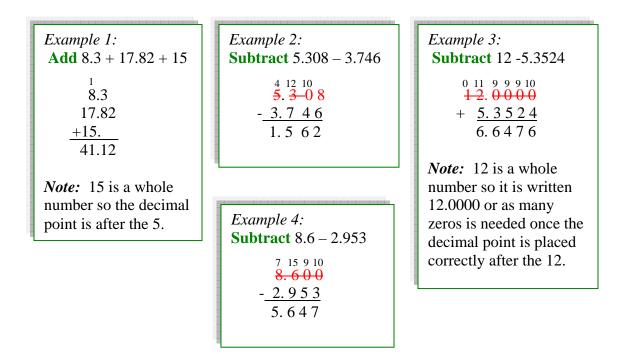# Decimals and Scientific Notation

In this lesson you will begin with a review of computations in decimals. You will also write decimals as fractions and vice versa. You will then write and compare rational numbers in scientific notation.

Add and Subtract Decimals
Multiply Decimals
Divide Decimals
Decimals to Fractions
Fractions to Decimals
Scientific Notation

Add and Subtract Decimals

To add and subtract decimals, be sure to **line up the decimal points** so that the place values will also line up – tenths with tenths, hundredths with hundredths, and so on.

Example 1:
Add 8.3 + 17.82 + 15

$$\begin{array}{r} \overset{1}{} \\ 8.3 \\ 17.82 \\ +15. \\ \hline 41.12 \end{array}$$

Note: 15 is a whole number so the decimal point is after the 5.

Example 2:
Subtract 5.308 – 3.746

$$\begin{array}{r} \overset{4\ \ 12\ \ 10}{} \\ 5.\,3\,0\,8 \\ -\ 3.7\ 4\ 6 \\ \hline 1.5\ 6\ 2 \end{array}$$

Example 4:
Subtract 8.6 – 2.953

$$\begin{array}{r} \overset{7\ \ 15\ 9\ 10}{} \\ 8.\,6\,0\,0 \\ -\ 2.9\ 5\ 3 \\ \hline 5.\,6\,4\,7 \end{array}$$

Example 3:
Subtract 12 -5.3524

$$\begin{array}{r} \overset{0\ \ 11\ \ 9\ 9\ 9\ 10}{} \\ 12.\,0\,0\,0\,0 \\ +\ 5.\,3\,5\,2\,4 \\ \hline 6.\,6\,4\,7\,6 \end{array}$$

Note: 12 is a whole number so it is written 12.0000 or as many zeros is needed once the decimal point is placed correctly after the 12.

Check through estimation.

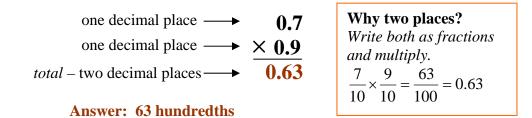|		
1. 8 + 18 + 15 = 41	Actual Answer is 41.12.	Estimate is very close.
2. 5 – 4 = 1	Actual Answer is 1.562.	Estimate is close.
3. 12 – 5 = 7	Actual Answer is 6.6476.	Estimate is close.
4. 9 – 3 = 6	Actual Answer is 5.657.	Estimate is close.

Multiply Decimals

Multiplying Decimals Less Than One

To place the decimal point when multiplying decimals, count the decimal places in each factor and total them. The total is the number of decimal places that will be in the product (answer to a multiplication problem).

Multiply 0.7 × 0.9   *Estimate: 1 × 1 = 1*

one decimal place $\longrightarrow$ **0.7**
one decimal place $\longrightarrow$ **× 0.9**
total – two decimal places $\longrightarrow$ **0.63**

Why two places?
Write both as fractions and multiply.
$$\frac{7}{10} \times \frac{9}{10} = \frac{63}{100} = 0.63$$

Answer: 63 hundredths

Check: The estimate, 1, is close to 0.63.

Multiply 0.12 × 0.36   *Estimate: 0 × 0 = 0*

two decimal places $\longrightarrow$ **0.12**
two decimal places $\longrightarrow$ **× 0.36**
72
36
total – four decimal places $\longrightarrow$ **0.0432**

Why four places?
Write both as mixed fractions and multiply.
$$\frac{12}{100} \times \frac{36}{100} = \frac{432}{10000} = 0.0432$$

Use 0 as a place holder before the numerals to give 4 decimal places.

Answer: 432 ten thousandths

Check: The estimate, 0, is close to 0.0432.

More Decimal Multiplication

To place the decimal point when multiplying decimals, count the decimal places in each factor and total them. The total is the number of decimal places that will be in the product (answer to a multiplication problem).

Multiply 5.23 × 7.9   *Estimate: 5 × 8 = 40*

two decimal places ⟶ **5.23**
one decimal place ⟶ **× 7.9**
$$\overline{\textbf{4707}}$$
3661
total – three decimal places ⟶ **41.317**

Answer: 41 and 317 thousandths

Check: The estimate, 40, is close to 41.317.

Why three places?
Write both as mixed fractions and multiply.

$$5\frac{23}{100} \times 7\frac{9}{10} =$$
$$\frac{523}{100} \times \frac{79}{10} = \frac{41317}{1000} =$$
$$41\frac{317}{1000} = 41.317$$

Multiply 46 × 2.8   *Estimate: 50 × 3 = 150*

zero decimal places ⟶ **46**
one decimal place ⟶ **× 2.8**
368
92
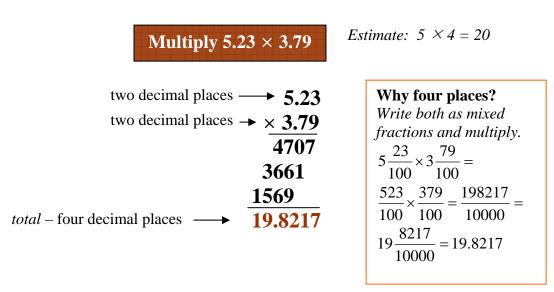*total – one decimal place* ⟶ **128.8**

Answer: 128 and 8 tenths

Check: The estimate, 150, is close to 128.8.

Why one decimal place?
Write both as mixed fractions and multiply.

$$\frac{46}{1} \times 2\frac{8}{10} =$$
$$\frac{46}{1} \times \frac{28}{10} = \frac{1288}{10} = 128.8$$

Extending Decimal Multiplication

To place the decimal point when multiplying decimals, count the decimal places in each factor and total them. The total is the number of decimal places that will be in the product (answer to a multiplication problem).
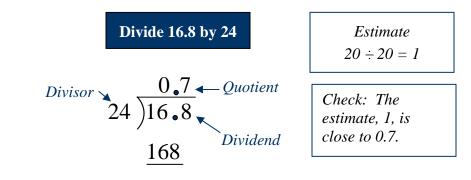
Multiply 5.23 × 3.79

Estimate: 5 × 4 = 20

two decimal places ⟶ **5.23**
two decimal places ➝ **× 3.79**
4707
3661
1569
total – four decimal places ⟶ **19.8217**

Why four places?
Write both as mixed fractions and multiply.

$$5\frac{23}{100} \times 3\frac{79}{100} =$$

$$\frac{523}{100} \times \frac{379}{100} = \frac{198217}{10000} =$$

$$19\frac{8217}{10000} = 19.8217$$

Answer: 19 and 8217 ten thousandths

Check: The estimate, 20, is close to 19.8217.
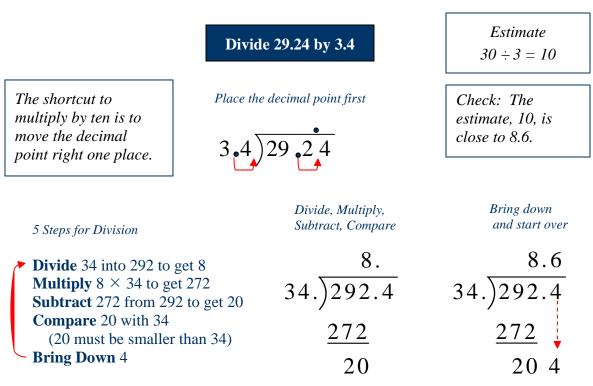
Divide Decimals

Dividing Decimals by Whole Numbers

To divide a decimal by a whole number, **place the decimal point** in the *quotient* right **above the decimal** in the *dividend*.

Divide 16.8 by 24

Divisor → $24 \overline{)16.8}$ ← *Quotient* 0.7

Dividend

168

Estimate
$20 \div 20 = 1$

Check: The estimate, 1, is close to 0.7.

Dividing By Decimals in Tenths

To divide a decimal by a decimal number, make the divisor a whole number by multiplying it by the power of ten needed to **move the decimal to the right of all of the digits in the divisor**, then multiply the dividend by the same power of ten.

Divide 29.24 by 3.4

Estimate
$30 \div 3 = 10$

The shortcut to multiply by ten is to move the decimal point right one place.

Place the decimal point first

$3.4 \overline{)29.24}$

Check: The estimate, 10, is close to 8.6.

5 Steps for Division

Divide 34 into 292 to get 8
Multiply 8 × 34 to get 272
Subtract 272 from 292 to get 20
Compare 20 with 34
   (20 must be smaller than 34)
Bring Down 4

Divide, Multiply, Subtract, Compare

$$34.\overline{)292.4} \quad 8.$$
$$\underline{272}$$
$$20$$

Bring down and start over

$$34.\overline{)292.4} \quad 8.6$$
$$\underline{272}$$
$$20\ 4$$
$$\underline{20\ 4}$$

Dividing By Decimals in Hundredths

To divide a decimal by a decimal number, make the divisor a whole number by multiplying it by the power of ten needed to **move the decimal to the right of all of the digits in the divisor**, then multiply the dividend by the same power of ten.
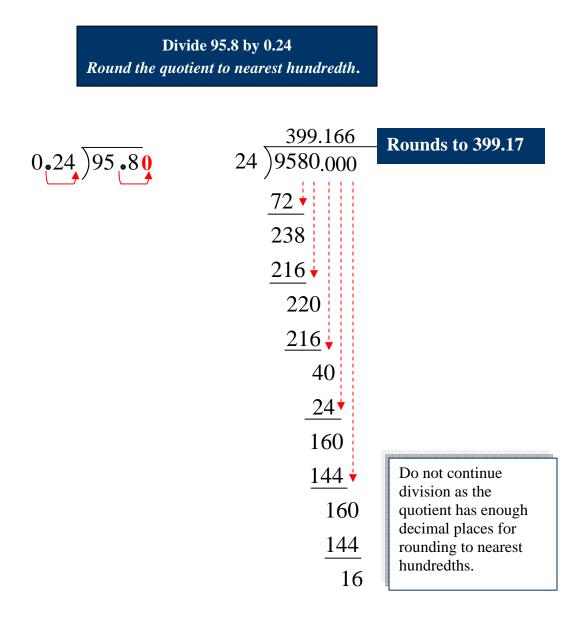
Divide 8.0124 by 1.32

$$1.32\,\overline{)8.01\,24}$$

$$\frac{8.0124}{1.32} \times \frac{100}{100} = \frac{801.24}{132}$$

$$
\begin{array}{r}
6. \\
132.\overline{)801.24} \\
\underline{792} \\
9\,2
\end{array}
\qquad
\begin{array}{r}
6.0 \\
132.\overline{)801.24} \\
\underline{792} \\
9\,2 \\
\underline{0}
\end{array}
\qquad
\begin{array}{r}
6.07 \\
132.\overline{)801.24} \\
\underline{792} \\
9\,2 \\
\underline{0} \\
924 \\
\underline{924}
\end{array}
$$

Dividing Decimals and Rounding Quotients

In division, sometimes the answer does not come out even. That's when we divide until the quotient has one extra decimal place, then round to the given place.

Divide 95.8 by 0.24
Round the quotient to nearest hundredth.

$$0.24 \overline{)95.80}$$

$$
\begin{array}{r}
399.166 \\
24 \overline{)9580.000} \\
\underline{72} \\
238 \\
\underline{216} \\
220 \\
\underline{216} \\
40 \\
\underline{24} \\
160 \\
\underline{144} \\
160 \\
\underline{144} \\
16
\end{array}
$$

Rounds to 399.17

Do not continue division as the quotient has enough decimal places for rounding to nearest hundredths.

Decimals to Fractions

Decimals may be written as fractions, and then simplified when possible.

4.53 is read as 4 and 53 hundredths which is $4\frac{53}{100}$.

Thus, $4.53 = 4\frac{53}{100}$

(Notice, <mark>two decimal places</mark> gives <mark>two zeros</mark> in the denominator of the fraction.)

7.5 is read as 7 and 5 tenths which is $7\frac{5}{10}$.

(Notice, <mark>one decimal place</mark> gives <mark>one zero</mark> in the denominator of the fraction.)

$$7\frac{5}{10} \div \frac{5}{5} = 7\frac{1}{2}$$

Thus, $7.5 = 7\frac{1}{2}$

6.250 is read as 6 and 250 thousandths which is $6\frac{250}{1000}$ and reduces to $6\frac{1}{4}$.

(Notice, <mark>three decimal places</mark> gives <mark>three zeros</mark> in the denominator of the fraction.)

$$6\frac{250}{1000} \div \frac{10}{10} = 6\frac{25}{100}$$
$$6\frac{25}{100} \div \frac{25}{25} = 6\frac{1}{4}$$

Thus, $6.250 = 6\frac{1}{4}$

Fractions to Decimals

To write a fraction as a decimal, you can divide to find the decimal.

Example 1: Find the decimal for 3/4.

To find the decimal for $\dfrac{3}{4}$, divide the denominator into the numerator and then add a decimal point and zeros until it comes out even.

$$\begin{array}{r} .75 \\ 4\overline{)3.00} \\ \underline{28} \\ 20 \\ \underline{20} \end{array}$$

$$\dfrac{3}{4} = 0.75$$

Some fractions do not make decimals that come out even and continue on forever when dividing. For them, divide and round to the given place.

Example 2: Find the decimal for 2/3 and round to nearest hundredth.

To find the decimal for $\dfrac{2}{3}$, divide to get one extra decimal place for rounding (in this case divide through thousandths), stop, and then round.

$$\begin{array}{r} .666 \\ 3\overline{)2.000} \\ \underline{18} \\ 20 \\ \underline{18} \\ 20 \\ \underline{18} \\ 2 \end{array}$$

$$\dfrac{2}{3} \approx 0.67$$

Sometimes repeating decimals are expressed with a bar over the repeating pattern in the decimal. Two - thirds may be written as $0.\overline{6}.$

Scientific Notation

Standard Form for Numbers in Scientific Notation

> **When a number is expressed in scientific notation, it is written as a product of two parts:**
> - **A number that is less than 10, but greater than or equal to 1**
> - **A power of ten.**
>
> **Ex. 56,000 expressed as scientific notation is 5.6×10^4**

Let's express a number given in scientific notation as a standard number.

Find the standard notation for 7×10^4.

Solution:
$10^4 = 10 \times 10 \times 10 \times 10 = 10,000$

Therefore, $7 \times 10,000 = 70,000$

The standard notation for 7×10^4 is 70,000.

Find the standard notation for 8.4×10^5.

Solution:
$10^5 = 10 \times 10 \times 10 \times 10 \times 10 = 100,000$

Therefore, $8.4 \times 100,000 = 840,000$

The standard notation for 8.4×10^5 is 840,000.

A shortcut to multiply by a power of ten is to start at the decimal point's location and move it **right** as many places as the given power.
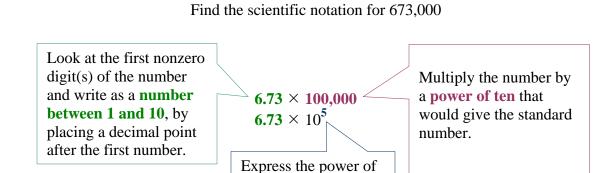
$8.4 \times 10^5 \rightarrow 840000 \rightarrow 840,000.$

Express Large Numbers in Scientific Notation

When a number is expressed in scientific notation, it is written as a product of two parts:
- **A number that is less than 10, but greater than or equal to 1**
- **A power of ten.**

Ex. 25,000 expressed as scientific notation is 2.5×10^4

Find the scientific notation for 673,000

Look at the first nonzero digit(s) of the number and write as a **number between 1 and 10**, by placing a decimal point after the first number.

$6.73 \times 100,000$
6.73×10^5

Multiply the number by a **power of ten** that would give the standard number.

Express the power of ten in exponent form.

A shortcut for finding the **exponent** is to **count** the decimal places from **after the first digit** to the **end (right) of the number**; thus, a positive exponent.
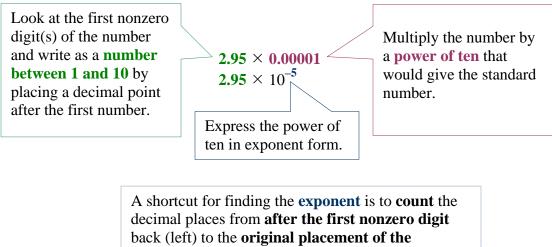
$673,000 \rightarrow 673000 \rightarrow 6.73 \times 10^5$

Express Small Numbers in Scientific Numbers

When a number is expressed in scientific notation, it is written as a product of two parts:
- **A number that is less than 10, but greater than or equal to 1.**
- **A power of ten.**

Ex. 0.00347 expressed as scientific notation is 3.47×10^{-3}

Find the scientific notation for 0.0000295

Look at the first nonzero digit(s) of the number and write as a **number between 1 and 10** by placing a decimal point after the first number.

2.95×0.00001
2.95×10^{-5}

Multiply the number by a **power of ten** that would give the standard number.

Express the power of ten in exponent form.

A shortcut for finding the **exponent** is to **count** the decimal places from **after the first nonzero digit** back (left) to the **original placement of the decimal**; thus, a negative exponent.

$0.0000295 \rightarrow 0.0000295 \rightarrow 2.95 \times 10^{-5}$

$0.0000295 = 2.95 \times 10^{-5}$