Lemniscate of Bernoulli

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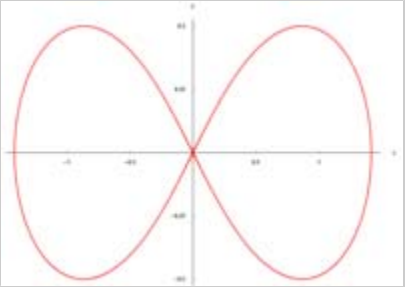

A lemniscate of Bernoulli

In mathematics, the **Lemniscate of Bernoulli** is a figure-eight shaped algebraic curve described by a Cartesian equation of the form:

$$(x^2 + y^2)^2 = a^2(x^2 - y^2)$$

Graphing this equation produces a curve similar to the ∞ symbol. The symbol itself is sometimes referred to as the lemniscate. Its Unicode representation is ∞ (∞).

The lemniscate was first described in 1694 by Jakob Bernoulli as a modification of an ellipse, which is the locus of points for which the sum of the distances to each of two fixed *focal points* is a constant. A lemniscate, by contrast, is the locus of points for which the *product* of these distances is constant. Bernoulli called it the *lemniscus*, which is Latin for 'pendant ribbon'.

The lemniscate can be obtained as the inverse transform of a hyperbola, with the inversion circle centered at the center of the hyperbola (bisector of its two foci).

Other equations

A lemniscate may also be described by the polar equation

$$r^2 = a^2 \cos 2\theta$$

or the bipolar equation

$$rr' = \frac{a^2}{2}$$

Arc length and elliptic functions

The determination of the arc length of arcs of the lemniscate leads to elliptic integrals, as was discovered in the eighteenth century. Around 1800, the elliptic functions inverting those integrals were studied by C. F. Gauss (largely unpublished at the time, but allusions in the notes to his *Disquisitiones Arithmeticae*). The period lattices are of a very special form, being proportional to the Gaussian integers. For this reason the case of elliptic functions with complex multiplication by the square root of minus one is called the *lemniscatic case* in some sources.

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