MULTIPLY FRACTIONS

This unit is about multiplication of fractions and mixed numbers. The technique of cancelling will also be discussed to show a way to simplify computations.

Estimate Products

Modeling Multiplication of Fractions

Mixed Numbers (Review)

Multiplication of Fractions

Estimate Products

When two numbers are multiplied together, the result is called the **product** of the two numbers.

Estimation is used to approximate an answer to a math problem when an exact answer is not required.

Example 1: Mr. Saddler is building a rectangular fence to enclose the field for his new horses. After measuring the length and width of the field, he found that the length of the field was $89\frac{7}{9}$ meters and the width was $75\frac{1}{5}$ meters.

Mr. Saddler's neighbor asked, "*About* how much area will the horses have to roam?"

Since his neighbor asked *about how much*, Mr. Saddler used estimation to answer his neighbor's question.

> *Step 1*: Determine the formula to use for finding the area of the field, and then set up the problem.
>
> $A = l \times w$
>
> $A = 89\frac{7}{9} \times 75\frac{1}{5}$

Step 2: Round each mixed number to the nearest whole number.

For $89\frac{7}{9}$ find the fraction that represents 1/2 in 9ths.

$\frac{1}{2}$ of 9 is 4.5 $\qquad 2\overline{)9.0}^{\,4.5} \qquad$ so, $\frac{1}{2} = \frac{4.5}{9}$

Since $\frac{7}{9}$ is greater than $\frac{4.5}{9}$, $89\frac{7}{9}$ rounds up to 90.

For $75\frac{1}{5}$ find the fraction that represents 1/2 in 5ths.

$\frac{1}{2}$ of 5 is 2.5 $\qquad 2\overline{)5.0}^{\,2.5} \qquad$ so, $\frac{1}{2} = \frac{2.5}{5}$

Since $\frac{1}{5}$ is less than $\frac{2.5}{5}$, $75\frac{1}{5}$ rounds to 75.

Step 3: Solve the problem using the rounded numbers.

$A = l \times w$

$A \approx 90 \times 75$ $\qquad \approx$ is the symbol for "approximately equal to"

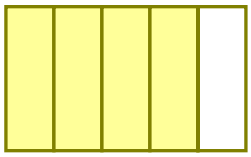
$A \approx 6750$

The field for the horses to roam will have an area of about 6750 square meters.

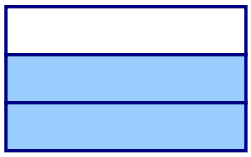
Modeling Multiplication of Fractions

In the model below, the fraction 4/5 is represented in Rectangle A (yellow) and 2/3 is represented in Rectangle B (blue).

Rectangle A $\frac{4}{5}$

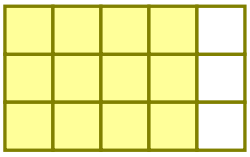


Rectangle B $\frac{2}{3}$



To show multiplication through the models, each rectangle is divided into 15ths, since 15 is a common denominator of 5 and 3.

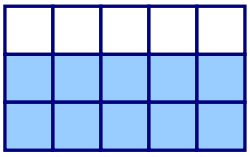
Rectangle A $\frac{4}{5}=\frac{12}{15}$



$\frac{4}{5}$

$\times$

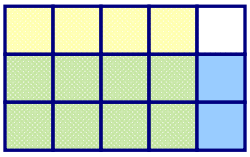
Rectangle B $\frac{2}{3}=\frac{10}{15}$



$\frac{2}{3}$

$=$

Rectangle A$\times$ Rectangle B*



$\frac{8}{15}$

The *eight* green squares show the areas that overlap between Rectangle A and Rectangle B; thus, the overlapping area (8/15) represents the product of 4/5 and 2/3.

$$\frac{4}{5}\times\frac{2}{3}=\frac{4\times 2}{5\times 3}=\frac{8}{15}$$

Mixed Numbers

Mixed numbers are numbers that have a whole number and a fraction.

Examples of Mixed Numbers: $2\frac{2}{3}$ $7\frac{5}{8}$ $29\frac{3}{4}$

Improper fractions are fractions where the numerator is larger than the denominator.

Examples of Improper Fractions: $\frac{8}{3}$ $\frac{61}{8}$ $\frac{119}{4}$

When working with fractions, it is necessary to know how to convert mixed numbers to improper fractions and vice versa.

Changing Mixed Numbers to Improper Fractions

Example 1: Express $1\frac{5}{12}$ as an improper fraction.

$$1\frac{5}{12} = 1+\frac{5}{12} = \frac{12}{12}+\frac{5}{12} = \frac{12+5}{12} = \frac{17}{12}$$

*A quick way to find the improper fraction is to multiply the denominator by the whole number, and add on the numerator. Then, place that number over the denominator.

$$1\frac{5}{12} = \frac{12 \times 1 + 5}{12} = \frac{12+5}{12} = \frac{17}{12}$$

The improper fraction for 1 5/12 is 17/12.

Example 2: Express $2\frac{4}{9}$ as an improper fraction.

$$2\frac{4}{9} = \frac{9 \times 2 + 4}{9} = \frac{18+4}{9} = \frac{22}{9}$$

The improper fraction for 2 4/9 is 22/9.

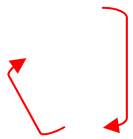
Changing Improper Fractions to Mixed Numbers

Example 3: Express $\frac{13}{10}$ as a mixed fraction.

Think of $\frac{13}{10}$ as $\frac{10}{10}+\frac{3}{10}$, then as $1+\frac{3}{10}$ because $\frac{10}{10}=1$, then as $1\frac{3}{10}$.

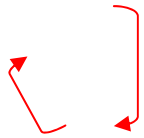
In this problem, the whole number is 1 ($\frac{10}{10}$) and the remaining part is $\frac{3}{10}$.

*A quick way to find the mixed number is to divide the numerator by the denominator and express the remainder as a fraction.



The mixed number for 13/10 is 1 3/10.

Example 4: Express $\frac{27}{15}$ as a mixed fraction.



The fraction must be simplified.

$$1\frac{12}{15} = 1\frac{12 \div 3}{15 \div 3} = 1\frac{4}{5}$$

The mixed number for 27/15 is 1 4/5.

Multiplication of Fractions

Multiplying Fractions

To multiply fractions, multiply the numerators and multiply the denominators. Simplify the fractions when necessary.

numerator – A numerator is the top part of a fraction. In the fraction 2/3, the numerator is two $\left(\frac{2}{3}\right)$.

denominator – A denominator is the bottom part of a fraction. In the fraction 2/3, the denominator is three $\left(\frac{2}{3}\right)$.

proper fraction – A proper fraction is a fraction where the numerator is less than the denominator. An example of a proper fraction is $\frac{7}{8}$.

*Recall that the answer to a multiplication problem is called the **product**.

Example 1: Find the product of the proper fractions, 2/3 × 8/9.

$$\frac{2}{3}\times\frac{8}{9} = \frac{2\times 8}{3\times 9} = \frac{16}{27}$$

Multiply the numerators.
Multiply the denominators.

Multiplication of fractions can be made easier by using canceling to simplify first, and then multiplying the numerators and the denominators.

Canceling

Look for a numerator and a denominator that will simplify.

Example 2: Find the product of proper fractions, 3/4 × 8/11.

In canceling, one number must be in the numerator and the other number must be in the denominator.

$$\frac{3}{4}\times\frac{8}{11} = \frac{3}{\cancel{4}_{1}}\times\frac{\cancel{8}^{2}}{11} = \frac{3\times 2}{1\times 11} = \frac{6}{11}$$

Cancel the 4 and 8 by 4.

Example 3: Find the product of 2/3 of 9.

Make the whole number 9 a fraction by placing it over 1. 9 = 9/1.

$$\frac{2}{3}\times\frac{9}{1} = \frac{2}{\cancel{3}_{1}}\times\frac{\cancel{9}^{3}}{1} = \frac{2\times 3}{1\times 1} = \frac{6}{1} = 6$$

Cancel the 3 and 9 by 3.

Multiplying Mixed Numbers

improper fraction – An improper fraction is a fraction where the numerator is larger than or equal to the denominator. An example of an improper fraction is $\frac{12}{5}$.

mixed number – A mixed number is a number that is a combination of a whole number and a fraction. An example of a mixed number is $2\frac{2}{5}$.

*To multiply mixed numbers, first change the mixed numbers to improper fractions.

Example 4: Find the improper fractions for 1 1/11 and 2 4/9.

*Multiply the denominator by the whole number, and then add on the numerator. Put that number over the denominator.

$$1\frac{1}{11}=\frac{11\times1+1}{11}=\frac{12}{11} \qquad 2\frac{4}{9}=\frac{9\times2+4}{9}=\frac{22}{9}$$

Example 5: Find the product of the mixed numbers, 1 1/11 × 2 4/9.

In the previous problem, the two mixed numbers are expressed as improper fractions. (1 1/11 = 12/11 and 2 4/9 = 22/9)

$$\frac{12}{11}\times\frac{22}{9} = \frac{\cancel{12}^{4}}{\cancel{11}_{1}}\times\frac{\cancel{22}^{2}}{\cancel{9}_{3}} = \frac{8}{3} = 2\frac{2}{3}$$

Cancel the 11 and 22 by 11.

Cancel the 12 and 9 by 3. (Think of a number that will divide into 12 and 9 evenly. That number is 3.

Multiplying Multiple Fractions

Example 6: Find the product of the proper fractions 9/16 × 5/8 × 2/3.

Simplify through canceling, and then multiply the numerators and denominators.

*With multiple fractions, cancel any numerator with any denominator.

Look for a numerator and a denominator that will simplify.

$$\frac{\cancel{9}^{3}}{20}\times\frac{5}{8}\times\frac{2}{\cancel{3}^{1}}$$

First cancel the 9 and 3.

$$\frac{\cancel{9}^{3}}{\cancel{20}^{4}}\times\frac{\cancel{5}^{1}}{8}\times\frac{2}{\cancel{3}^{1}}$$

Now cancel the 5 and 20.

$$\frac{\cancel{9}^{3}}{\cancel{20}^{4}}\times\frac{\cancel{5}^{1}}{\cancel{8}^{4}}\times\frac{\cancel{2}^{1}}{\cancel{3}^{1}}$$

Last cancel the 2 and 8.

$$\frac{\cancel{9}^{3}}{\cancel{20}^{4}}\times\frac{\cancel{5}^{1}}{\cancel{8}^{4}}\times\frac{\cancel{2}^{1}}{\cancel{3}^{1}}=\frac{3}{16}$$

Multiply the cancelled numerators and denominators.