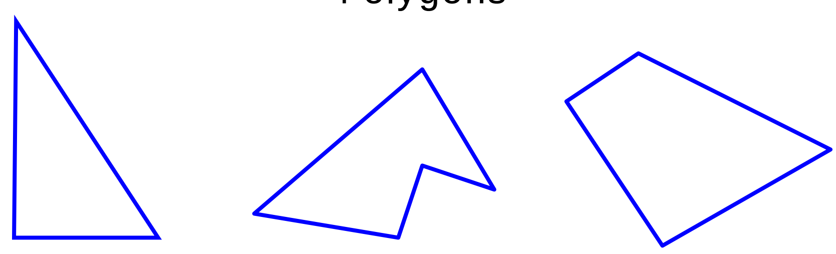
POLYGONS AND SOLIDS

This unit is about polygons such as triangles, quadrilaterals, and multi-sided polygons. In addition, the properties of solids are examined. Nets are used to show how to lay out solids in two dimensions. The unit concludes with solving problems in a logical and systematic method.

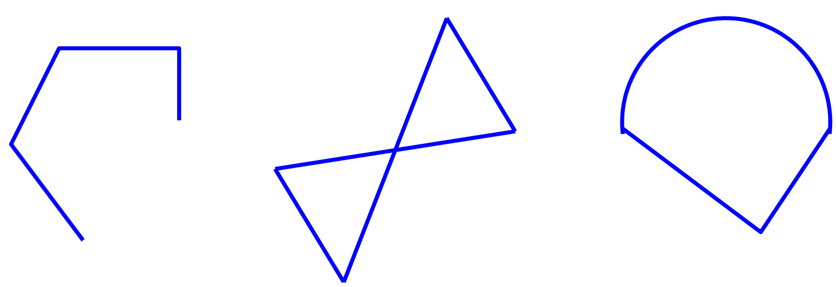
Polygons

polygon - A polygon is a closed figure made up of line segments that lie in the same plane and each side of the polygon intersects with two other sides at its endpoints.

Polygons

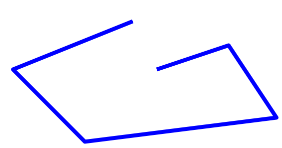


Not Polygons



Path is open.	Two sides intersect at a point other than the endpoints.	One side is not a line segment.

Example 1: Is the shape a polygon? Explain why or why not.



The shape is not a polygon because the path is open.

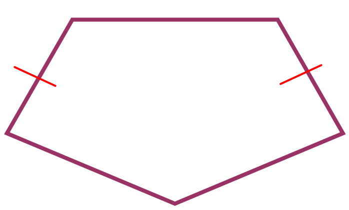
In general a polygon with “n” sides is called an “n-gon”.   The names of several common polygons and their number of sides are listed in the table below.

Number of Sides	**Polygon**
3	triangle
4	quadrilateral
5	pentagon
6	hexagon
7	heptagon
8	octagon
9	nonagon
10	decagon
12	dodecagon
n	*n*-gon

regular polygon – A regular polygon is a convex polygon with all sides and angles congruent.

Example 2: Name the polygon, and then state whether it is regular or not. Explain the answer.

*Note – The red hash marks designate congruent sides.



The polygon is a pentagon because it has 5 sides.

It is NOT a regular pentagon because all sides are not congruent.

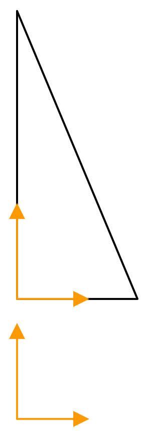
Triangles

Classifying Triangles by Types of Angles

A triangle may be classified by the types of angles that are formed by its three sides.

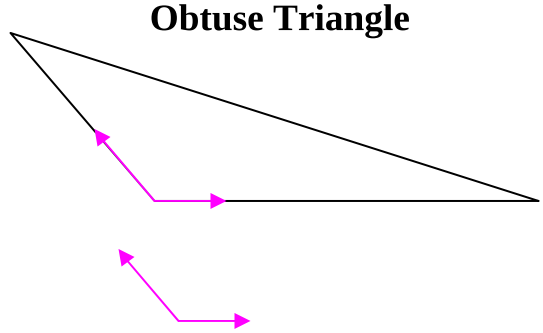
- A **right** triangle is a triangle that has ONE right angle.
- An **obtuse** triangle is a triangle that has ONE obtuse angle.
- An **acute** triangle is a triangle with ALL acute angles.

Right Triangle



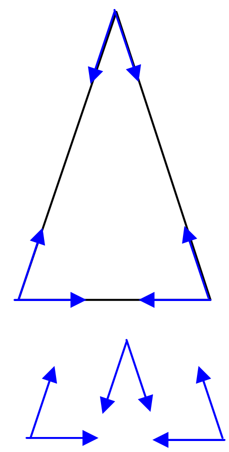
right angle (90°)

Obtuse Triangle



obtuse angle (greater than 90°)

Acute Triangle



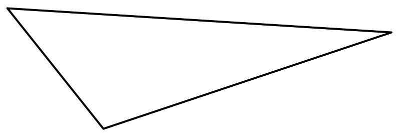
All are acute angles, angles that measure less than 90°.

Classifying Triangles by Length of Sides

A triangle may be classified by the lengths of its three sides.

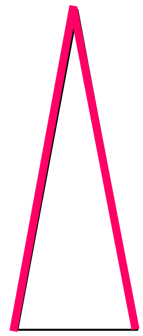
- A **scalene** triangle is a triangle where **all three sides** measure **different** lengths.
- An **isosceles** triangle is a triangle where **two sides** are the **same** length.
- An equilateral triangle is a triangle where **ALL three sides** are the **same length**.

Scalene Triangle



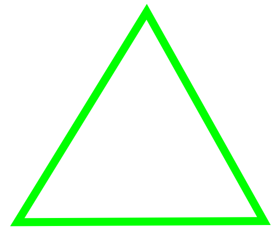
All sides are different lengths.

Isosceles Triangle



Two sides are the same lengths.

Equilateral Triangle

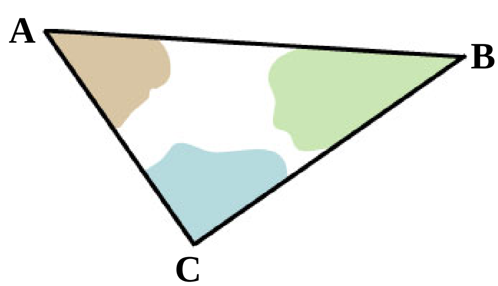


All three sides are the same lengths.

*Note: In a scalene triangle, all of the angles are different in measure. In an isosceles triangle, two of the angles are congruent. An equilateral triangle is equiangular; that is, all the angles are congruent.

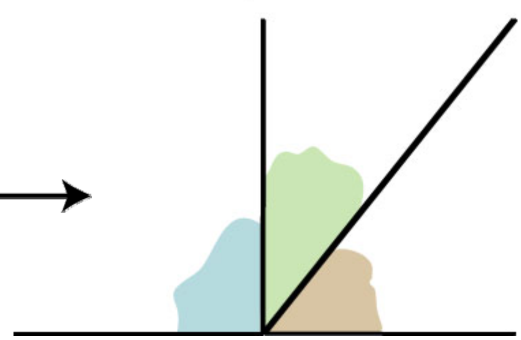
Sum of the Angles in a Triangle

Look at the angles in the triangle below. Cut the angles away from the triangle.



Place the angles side by side at one vertex point so that there are no gaps between the angles.

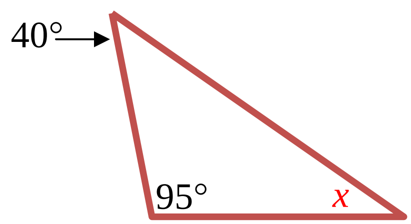
180° angle



The outer rays of three angles of the triangle together make up a 180° angle, a straight angle.

In a triangle, the three angles will always total 180degrees.

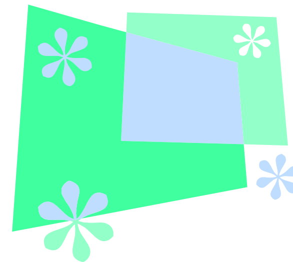
Example: If two angles in a triangle measure 95° and 40°, what is the measure of the third angle?



$95+40+x=180$	The sum of the three angles equals 180 degrees.
$135+x=180$	Simplify. $95+40=135$
$135+x=180$	
$-135 \quad -135$	Subtract 135 from both sides of the equation.
$x=45$	

The third angle of the triangle measures 45 degrees.

Quadrilaterals



quadrilateral - A quadrilateral is a polygon with four sides.

Some quadrilaterals are given other names because of the special angles and line segments that make up the shape.

Specific quadrilaterals are shown below and their properties are listed within the shape.

Trapezoid

- One pair of opposite sides are parallel.

Parallelogram

- Both pairs of opposite sides are parallel.
- Both pairs of opposition sides are congruent.

Rectangle

- Both pairs of opposite sides are parallel.
- Both pairs of opposite sides are congruent.
- All four angles are right angles.

Rhombus

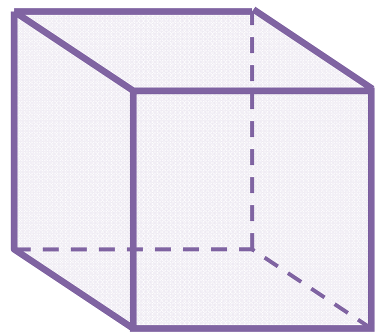
- Both pairs of opposite sides are parallel.
- All sides are congruent.

Square

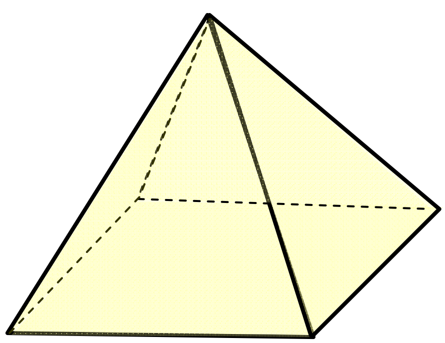
- Both pairs of opposite sides are parallel.
- All four sides are congruent.
- All four angles are right angles.

Solids

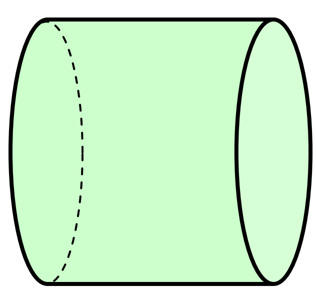
cube – A cube is a prism in which all of its faces are congruent squares.



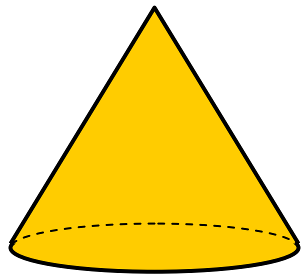
pyramid - A pyramid is a polyhedron where all of the triangular faces meet at a point and the base is either a square or a rectangle.



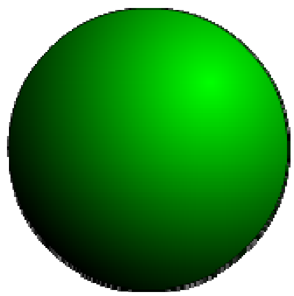
cylinder – A cylinder is a solid with congruent circular bases and a curved rectangle as its lateral face.



cone – A cone is a solid that has one circular base, a vertex point that does not lie in the same plane as the base, and a curved lateral surface area that is made up of all points that lie on segments connecting the vertex and the edge of the base.



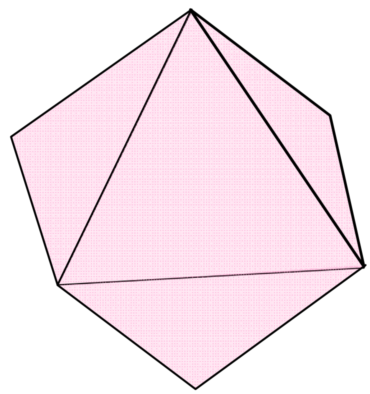
sphere – A sphere is a set of points in space that are ALL equidistant from a given point.



Three-Dimensional Figures

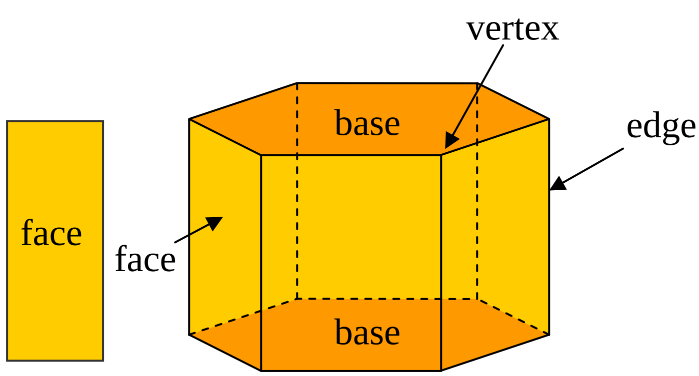
polyhedron – A polyhedron is a solid with all flat faces that enclose a single region of space.

Polyhedron



polyhedra – Polyhedra is the plural of polyhedron.

prism – A prism is a polyhedron with two congruent bases that are polygons contained in parallel planes.



faces – The faces of a prism are its flat surfaces.

bases of a prism – The bases of a prism are its congruent faces that are polygons contained in parallel planes.

edges – Edges are the line segments formed at the intersection of faces.

vertices (plural of vertex) – Vertices are the points where edges intersect.

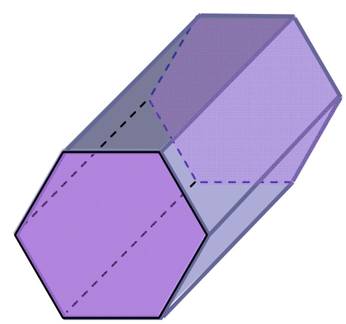
Example: How many faces, edges, and vertices does the hexagonal prism shown below have?

*Note: The name of this solid (hexagonal prism) is based on the two bases that are hexagons.

The hexagonal prism has **8** faces.

2 hexagonal bases (front and back)
\+ **6** faces that connect the two hexagonal bases (sides)

The hexagonal prism has **18** edges.



6 edges that connect the faces of the body
\+ **6** edges that connect one base to the faces of the body
\+ **6** edges that connect the other base to the faces of the body

The hexagonal prism has **12** vertices.

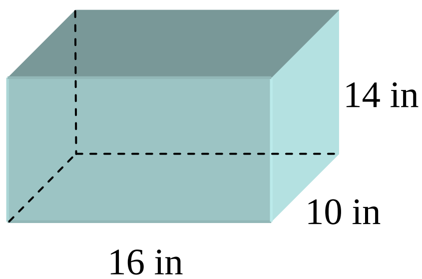
6 vertices that form at the connection of two edges of the body with a base edge
6 vertices that form at the connection of two edges of the body with the edge of the other base.

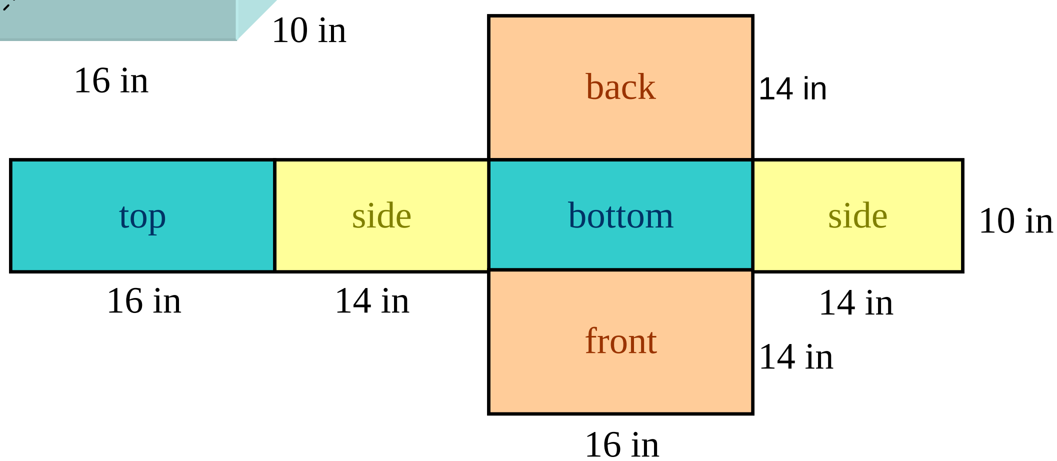
Nets

A **net** is a two-dimensional representation of a solid.

Rectangular Prism

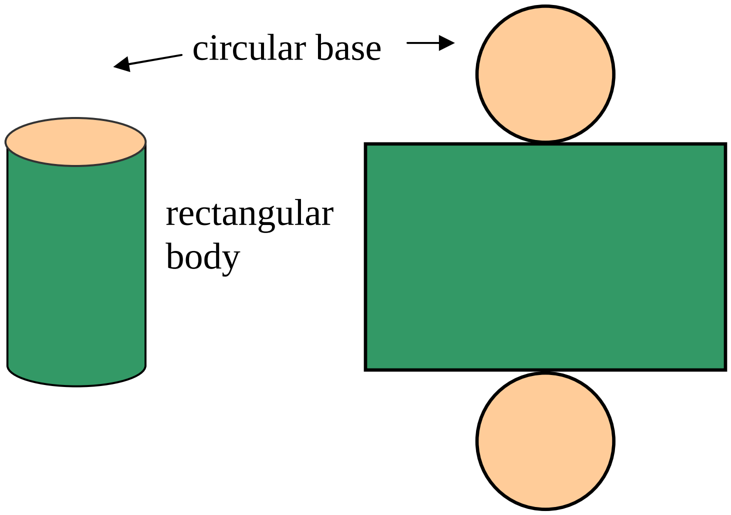
A rectangular prism and its net are shown below.





Cylinder

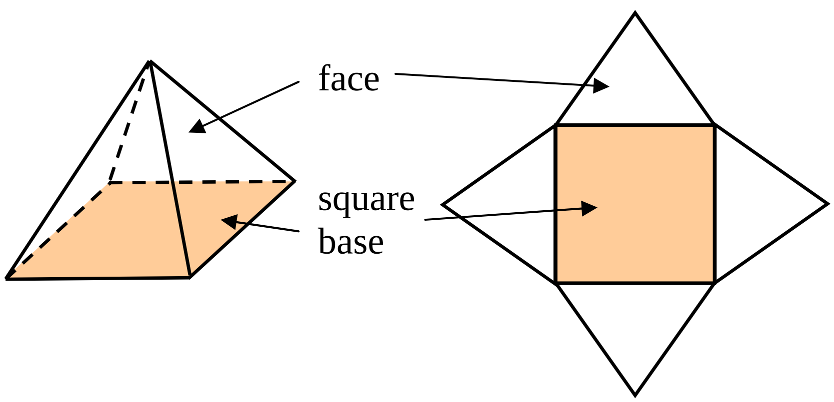
A cylinder and its net are shown below.



*Notice that the curved body is simply a rectangle when lain flat. The two bases are circles.

Pyramid

A pyramid and its net are show below.



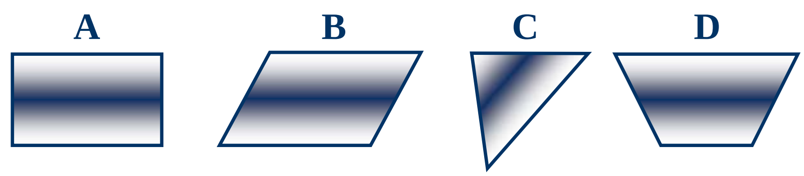
*Notice that the pyramid has a square base (sometimes it's rectangular) and four faces that are each triangular in shape.

Use Logical Reasoning

Many types of problems may be solved by using logical reasoning. There are four basic steps involved in using this strategy.

- Explore the problem.
- Plan the problem.
- Solve the problem.
- Examine the answer.

Example: Which figure does not belong in the group below?



Step 1: Explore the problem.

What is given? → Four different polygons

What is asked? → Which polygon does not belong?

Step 2: Plan the problem.

Look at the polygons and decide:

- What characteristics do polygons have in common?
- What characteristic distinguishes one polygon from the rest of the group?
- Use logical reasoning.

Step 3: Solve the problem.

- All figures are about the same size.
- All figures are the same color pattern.
- All of the figures are polygons.
- Both pairs of opposite sides are parallel for figures A and B only.
- Figures A, B, and D have four sides.
- Figure C has three sides.

Therefore, Figure C does not belong in the group.

Step 4: Examine the answer.

All four figures are polygons and have the same color pattern. There is only one figure that has three sides. Therefore, the answer is reasonable.