Cardioid

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In geometry, the **cardioid**, literally *heart shape*, is an epicycloid which has one and only one cusp. That is, a cardioid is a curve that can be produced as a locus — by tracing the path of a chosen point of a circle which rolls without slipping around another circle which is fixed but which has the same radius as the rolling circle.

The cardioid is also a special type of limaçon: it is the limaçon with one cusp. (The cusp is formed when the ratio of a to b in the equation is equal to one.)

The name comes from the heart shape of the curve (Greek *kardioeides* = *kardia*:heart + *eidos*:shape). Compared to the heart symbol (♥), though, a cardioid does not come to a sharp point. It is rather shaped more like the outline of the cross section of a plum.

The cardioid is an inverse transform of a parabola.

The large, central, black figure in a Mandelbrot set is a cardioid. This cardioid is surrounded by a fractal arrangement of circles.

Contents

Equations

Since the cardioid is an epicycloid with one cusp, its parametric equations are

$$x(\theta) = \cos\theta + \frac{1}{2}\cos 2\theta,$$

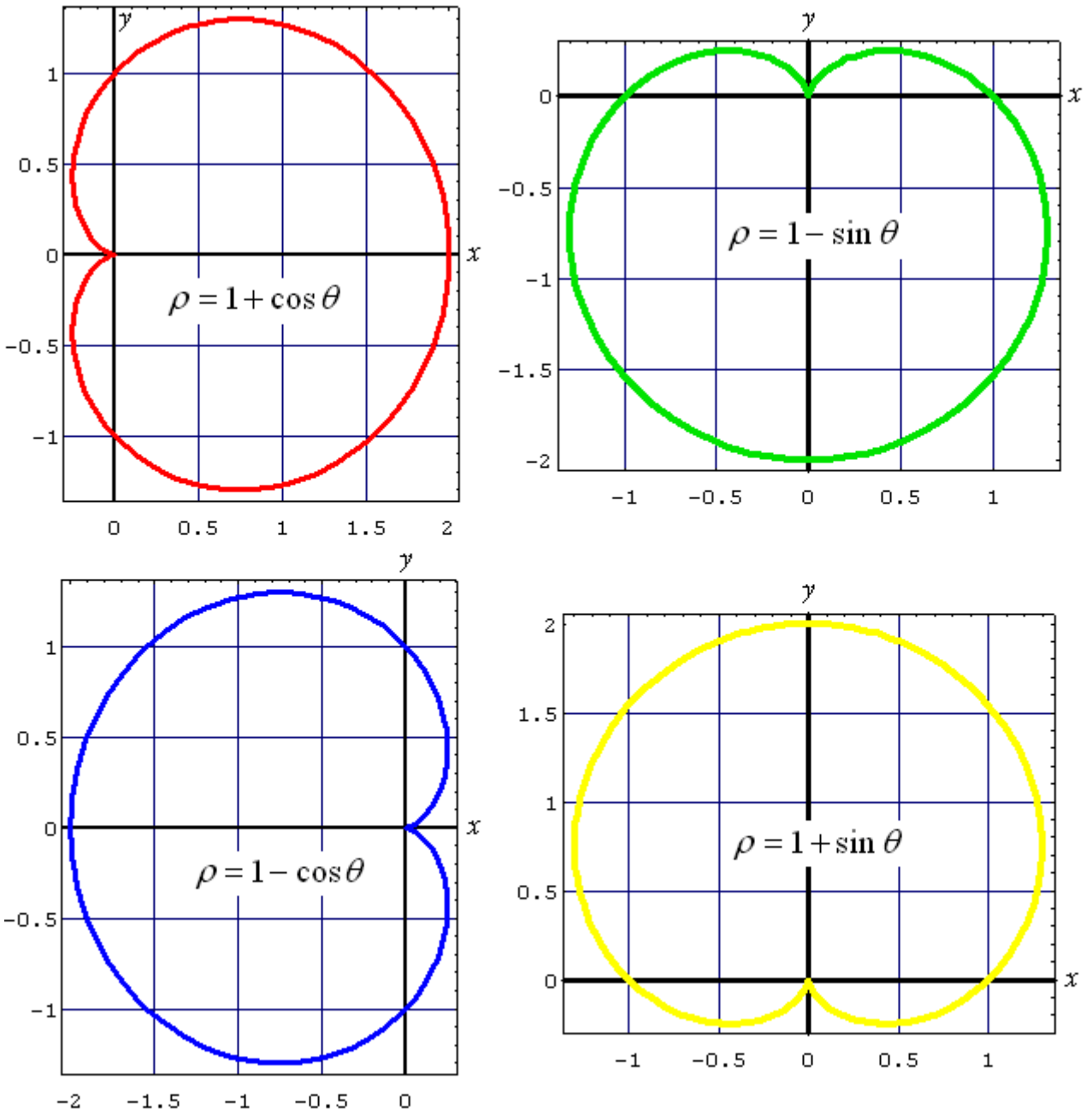
$$y(\theta) = \sin\theta + \frac{1}{2}\sin 2\theta.$$

The same shape can be defined in polar coordinates by the equation

$$\rho(\theta) = 1 + \cos\theta.$$

For a proof, see cardioid proofs.

Graphs



Four graphs of cardioids oriented in the four cardinal directions, with their respective polar equations.

Area

The area of a cardioid which is congruent to

$$\rho(\theta) = a(1 - \cos\theta)$$

is

$$A = \frac{3}{2}\pi a^2.$$

See proof.

See also

- Wittgenstein's rod
- microphone#Directionality

References

- Hearty Munching on Cardioids (http://www.cut-the-knot.org/ctk/Cardi.shtml) at cut-the-knot
- Xah Lee, *Cardioid (http://www.xahlee.org/SpecialPlaneCurves_dir/Cardioid_dir/cardioid.html)* (1998) *(This site provides a number of alternative constructions)*.
- Jan Wassenaar, *Cardioid (http://www.2dcurves.com/roulette/rouletteca.html)*, (2005) in *862 two-dimensional mathematical curves*.

Category: Curves