Percent

In this lesson you will work with various aspects of percent problems. You will review the basics of percents and then apply those skills to problem-solving scenarios.

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Percent to Decimal and Decimal to Percent

Express Percents as Decimals Using a Shortcut

Example 1: Write 89% as a decimal.

Think of the decimal point being at the end of the decimal, and then divide the percent by 100 or just move the decimal point **two places** to the **left**.

89.% = 0.89

Example 2: Write 6.25% as a decimal. Move the decimal point two places to the left.

6.25% = 0.0625 (Use zeros as place holders.)

Example 3: Write 275% as a decimal.

275. % = 2.75

Express Decimals as Percents Using a Shortcut

Example 1: Write 0.34 as a percent.

When expressing decimals as percent, multiply the decimal by 100 or just move the decimal point **two places** to the **right**.

0.34 = 34.% or just 34%

Example 2: Write 8.5 as a percent.

8.5 = 850% (Use a zero as a place holder.)

Percents to Fractions

Fractions have percent equivalences and vice versa. Let's look at some examples of expressing a percent as a fraction.

Example 1: Express 35% as a fraction.

$35\ \% = \dfrac{35}{100}$ Meaning of percent

$\dfrac{35}{100} = \dfrac{7}{20}$ Simplify fraction

$35\% = \dfrac{7}{20}$

Example 2: Express $8\dfrac{1}{2}\%$ as a fraction.

$8\dfrac{1}{2}\% = \dfrac{8\frac{1}{2}}{100}$ Meaning of percent

$8\dfrac{1}{2} \div 100$ Use division of fractions

$\dfrac{17}{2} \div \dfrac{100}{1}$ Write division problem in fraction form

$\dfrac{17}{2} \cdot \dfrac{1}{100} = \dfrac{17}{200}$ Invert second fraction and multiply

$8\dfrac{1}{2}\% = \dfrac{17}{200}$

Fractions to Percents

Fractions have percent equivalences and vice versa. Let's look at some examples of expressing a fraction as a percent.

Example 1: Express $\frac{5}{8}$ as percent.

$5 \div 8 = 0.625$	Express fraction as decimal by dividing numerator by denominator.
$0.625 = 62.5\%$	Multiply decimal by 100 using shortcut of moving the decimal point right two places.

$$\frac{5}{8} = 62.5\%$$

Example 2: Express $2\frac{2}{3}$ as percent.

$2\frac{2}{3} = \frac{8}{3}$	Express mixed fraction as improper fraction.
$8 \div 3 = 2.666...$	Express fraction as decimal by dividing numerator by denominator.
$2.666... = 266.67\%$	Multiply decimal by 100 using shortcut of moving the decimal point right two places and round repeating decimal to nearest hundredth.

$$2\frac{2}{3} = 266.67\%$$

Example 3: Express $\frac{3}{400}$ as percent.

$3 \div 400 = 0.0075$	Express fraction as decimal by dividing numerator by denominator.
$0.0075 = 0.75\%$	Multiply decimal by 100 using shortcut of moving the decimal point right two places.

$$\frac{3}{400} = 0.75\%$$

Comparing Percents, Fractions, and Decimals

Percents, fractions, and decimals may all be used to represent the same quantity. Let's take a look at how to apply this connection.

45%

$\frac{9}{20} \longleftrightarrow 0.45$

$$45\% = 0.45 = \frac{9}{20}$$

Example: Rearrange the given numbers to be in order from least to greatest.

$\frac{3}{4}$, 15 out of 16, 0.075, 79%

To solve, express each number as a decimal and then compare.

$\frac{3}{4} =$	0.75
15 out of 16 = 15 ÷ 16 =	0.9375
0.075	No change
79% =	0.79

List the decimals and add enough zeros to compare.

0.7500
0.9375
0.0750
0.7900

Least to greatest as decimal: 0.0750, 0.7500, 0.7900, 0.9375

Least to greatest as original numbers: 0.075, $\frac{3}{4}$, 79%, 15 out of 16

Estimating with Percent

Often times when we use percent, we only need to estimate an answer to have an idea of a value that is close to the actual. There are many ways to estimate and you may choose different methods or combinations of methods depending on the type of numbers within the problem.

Example 1: Estimate 40% of 876.

1% of 876 is 8.76	$876 \times 0.01 = 8.76$
$8.76 \approx 9$	Round 8.76 to nearest whole number 9
$40 \times 9 = 360$	If 1% is approximately 9, then multiply 9 by 40 to get an estimate of 40% of 876.

40% of 876 ≈ 360

Example 2: Estimate 73% of 589.

73% is about 75% or $\frac{3}{4}$.	Round percent to nearest percent equal to a simple fraction.
$589 \approx 600$	Round 589 to 600 to multiply by $\frac{3}{4}$.
$\frac{3}{4} \times 600$	$\frac{3}{\cancel{4}_{1}} \times \frac{\cancel{600}^{150}}{1} = 450$

73% of 589 ≈ 450

Example 3: When eating out at a restaurant, it is standard to give a tip of 15% for good service. If the family meal costs $43.28, how much should be left as a tip?

Estimate 15% of $43.28.

$43.28 is about $44 (easier to round up to get an even number).

10% of $44 is $4.40 →	A quick way to find 10% of a number is to move the decimal one place to the left or $44 \times 0.10 = 4.4$.
5% of $44 is $2.20 →	5% is half of 10%, therefore half of $4.40 is $2.20.
10% + 5% = 15% →	$4.40 + $2.20 = $6.60

Thus, a tip that might be left for this meal could be $6.50, $6.60, or $7.00.

Percents ("is / of" Method)

Percent means per hundred. Thus when we say 27% we mean 27 out of 100. Percents can be written as equivalent decimals and fractions.

$27\% = 0.27$ move the decimal point 2 places left

$27\% = \frac{27}{100}$ put 27 over 100 since percent means per hundred

Percents may be written as simplified fractions.

$75\% = \frac{75}{100} = \frac{3}{4}$ 75% means 3 out of 4

Make sure to always simplify your ratios!

Percents greater than 100% represent whole numbers or mixed numbers.

$$200\% = \frac{200}{100} = \frac{2}{1} = 2$$

$$350\% = \frac{350}{100} = 3\frac{50}{100} = 3\frac{1}{2}$$

To write a percent as a decimal, write the number without the percent sign (%) and move the decimal 2 places LEFT.

Example 1: 45% = .45

3.2% = .032

120% = 1.20 or 1.2

There are three basic types of percent problems. They are finding part (percent of a number), finding percent, and finding base.

Proportions can be used to solve percent problems. Study the proportion below, and the following examples will clarify any questions.

$$\frac{\%}{100} = \frac{\text{is}}{\text{of}}$$

When finding the part or base, use the ratio, $\frac{\text{is}}{\text{of}}$, to set up a proportion. The part will follow the word "is" and the base will follow the word "of".

Finding Part

Example 2: 20% of 60 is what number?

$$\frac{\%}{100} = \frac{\text{is}}{\text{of}}$$

20% is given --- 60 follows "of" --- "what number" follows "is"

$$\frac{20}{100} = \frac{x}{60}$$

-the "is" number is what we are looking for

$$20 \times 60 = 100x$$

-cross multiply

$$1200 = 100x$$

$$12 = x$$

12 is 20% of 60.

Finding Percent

Example 3: What percent of 80 is 60?

$$\frac{\%}{100} = \frac{\text{is}}{\text{of}}$$

% is not known--- 80 follows “of” --- 60 follows “is”

$$\frac{x}{100} = \frac{60}{80}$$ -the % is missing

$$x \times 80 = 100 \times 60$$

$80x = 6000$

$x = 75$ **75% of 80 is 60.**

Finding Base

Example 4: 15% of what number is 75?

$$\frac{\%}{100} = \frac{\text{is}}{\text{of}}$$

15% is given--- “what number” follows “of” --- 75 follows “is”

$$\frac{15}{100} = \frac{75}{x}$$ -cross multiply and divide

$15x = 75(10)$

$15x = 7500$

$x = 500$ **15% of 500 is 75**

Percent Equations

In the previous lesson you learned how to solve the three types of percent problems using proportions. Now we will examine how they can be solved as equations.

A general equation that is used for percent equations is ***P* = *R* · *B*** where ***P*** represents ***P***ercentage, ***R*** represents ***R***ate, and ***B*** represents ***B***ase.

Let's look at how the formula is derived.

$$\frac{P}{B} = \frac{R}{100}$$ Relationship of P, R, and B in a proportion.

$$\frac{P}{B} \bullet B = \frac{R}{100} \bullet B$$ Multiply both sides of the proportion by B.

$$P = \frac{R}{100} \bullet B \qquad \frac{P}{\underset{1}{\cancel{B}}} \bullet \frac{\overset{1}{\cancel{B}}}{1} = \frac{P}{1} = P$$

$$P = R \bullet B$$ R represents the decimal form for $\frac{R}{100}$.

Example 1: What is 29% of 83?

Given:

29% is the rate (Rate is the percent in the problem.)
83 is the base (General rule: the number that follows "of" is the base.)

Finding:

P will represent the percentage (part)

What **is** 29% **of** 83?

$$\begin{array}{ccccc} \downarrow & & \downarrow & & \downarrow \\ P & = & 0.29 & \times & 83 \end{array}$$

$P = 24.07$

24.07 is 29% of 83.

Example 2: 45 is 60% of what number?

Given:

60% is the rate (Rate is the percent in the problem.)
45 is the percentage (part)

Finding:

B will represent the base (the unknown follows "of")

45 **is** 60% **of** what number?

$$\downarrow \quad \downarrow \quad \downarrow$$

$$45 = 0.6 \times B$$

$$\frac{45}{0.6} = \frac{0.6\,B}{0.6}$$

$$75 = B$$

45 is 60% of 75.

Example 3: 12 is what percent of 18?

Given:

12 is the percentage (part)
18 is the base (18 follows "of")

Finding:

R will be the rate (unknown number)

12 **is** *R* **of** 18?

$$\downarrow \quad \downarrow \quad \downarrow$$

$$12 = R \times 18$$

$$\frac{12}{18} = \frac{18\,R}{18}$$

$0.666... = R$
$66.7\% = R$ (Multiply by 100 for percent, 66.66 rounds to 66.7)
$66\frac{2}{3}\% = R$ (Multiply by 100 for percent, mixed fraction answer)

12 is $66\frac{2}{3}\%$ of 18.

Work for the division in Example 3.

$$\begin{array}{r} .6666 \\ 18\overline{)12.0000} \\ \underline{108} \\ 120 \\ \underline{108} \\ 120 \\ \underline{108} \\ 120 \\ \underline{108} \\ 12 \end{array} \quad \textit{or} \quad \begin{array}{r} .66 \\ 18\overline{)12.00} \\ \underline{108} \\ 120 \\ \underline{108} \\ 12 \end{array} \quad \tfrac{12}{18} = .66\tfrac{2}{3} = 66\tfrac{2}{3}\%$$

Percent Change

In our daily living we see many changes transpire through the years. In the financial world changes can be charted through percent. We will now look at how to represent changes of increase and of decrease in percent.

Percent of change is the ratio of the **amount of change** to **the original amount**.

$$\text{Percent Change} = \frac{\text{Amount of Change}}{\text{Original Amount}}$$

Percent of Increase

Example: Union High School's enrollment increased from 525 students last year to 562 students this year. What is the percent of increase in the number of students this year?

Step 1: Subtract to find the amount of change.

$562 - 525 = 37$

Step 2: Use the percent equation, $P = R \cdot B$, to find the percent of increase.

P – Percentage – Amount of Change (37)

B – Base – Original Amount (525)

$$\begin{aligned} P &= R \cdot B \\ 37 &= R \cdot 525 \\ \frac{37}{525} &= \frac{525R}{525} \\ R &= 0.07 \\ R &= 7\% \text{ (Multiply 0.07 by 100 for percent.)} \end{aligned}$$

There is a 7% increase in students this year.

Percent of Decrease

Example: The town population decreased from 4000 to 3350 in the last census year. Find the percent of decrease.

Step 1: Subtract to find the amount of change

4000 – 3350 = 650

Step 2: Use the percent equation, $P = R \cdot B$, to find the percent of decrease.

P – Percentage – Amount of Change (650)

B – Base – Original Amount (4000)

$$P = R \cdot B$$

$$650 = R \cdot 4000$$

$$\frac{650}{4000} = \frac{4000R}{4000}$$

$$R = 0.1625$$

$$R = 16.25\% \text{ (Multiply 0.1625 by 100 for percent.)}$$

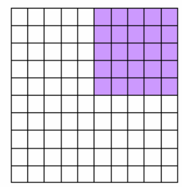
There is a 16.25% decrease in the town population.

Using Percents

Lyle went shopping with his mom and found a jacket that was on sale marked **25% off**. The jacket cost approximately $40.00. He wanted to figure the savings quickly.

Here is how Lyle figured his discount.

He thought of 25% as a fraction $\frac{25}{100}$ which he remembered from math class was the same as $\frac{1}{4}$.



He multiplied $\$40 \times \frac{1}{4}$ and figured that the discount was $10.

He subtracted $\$40 - \$10$ and told his mom the jacket only cost $30 on sale.

Lyle's mother had a calculator in her purse and decided to check his math. She found the discount another way. She thought of 25% as the decimal, 0.25, and then multiplied by $40.00. She also figured $10 for the discount. Since that was a good discount on the jacket and the final price of $30 was reasonable, Lyle took home a new jacket!

$$\begin{array}{r} 40 \\ \times\ \underline{0.25} \\ 200 \\ \underline{80\ } \\ 10.00 \end{array} \qquad \begin{array}{r} \$40 \\ \underline{-10} \\ \$30 \end{array}$$

Discounts / Markdowns

A **discount**, or **markdown**, is the amount of money you save by buying an item at a discounted price, or sale price. To find the discount or markdown when the percent of reduction is given, first express the percent as a decimal and multiply it by the regular price. The result will be the discount amount that will be subtracted from the original price.

Example: Annie purchased a sweater at Gaylord's. The regular price was $39.95. The markdown rate was 20%. What was her discount and sale price?

Step 1: Change 20% into **0.20**.

Step 2: Multiply 0.20 and the regular price.

$$0.20 \times 39.95 = \mathbf{7.99}$$

Step 3: Subtract 7.99 from the original price of 39.95.

$$39.95 - 7.99 = \mathbf{\$31.96}$$

Annie paid $31.96 for her sweater.

Commission

In some jobs, a person's pay depends upon the amount of goods, or services, the person sells. The salesperson receives a **commission,** or specified amount of money for sales made during a pay period. Commission is usually expressed as a percentage of sales and has the purpose of encouraging salespeople to sell more goods or services. The percent of total sales paid as commission is the **commission rate**.

Example: Mr. Green sells $25,000 worth of computer equipment. His rate of commission is 1.4%. What is his commission on that sale?

$$\text{Commission} = \text{Sales} \times \text{Commission Rate}$$

Step 1: Change the percent to a decimal

$1.4\% = 0.014$

Step 2: Multiply the sales by 0.014

$25{,}000 \times 0.014 = 350.00$

Mr. Green's commission on his sales would be $350.00

Salary Plus Commission

Some people receive a fixed salary in addition to commission. The following example demonstrates how to find the total pay of someone who receives a fixed salary plus a commission on goods sold.

Example: Jerry sells auto parts. He receives a salary of $450.00 a month plus a commission of 7% of his total sales. What was his total pay for the month if he had sales totaling $4765.00?

Step1: Change 7% to a decimal.

7% = 0.07

Step 2: Find the commission.

Sales $\times$ commission rate = commission

4765 $\times$ 0.07 = 333.55 $\longrightarrow$ $333.55 commission

Step 3: Add the salary to the commission amount

450 + 333.55 = 783.55

Jerry's total pay for the month was $783.55

Sales Tax

When you make a purchase at the store, **sales tax** may be added to the total amount of the purchase. A sales tax is a percentage of the total sales and is collected on behalf of the state, county, or local government. The percentage of sales tax varies between states.

To find the amount of sales tax on your purchase you will multiply the amount of your purchase by the decimal value of the percent. If you are trying to find the total amount of the purchase then you want to **add** the sales tax to your purchase amount.

Example 1: Greg purchased a television set for $296.50. How much sales tax did he have to pay if the tax rate was 5%?

Think: 5% = 0.05

purchase price	×	rate	=	sales tax
296.50	×	0.05	=	14.83

Greg will have to pay $14.83 sales tax for his purchase.

Example 2: Amy purchased a computer totaling $598.50. In her state sales tax is 7%. Determine her final cost.

Step 1: Find the sales tax

Think: 7% = 0.07

purchase price	×	rate	=	sales tax
598.50	×	0.07	=	**41.90**

Step 2: Find the total cost of the purchase by adding the amount of sales tax to the purchase price.

598.50	+	41.90	=	**$640.40**

Amy's total price for her computer will be $640.40.

Simple Interest

Calculating interest is a very important application of percent. Interest is used when saving money through a financial institution. Interest is also used when making a loan from a bank which could be a house mortgage or a car loan.

We will look at the simple interest formula which is the basis for more complicated types of interest like compound interest or interest on car loans.

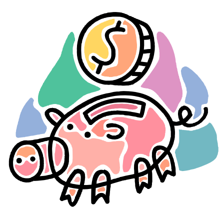
The simple interest formula is ***I = p r t***, where ***I*** represents ***i***nterest, ***p*** represents ***p***rincipal, ***r*** represents ***r***ate, and ***t*** represents ***t***ime.

Example: Find the interest on $2,500 at an annual interest rate of $6\frac{1}{2}\%$ for 18 months.

When we calculate the interest, rate and time must agree over the same type of time. In this problem since the interest rate is an annual yearly rate, the time must also be expressed in years.

Rate: $6\frac{1}{2}\% = 6.5\% = 0.065$ (express as decimal)

Time: 18 months $= \frac{18}{12} = 1.5$ years



$$I = p \cdot r \cdot t$$

$$I = 2500 \cdot 0.065 \cdot 1.5$$

$$I = \$243.75$$

The interest on $2500 for 18 months at $6\frac{1}{2}$ % is $243.75.

Compound Interest

If you deposit money in a savings account, the bank will pay you interest. It will most likely be *compound interest*. **Compound interest** is interest earned on both the principal and any interest that has been earned previously.

Compound interest is computed on the principal plus any interest already earned in a previous period. Interest may be compounded:

- annual (once a year)
- semiannually (twice a year)
- quarterly (four times in a year)
- monthly, daily, or continuously

To calculate compound interest, you are advised to use a **calculator** as the numbers grow large with several digits.

Example 1: Theresa's savings account pays 5% annual interest compounded quarterly. At the end of 1 year, find the total savings and how much interest Theresa will earn on $1,500.

NOTE: Each quarter is $\frac{1}{4}$ of a year. To multiply by $\frac{1}{4}$ on the calculator, divide by 4. You may also multiply by the decimal equivalent 0.25.

Step 1: **$18.75** = $1,500 × 0.05 ÷ 4	Use formula $I = p \times r \times t$ to find the interest at the end of the Quarter 1.
Step 2: $1,500 + $18.75 = $1,518.75	Add $18.75 to the principal.

$1,518.75 is your new principal.

To find the interest at the end of Quarter 2, use $I = p \times r \times t$ again using your new principal (**$18.984375** = $1,518.75 × .05 ÷ 4). Add $18.984375 to the new principal ($1518.75 + $18.984375 = **$1,537.734375**).

$1,537.734375 is now your new principal.

This process is then repeated for Quarter 3 and Quarter 4. Refer to the table below. Notice that at the end of each quarter, interest is computed and added to the account.

Quarter	Principal ($)	Compound Interest ($)	Total at End of Quarter ($)
1	1,500	1,500 × 0.05 ÷ 4 = **18.75**	1,500 + 18.75 = **1,518.75**
2	**1,518.75**	1,518.75 × 0.05 ÷ 4 = **18.984375**	1,518.75 + 18.984375 = **1,537.734375**
3	**1,537.734375**	1,537.734375 × 0.05 ÷ 4 = **19.22167969**	1,537.734375 + 19.22167969 = **1,556.956055**
4	**1,556.956055**	1,556.956055 × 0.05 ÷ 4 = **19.46195068**	1,556.956055 + 19.46195068 = **1,576.418006**

The savings total at the end of one year is $1,576.42.

Therefore, Theresa earns $76.42 in interest. ($1576.42 – $1500)

Example 2: Marla invested money in a certificate. At the end of 1 year, find the total savings and how much interest Marla will earn on \$175.

principal: \$175
annual rate: 6%
compounded semiannually
time: 1 year

NOTE: Semiannually is $\frac{1}{2}$ of a year or 0.5 yr. To multiply $\frac{1}{2}$ on your calculator, divide by 2. You may also multiply by the decimal equivalent 0.50.

Step 1: Calculate interest and new principal at the end of 6 months.

$$I = p \cdot r \cdot t$$

$$I = 175 \cdot 0.06 \cdot 0.5$$

$$I = \mathbf{5.25}$$

New Principal = Previous Principal + Interest

New Principal = 175 + 5.25

New Principal = **180.25**

Step 2: Calculate interest and principal at the end of the year.

$$I = p \cdot r \cdot t$$

$$I = 180.25 \cdot 0.06 \cdot 0.5$$

NOTE: The new principal is used to calculate the interest for the second half of the year.

$$I = \mathbf{5.41}$$

Rounded to nearest cent.

New Principal = Previous Principal + Interest

New Principal = 180.25 + 5.41

New Principal = **\$185.66**

Refer to the table below. Notice that at the end of every 6 months, interest is computed and added to the account.

Semiannually	**Principal ($)**	**Compound Interest ($)**	**Total at End of Every 6 months ($)**
0.5 year	175	$175 \times .06 \div 2 =$ **5.25**	$175 + 5.25 =$ **180.25**
1 year	**180.25**	$180.25 \times .06 \div 2 =$ **5.4075 = 5.41**	$180.25 + 5.41 =$ **185.66**

The certificate matures and is now valued at $185.66.

The interest earned for the year was $10.66. ($185.66 – 175.00)