Algebraic Expressions

This lesson will lay the foundation for the studies of Algebra. In this lesson you will be introduced to algebraic terminology. You will learn that there is an order to evaluate complicated expressions and also be introduced to the world of variables.

Order of Operations

In order to find the numerical value (**evaluate**) of any combination of numbers and operations (**expression**) correctly, mathematicians have established the order of operations which tells us which operations to do first in any mathematical problem.

P (parentheses)

E (exponents – powers)

M (multiply)
D (divide)
} work left to right

A (add)
S (subtract)
} work left to right

The following saying can be used to help remember the order of operations:

> **P**lease
> **E**xcuse
> **M**y **D**ear
> **A**unt **S**ally

*Remember that if "multiplication and division" or "addition and subtraction" are the only two operations in the expression, work the problem from left to right.

Examples:

1)	$6 \times 4 + 2$	1. Multiply 6×4
	$24 + 2$	2. Add $24 + 2$
	26	
2)	$4(6+3)-5\cdot 2$	1. Parentheses $(6 + 3)$
	$4(9)-5\cdot 2$	2. Multiply $4(9)$
	$36-5\cdot 2$	3. Multiply $5\cdot 2$
	$36 - 10$	4. Subtract
	26	

3)	$5[(3+12)-2(4)]$	1. Work within [ ] a. Multiply $2 \cdot 4$
	$5[(3+12)-8]$	b. Add 3 + 12
	$5[15-8]$	c. Subtract 15 – 8
	$5[7]$	2. Multiply 5[7]
	35	
4)	$4[3(3+2)^2]$	1. Work within [ ] a. Parenthesis (3 + 2)
	$4[3(5)^2]$	b. Powers $(5)^2$
	$4[3 \cdot 25]$	c. Multiply $3 \cdot 25$
	$4[75]$	2. Multiply 4[75]
	300	
5)	$4 \cdot 5 - 18 \div 6 + 2 \cdot 3$	1. Multiply and divide left to right
	$20-3+6$	2. Add and subtract left to right
	$17+6$	3. Add
	23	

You will continue to use the **order of operations** throughout the remainder of this lesson and throughout any other mathematics courses you continue to take.

Introduction to Variables and Expressions

Did you know?

Most countries in the world use the Celsius scale to measure temperature. The Celsius scale measures temperature from $0°$ freezing to $100°$ boiling. On the other hand, in the United States we use the Fahrenheit scale, most of the time. The Fahrenheit scale measures temperature from $32°$ freezing to $212°$ boiling. It is possible to convert between temperature scales by using algebra. If the Celsius temperature is multiplied by $\frac{9}{5}$ and then added to 32, the Fahrenheit temperature can be determined.

Algebra can be thought of as a language of symbols. For example we already know the symbols for addition (+) and multiplication ($\times$ or $\cdot$) so we could write the temperature relationship from above as follows:

$$\frac{9}{5} \cdot \textbf{Celsius} + 32$$

In arithmetic we could write the same expression as:

$$\frac{9}{5} \cdot \square + 32$$

Where $\square$ represents the Celsius temperature and is serving as a place holder.

In algebra when a problem has missing or "unknown" information, the place holders used are called **variables**. Variables are letters such as "*x*, *n*, or *a*" that represent the unknown value. (You may use any letter as a variable; these were just a few examples.)

***When choosing a variable to represent an unknown value, make sure not to use the letter "o" because it could be mistaken for the number zero.**

Let’s take a look at how we could represent the above using algebra and a variable.

$\frac{9}{5}\cdot$ **Celsius** + 32	Words and symbols
$\frac{9}{5}\cdot\ \square\ + 32$	Arithmetic
$\frac{9}{5}\mathbf{C}+32$	Algebra (variable)

$\frac{9}{5}\cdot C+32$ is called an **algebraic expression** because it contains a combination of variables, numbers and at least one operation.

Algebraic expressions can be **evaluated** by replacing the variable with numbers.

For example, if given the expression *a* + *b* – 24 and asked to evaluate it for the given values *a* = 19 and *b* = 20, you would:

a) replace “*a*” and “*b*” with the given values
b) evaluate the expression using the order of operations

Let’s try the above example. You will be given this type of problem in the following form.

Example 1: Evaluate *a* + *b* – 24 if *a* = 19 and *b* = 20

a + *b* – 24

19 + 20 – 24 — 1) replace “*a*” and “*b*” with the given values

39 – 24 — 2) evaluate using the order of operations

15

Example 2: Evaluate 5a + *bc* – c if *a* = 4, *b* = 2, and *c* = 3

$5a + bc - c$	
5(4) + (2)(3) – 3	1) replace "*a*", "*b*", and "*c*" with the given values
20 + (2)(3) – 3 2)	2) multiply 5(4)
20 + 6 – 3	3) multiply 2(3)
26 – 3	4) add 20 + 6
23	5) subtract 26 - 3

As you become more familiar with evaluating expressions, you will be able to perform more than one operation per step. For example, in the above example it would have been okay in step 2 to also multiply 2 and 3. Again you will be able to do this after more practice.

Throughout this course and any of the more advanced mathematics courses you will take, it will be necessary to interpret verbal sentences into algebraic sentences. For this you will need to know the words and phrases that suggest the operation to use.

The chart below lists some of the most common phrases that will be used.

Addition	***Subtraction***	***Multiplication***	***Division***
sum	difference	product	quotient
plus	minus	times	divided by
increased by	decreased by	of	ratio
more than	less than	twice (× 2)	per
total	subtract	multiplied by	average

To translate verbal phrases into algebraic expressions:

a) chose a variable to represent the unknown (if it is not given)

b) determine what operation will be used based on the phrase

c) write the algebraic expression

Example 3: Eight more points than Rachel's score.

a) Let "r" represent Rachel's score

b) "More than" suggests addition

c) The algebraic expression is $r + 8$ *or* $8 + r$

Example 4: Four times as much money as Pete.

a) Let "p" represent Pete's money

b) "Times" suggests multiplication

c) The algebraic expression is $p \times 4$, $4 \times p$, or more commonly seen in algebra as $4p$.

*At this point it should be noted that anytime a variable or variables are multiplied with a number it will be written with the number first and then the variable or variables following. Study the examples given below.

$6x$, $7y$, $3xyz$