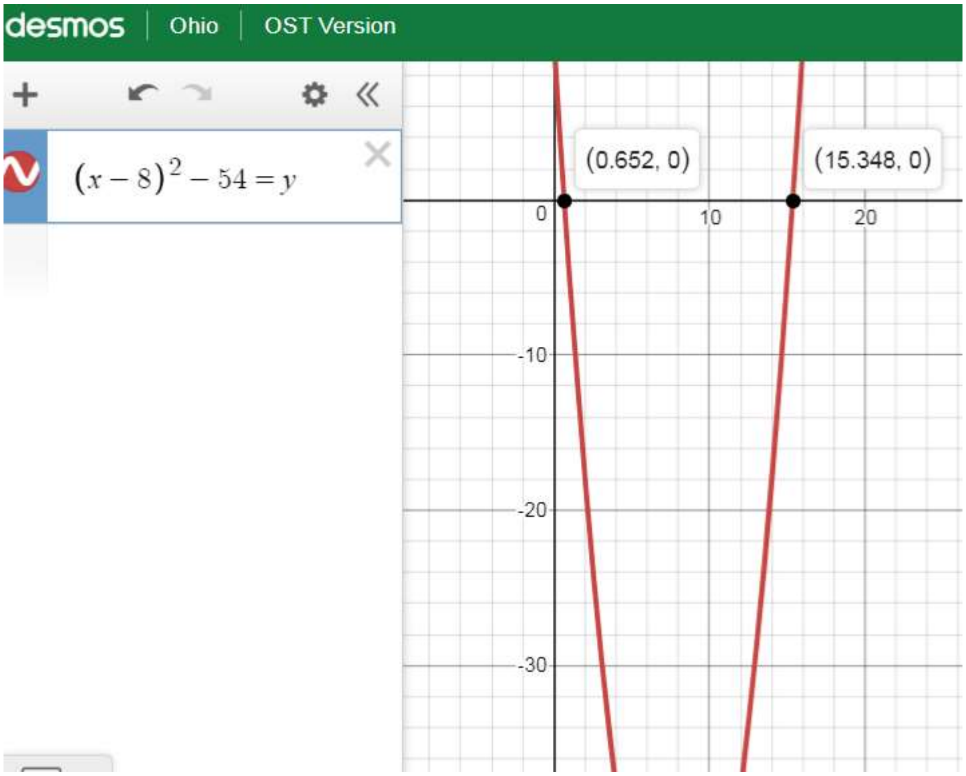
-48 -48

$(x - 8)^2 - 54 = 0$

2nd: replace 0 with *y*:

$(x - 8)^2 - 54 = y$

3rd: graph and identify the *x*-intercepts:



For the parabola, the *x*-intercepts listed are probably rounded decimals: (0.652, 0) and (15.348, 0). Thus, the solutions for the original equation are approximately: $x = 15.348$ or 0.652

Check your answers your TI-30XIIS calculator by substituting those solutions in for *x*:

$(15.348 - 8)^2 - 6 = 48$ $\qquad$ $(0.652 - 8)^2 - 6 = 48$

$47.993 \approx 48$ $\qquad$ $47.993 \approx 48$

≈ means approximately equal to

The exact values of the *x*-intercepts (solutions) are $8 + 3\sqrt{6}$ and $8 - 3\sqrt{6}$.

The answers may be written as $x = 8 \pm 3\sqrt{6}$.

Check those values using your TI-30XIIS calculator.

$8 + 3\sqrt{6} \approx 15.348$

$8 - 3\sqrt{6} \approx 0.652$

Example D: Solve the equation, $2x^2 + 15x - 51 = 4x - 11$

1st: get everything on one side:

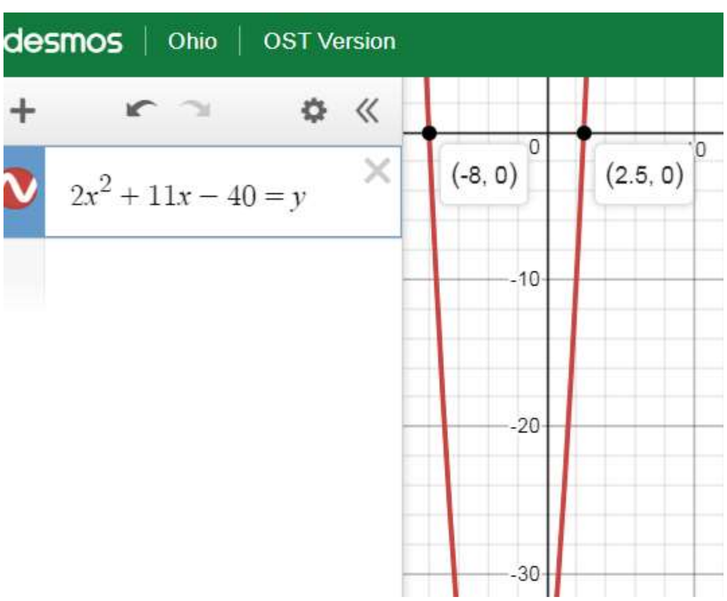
$2x^2 + 15x - 51 = 4x - 11$

$\quad -4x \; +11 \quad -4x \; +11$

$2x^2 + 11x - 40 = 0$

2nd: replace 0 with *y*: $2x^2 - 3x - 40 = y$

3rd: graph and identify the *x*-intercepts:



By looking at the x-intercepts, the solutions are $x = -8$ and $x = 2.5$.

Check your answers your TI-30XIIS calculator by substituting those solutions in for x:

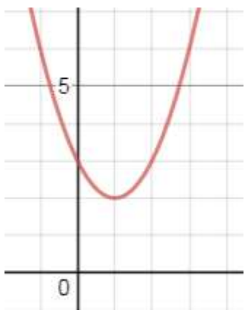
$$2 \bullet (-8)^2 + 15 \bullet (-8) - 51 = 4 \bullet (-8) - 11 \qquad 2 \bullet 2.5^2 + 15 \bullet 2.5 - 51 = 4 \bullet 2.5 - 11$$

$$-43 = -43 \qquad -1 = -1$$

Determining the Number of Solutions from a Graph

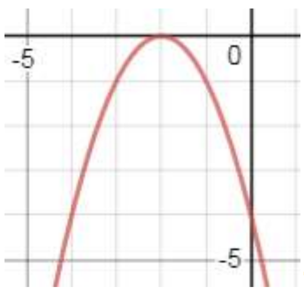
Number of solutions for Quadratic Equations:

No solutions: there are no x-intercepts: the graph never crosses the x-axis



→ no x-intercept → no solutions

One solution: the vertex lies on the x-axis



→ x-intercept = -2 → the solution = -2

Two solutions: the parabola crosses the vertex twice

See Examples A, B, C, and D

Example E: Solve the equation: $5x^2 = 3x - 8$

1st: get everything on one side:

$5x^2 = 3x - 8$ or $5x^2 = 3x - 8$

$-5x^2$ $-5x^2$ $-3x + 8$ $-3x + 8$

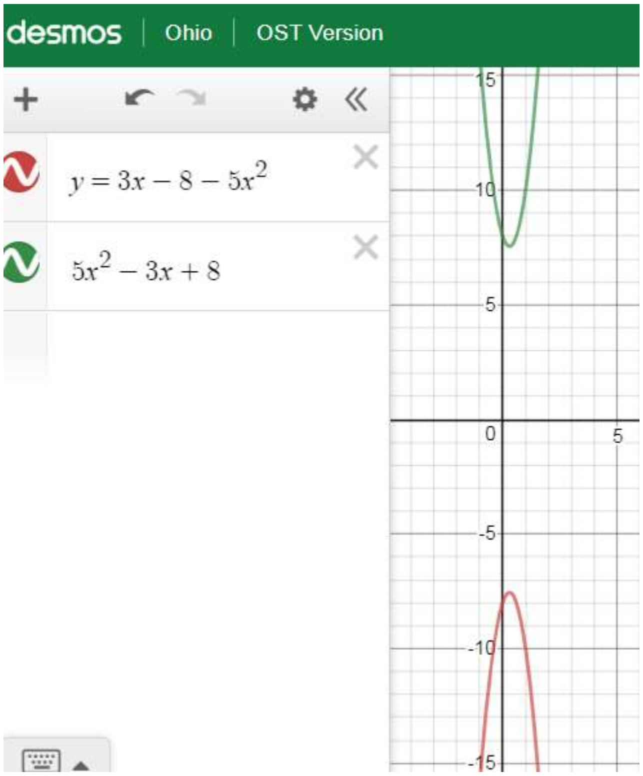
$0 = 3x - 8 - 5x^2$ $5x^2 - 3x + 8 = 0$

$0 = -5x^2 + 3x - 8$

2nd: replace 0 with y:

$y = 3x - 8 - 5x^2$ or $5x^2 - 3x + 8 = y$

3rd: graph and identify the x-intercepts:

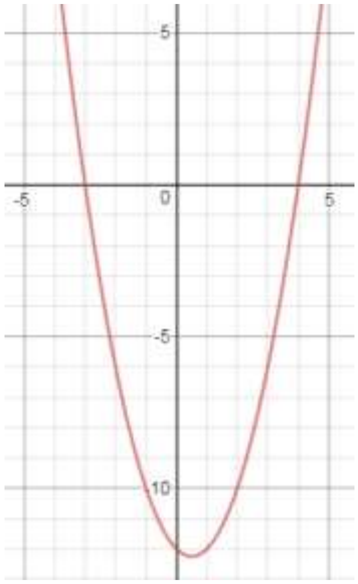


Both equations are graphed. Neither crosses the *x*-axis, so there are no *x*-intercepts. Thus the answer for the original equation is 'No solutions'.

Click on the video for further explanation and practice: Solve Quadratic equations by graphing

Let’s practice.

1.) The graph of $y = x^2 - x - 12$ is shown.



What are the solutions for $x^2 - x - 12 = 0$?
Choose all that apply:

A. 0

B. 3

C. 4

D. -3

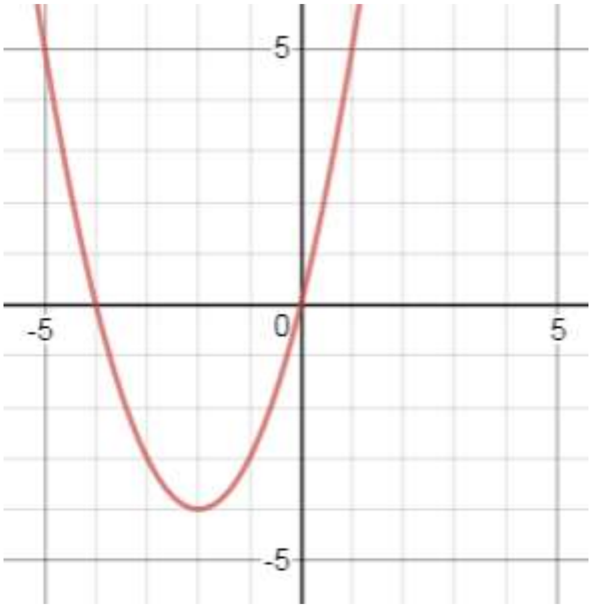
E. -4

F. 12

G. -12

H. no solutions

2.) The graph of $y = x^2 + 4x$ is shown.



What are the solutions for $x^2 + 4x = 0$?
Choose all that apply:

A. 0

B. -4

C. 4

D. -2

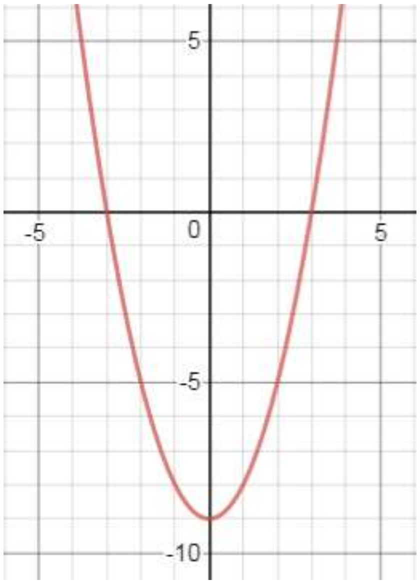
E. -2

F. 5

G. -5

H. no solutions

3.) The graph of $y = x^2 - 9$ is shown.



What are the solutions for $x^2 - 9 = 0$?
Choose all that apply:

A. 0

B. 9

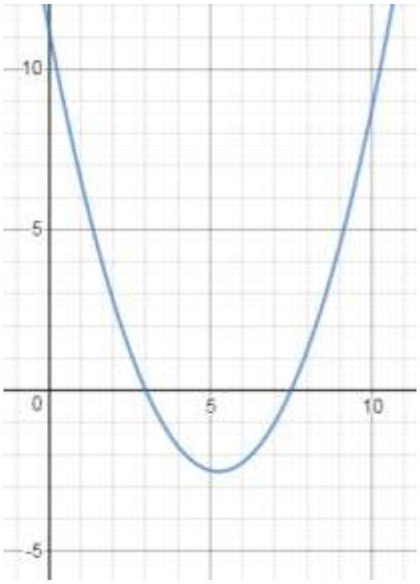
C. -9

D. 3

E. -3

F. no solutions

4.) The graph of $y = \frac{1}{2}x^2 - \frac{21}{4}x + \frac{45}{4}$ is shown.



What are the solutions for $\frac{1}{2}x^2 - \frac{21}{4}x + \frac{45}{4} = 0$?

Choose all that apply:

A. 0

B. 5.25

C. 3

D. -2.5

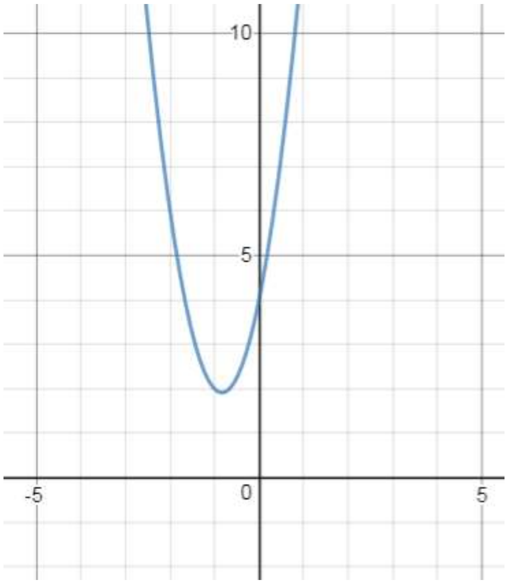
E. 11

F. 7.5

G. -5

H. no solutions

5.) The graph of $y = 3x^2 + 5x + 4$ is shown.



What are the solutions for $3x^2 + 5x + 4 = 0$?
Choose all that apply:

A. 0

B. 4

C. 3

D. 5

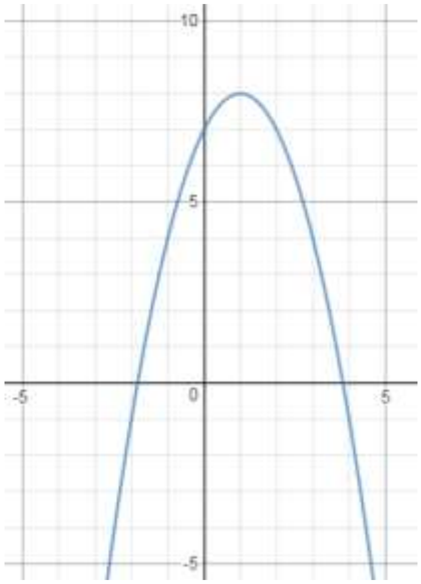
E. -3

F. -4

G. -5

H. no solutions

6.) The graph of $y = -x^2 + 2x + 7$ is shown.



What are the solutions for $-x^2 + 2x + 7 = 0$?

A. $2 \pm 4\sqrt{5}$

B. $3 \pm 5\sqrt{2}$

C. $1 \pm 2\sqrt{2}$

D. $4 \pm 3\sqrt{5}$

7.) Solve the equation: $x^2 - 36 = 0$

A. $x = \pm 36$

B. $x = \pm 6$

C. $x = \pm 18$

D. No solutions

8.) Solve the equation: $x^2 - 7x = 18$

A. $x = 7$ and $x = 18$

B. $x = -9$ and $x = 2$

C. $x = -7$ and $x = -18$

D. $x = 9$ and $x = -2$

9.) Solve the equation: $(x + 4)^2 + 8 = 17$

A. $x = -1$ and -7

B. $x = -4$ and 8

C. $x = 6$ and -2

D. $x = 8$ and -17

10.) Solve the equation: $2x^2 + 9 = 35$

A. $x = \pm\sqrt{26}$

B. $x = \pm\sqrt{35}$

C. $x = \pm\sqrt{13}$

D. no solutions

11.) An equation is shown.

$2x^2 - 5x - 3 = 0$

What values of x make the equation true?

$x =$ -0.5

$x =$ 3

12.) Solve the equation $x^2 + 6x = -\frac{11}{4}$

A. $x = -3$ and $x = 2$

B. $x = -2$ and $x = 3$

C. $x = \frac{1}{2}$ and $x = -\frac{11}{2}$

D. $x = -\frac{1}{2}$ and $x = -\frac{11}{2}$

13.) An equation is shown.

$16x^2 + 10x - 27 = -6x + 5$

What are the solutions to this equation?

$x =$ -2

$x =$ 1

14.) An equation is shown.

$x^2 - 6x + 9 = 0$

How many solutions does this equation have?

A. no solutions

B. one solution

C. two solutions

D. infinite solutions

15.) An equation is shown.

$-3x^2 - 5x = 11$

How many solutions does this equation have?

A. no solutions

B. one solution

C. two solutions

D. infinite solutions

16.) An equation is shown.

$(x + 6)(x - 8) = 0$

What are the solutions to this equation?

$x =$ -6

$x =$ 8

Solving Linear Equations by Graphing

In units 2 and 3, we studied how to solve linear equations by following these steps:

1. Eliminate parentheses by using the distributive property
2. Simplify each side by combining like terms
3. Get the variables on the same side (use the addition or subtraction property of equality)
4. Get the variable by itself using the inverse of the properties of equalities

Example F: Solve the equation for *x*:

2(3x + 4) – 2 + 8x = 4x + 73

2(3x + 4) – 2 + 8x = 4x + 73 *Eliminate parentheses by using the distributive property

6x + 8 – 2 + 8x = 4x + 73 *Combine like terms on the left side of the equation

14x + 6 = 4x + 73

-4x -4x * Get the variables on the same side

10x + 6 = 73

-6 -6 *get the variable by itself

10x = 67

$\div 10 \quad \div 10$

$x = 6.7$

Another way to solve this equation is to follow the same steps mentioned earlier in this unit for quadratic equations:

1st: get all the terms on one side of the equation and 0 on the other side
2nd: replace 0 with *y*
3rd: graph the function and identify the *x*-intercepts

Let's re-try **Example F** using this strategy:

$2(3x + 4) – 2 + 8x = 4x + 73$

$-73 \quad -4x \quad -4x \quad -73$ ← get all the terms on one side

$2(3x + 4) – 2 + 8x - 4x - 73 = 0$ ← no like terms combined

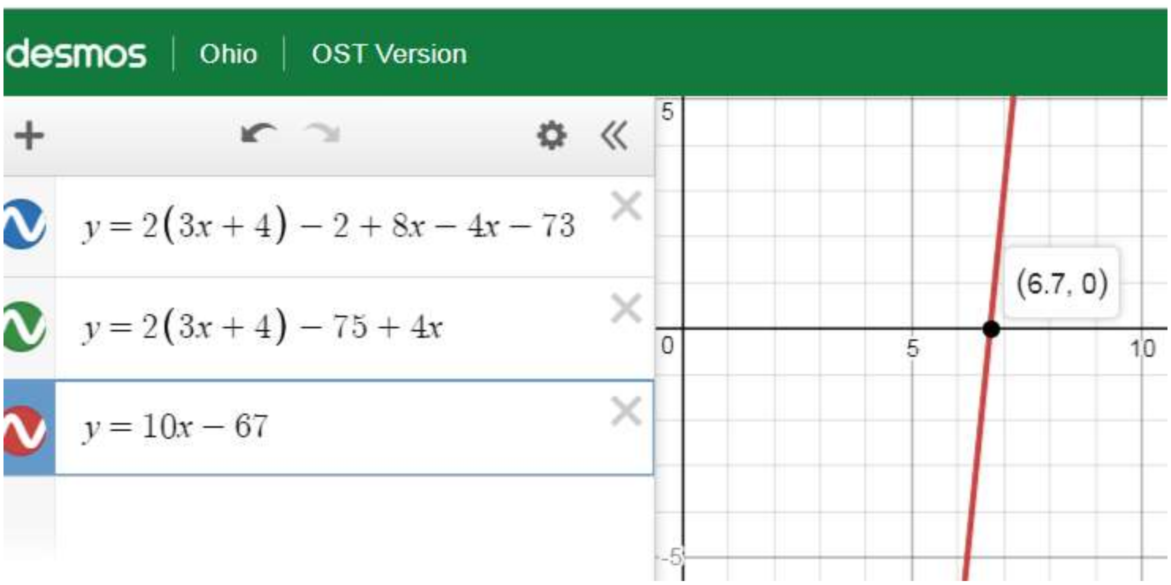
$2(3x + 4) - 75 + 4x = 0$ ← some like terms combined

$10x – 67 = 0$ ← all like terms combined

2nd: replace 0 with *y*

3rd: graph the function and identify the *x*-intercepts

You can use any of the three equations listed above, whether you combine like terms or not. Each has the same *x*-intercept when graphed on Desmos:



The *x*-intercept is (6.7, 0), so the solution is $x = 6.7$.

Use your calculator to check your solution:
$2(3 \cdot 6.7 + 4) - 2 + 8 \cdot 6.7 = 4 \cdot 6.7 + 73$
$99.8 = 99.8$
It does not necessarily work when you have an equation with no solutions or infinite solutions (*x* = all real numbers).

Example G:

6x + 5 = 2(3x + 4)

6x + 5 = 6x + 8

-6x -6x

5 = 8 → false statement → no solutions

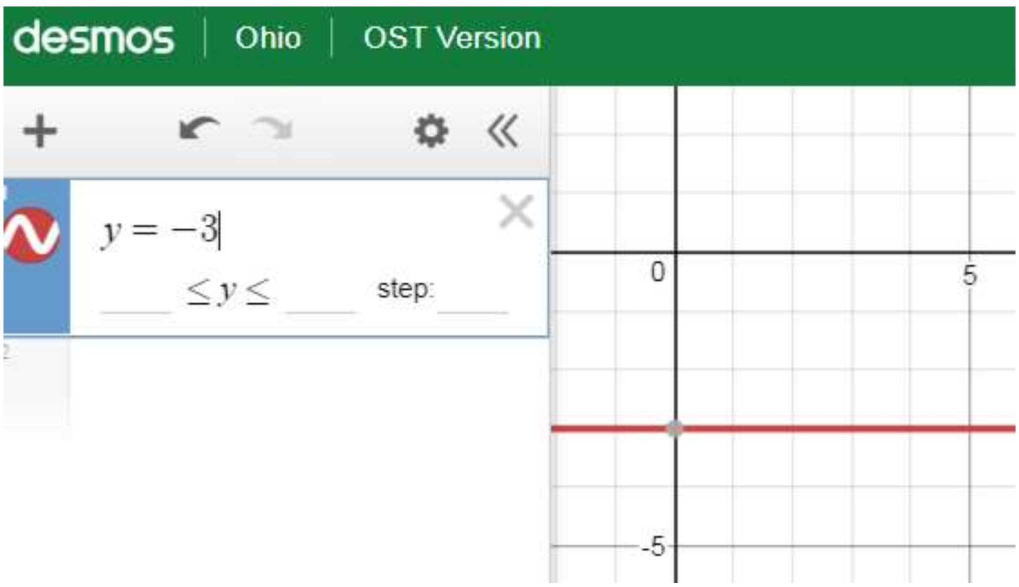
If you move everything to one side:

6x + 5 = 6x + 8

-6x -8 -6x -8

-3 = 0

Replace 0 with *y* → -3 = *y*



no *x*-intercept → no solutions

Example H:

8x + 4 = 4(2x + 1)

8x + 4 = 8x + 4

-8x -8x

4 = 4 → true statement → x = all real numbers

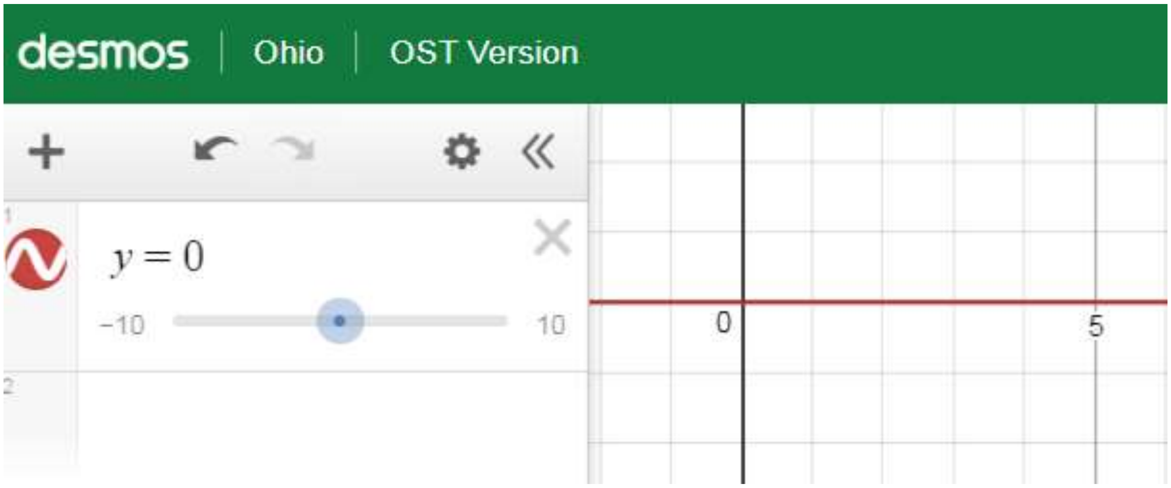
If you move everything to one side:

8x + 4 = 8x + 4

-8x -4 -8x -4

0 = 0

Replace 0 with y → $y = 0$



$y = 0$ → runs along the entire x-axis → infinite solutions→ x = all real #s

Let's practice. Solve the equation for the variable.

17.) $-2x + 58 = 8x - 41$

Answer $x = 9.9$

18.) $4(2x + 9) - 3x = 62$

Answer $x = 5.2$

19.) $2(3x + 8) + 2x = 10x - 45$

Answer $x = 30.5$

20.) Determine the solution for: $6x + 8 + 10x = 4(3x - 8)$

A. -10
B. 8
C. 12
D. -5