Required Materials for Math Integrated Math I
Please print or save this document for future reference.

There are practice worksheets in many of the units that provide more practice on specific topics. The review worksheets are provided to give extra practice in skill areas presented in the unit. The worksheets are optional unless otherwise specified by the instructor. The worksheets are Adobe Acrobat files. Click on the pencil icon to open the document. Save the document to a folder on the computer, and then enter answers for the problems in the textboxes. Once the document is completed, make sure to SAVE it again, and then send the document to the instructor via email. The answer key provided is for the instructors only and is password protected.

SURFACE AREA

This unit is about three-dimensional geometric shapes that will be referred to as solids. Each solid has a two-dimensional net that can be used to construct the solid. Formulas will be developed and used to find the surface area of the following solids: rectangular prisms, cubes, other prisms named by their bases, cylinders, and pyramids.

Surface Area of a Rectangular Prism

Surface Area of a Cube

Surface Area of a Triangular Prism

Surface Area of a Pyramid

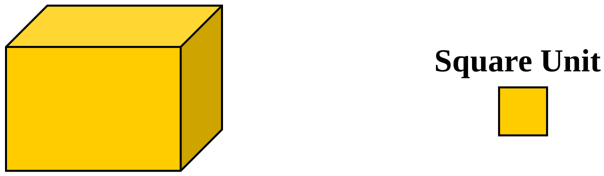
Surface Area of a Cylinder

Surface Area of a Cone

Surface Area of a Sphere

Surface Area of a Rectangular Prism

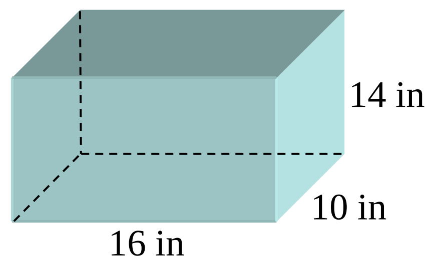
The **surface area** of a solid is the sum of the areas of all surfaces of a figure. Surface area is measured in square units.

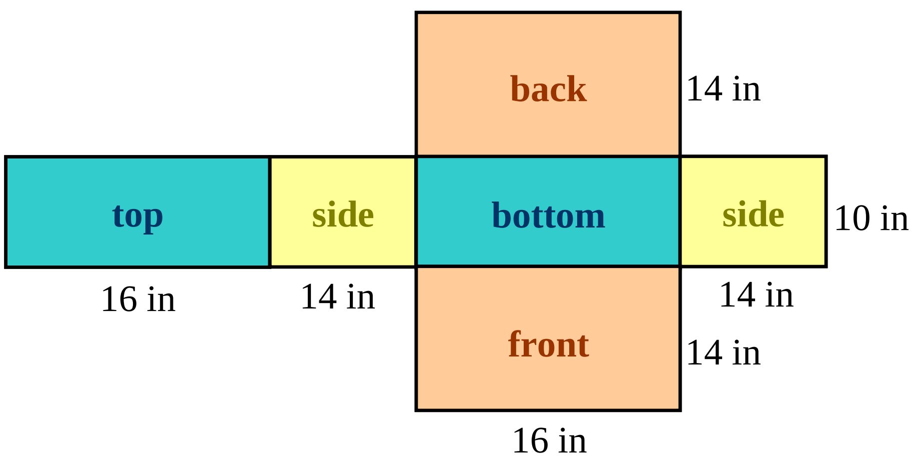


A **net** is a two-dimensional representation of a solid. The surface area of a solid is equal to the area of its net.

Example: Find the surface area of a rectangular prism that measures 16 inches by 10 inches by 14 inches.

Recall: The surface area of the rectangular prism is the sum of the areas of the six faces. It may be helpful to examine its net.





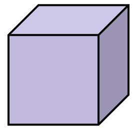
Since all of the faces are rectangles, we can use $A = lw$ to calculate the area of the faces.

Faces	$A = lw$	**Area**
Front and Back	$A = 16 \times 14$ $A = 224$	Two Faces $= 224(2) = 448$
Top and Bottom	$A = 16 \times 10$ $A = 160$	Two Faces $= 160(2) = 320$
Left and Right Sides	$A = 14 \times 10$ $A = 140$	Two Faces $= 140(2) = 280$
Total	Add up the areas.	**1048 in^2**

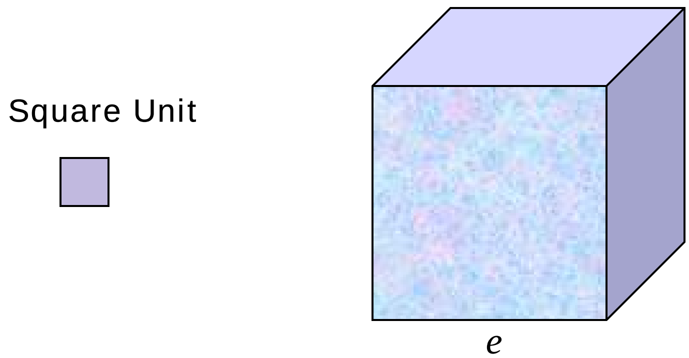
The surface area of the rectangular prism is 1048 square inches.

*Note: An abbreviated way to express square inches is in^2.

Surface Area of a Cube



A cube is a special rectangular prism with all of its edges measuring the same and all of its faces having the same area. The surface area of a cube is the total area of all of the square faces measured in square units.



The area of one face of a cube is a square.

Let e represent the length of one edge.

Then, the area of one face can be represent by $e \times e$ or e^2.

Since a cube has six faces, the total surface area of a cube is $6 \times e^2$.

"T" will be used to represent surface area in the formulas developed in this unit.

The formula for the surface area of a cube is:

The surface area (T) of a cube is:

$$\boldsymbol{T = 6e^2}$$

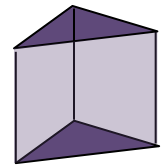
Example: Find the surface area of a cube with an edge measuring seven inches.

$$T = 6e^2$$
$$T = 6 \times 7^2$$
$$T = 6 \times 49$$
$$T = 294$$

Check: Area of one face is 7 × 7 or 49 square inches.

Area of six faces is 49 × 6 or 294 square inches.

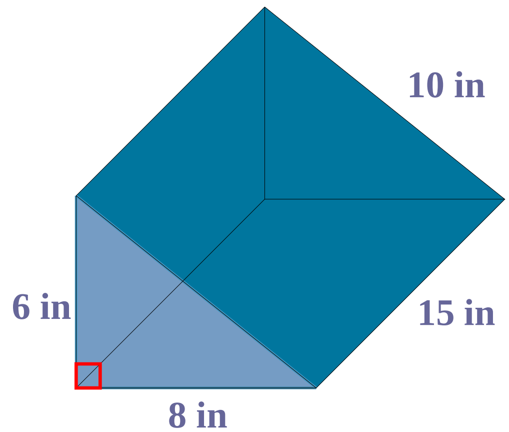
Surface Area of a Triangular Prism



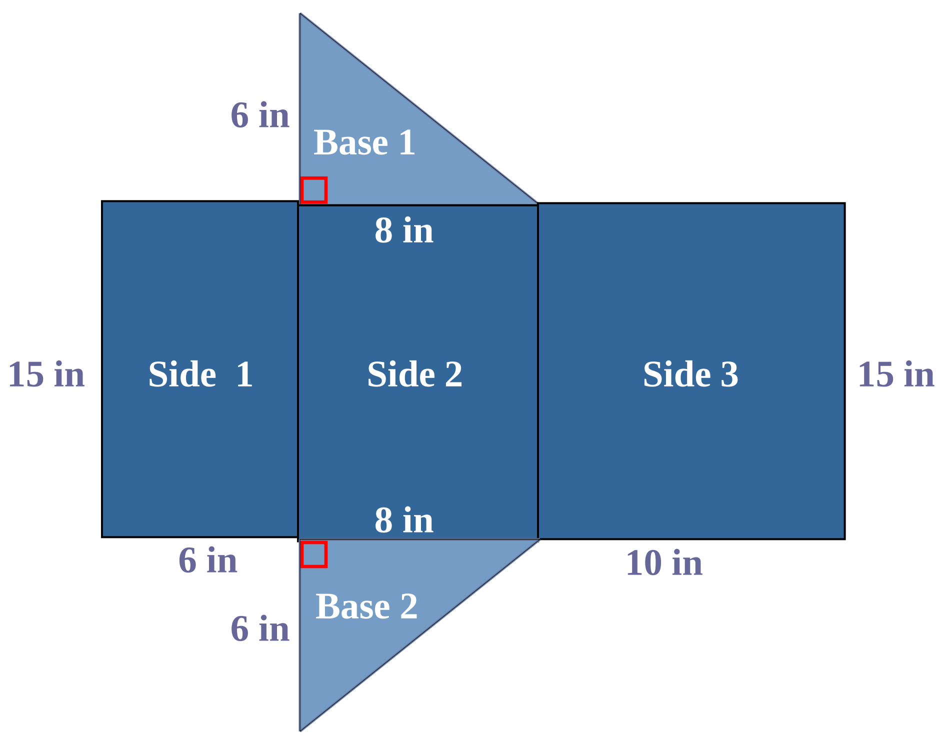
Prisms have sides shaped like rectangles and parallel bases that vary in shape. The name of a prism is determined by the shape of its bases.

A **triangular prism** is a prism that has parallel bases shaped like triangles and the sides are shaped like rectangles; thus, it is named triangular prism.

Example: Find the surface area of a triangular prism that is 15 inches long and has a base which is a right triangle. The dimensions of the right triangle are a base measuring eight inches and a height measuring six inches.



The net is shown below to help visualize the prism in two dimensions.



Step 1: Find the area of the two triangular faces.

The formula to find the area of one triangle is $A = \frac{1}{2}bh$.

$$A = \frac{1}{2} \times 8 \times 6$$
$$A = 24$$

*Both triangular faces have the same area.

The area of both triangular faces is $2 \times 24 = 48$ square inches.

Step 2: Find the area of the three rectangular faces.

The formula to find the area of a rectangle is $A = lw$.

Rectangle 1: The largest rectangular face measures 15 inches by 10 inches.

$$A = l \times w$$
$$A = 15 \times 10$$
$$A = 150 \text{ sq in}$$

Rectangle 2: The rectangular face on which the prism is resting measures 15 inches by 7 inches.

$$A = l \times w$$
$$A = 15 \times 8$$
$$A = 120 \text{ sq in}$$

Rectangle 3: The back rectangular face measures 15 inches by 6 inches.

$$A = l \times w$$
$$A = 15 \times 6$$
$$A = 90 \text{ sq in}$$

Step 3: Find the **total** area of all five faces.

$$T = 2 \text{ Triangular Areas} + 3 \text{ Rectangular Areas}$$
$$T = \quad 48 \quad + \; 150 \; + \; 120 \; + \; 90$$
$$T = 408 \text{ in}^2$$

The surface area of the triangular prism is 408 square inches.

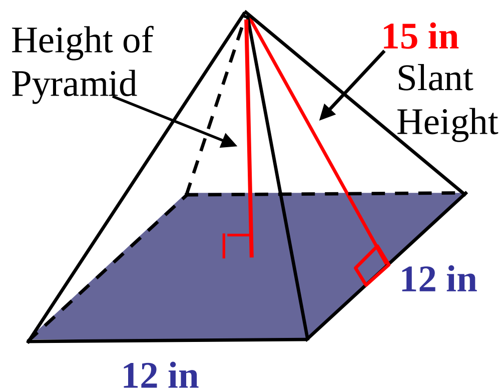
Surface Area of a Pyramid

To find the surface area of a pyramid, again, just add the areas of the faces of the solid. Notice that the sides of a pyramid are triangles.

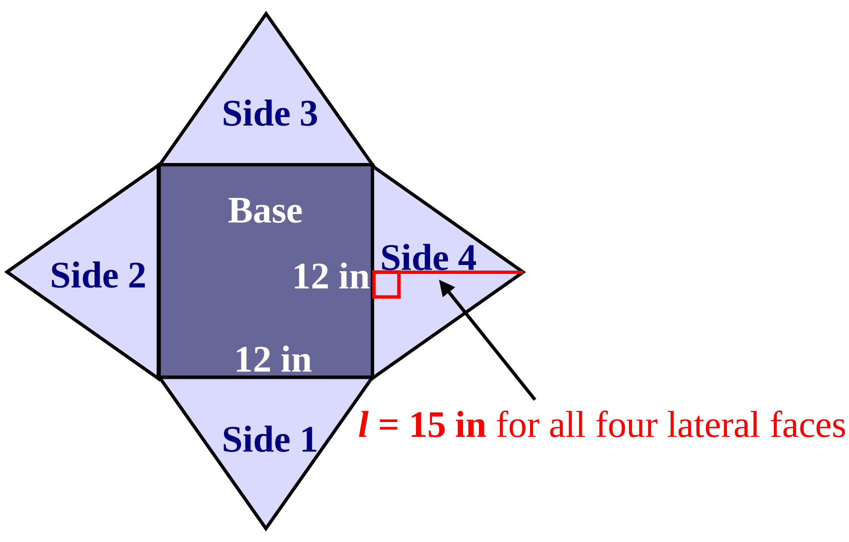
Please pay particular close attention to the difference between the height of a pyramid and the height along the side of the pyramid which is called the **slant height**.

*The slant height is used when calculating surface area, not the height of the pyramid itself.

Example: Find the surface area of a pyramid with a square base measuring 12 inches on one edge and has a slant height of 15 inches.



The net is shown below to help visualize the pyramid in two dimensions.



Step 1: Find the area of the base.

$$A = bh$$
$$A = 12(12)$$
$$A = 144 \text{ sq in}$$

Step 2: Find the area of the triangular faces.

Since each triangular face has a base of 12 inches and a height of 15 inches, the areas of all four triangles are the same.

Find the area of one triangle, and then multiply by four.

$A = \frac{1}{2}bh$	Area of a triangle
$A = \frac{1}{2}(12)(15)$	Substitute (Use slant height as height.)
$A = 90$	Simplify

The area of one triangular face is 90 square inches.

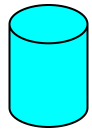
The area of all four triangles = 90 × 4 = 360 square inches.

Step 3: Find total area of the five faces.

$$T = \text{Area of Square Base} + \text{Area of the Four Triangular Sides}$$
$$T = 144 + 360$$
$$T = 504 \text{ sq in}$$

The surface area of the pyramid is 504 square inches.

Surface Area of a Cylinder



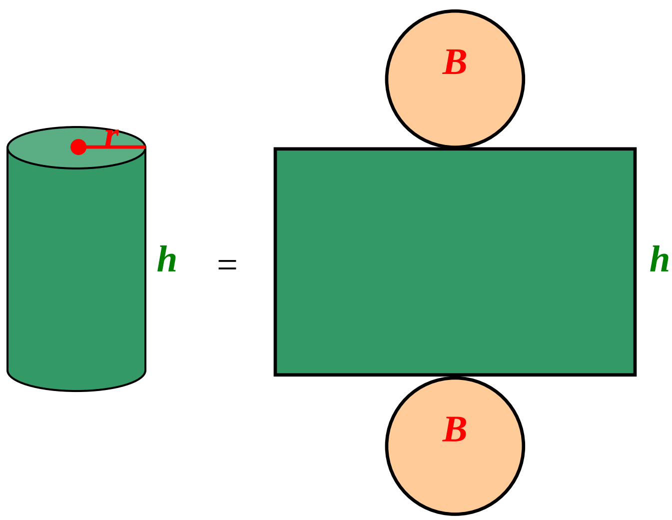
To find the surface area of a cylinder, a little more thinking is involved.

The top and bottom of a cylinder are circles.

The side of a cylinder is one continuous curved surface. When it is laid flat, it is shaped like a rectangle.

Notice that the length of the rectangle is the same as the circumference (distance around) of the circular base.

The area of the rectangular face is determined by multiplying the circumference of the base circle (length of the rectangle) by the height of the cylinder (width of the rectangle).



The formula for the surface area of a cylinder can be developed as follows:

$$
\begin{aligned}
T &= \text{Circular Face} + \text{Circular Face} + \text{Curved Surface (Rectangle)} \\
T &= \pi r^2 + \pi r^2 + C \times h \\
T &= \pi r^2 + \pi r^2 + 2\pi r \times h \\
T &= 2\pi r^2 + 2\pi rh
\end{aligned}
$$

The surface area (T) of a cylinder is:

$$\boldsymbol{T = 2\pi r^2 + 2\pi rh}$$

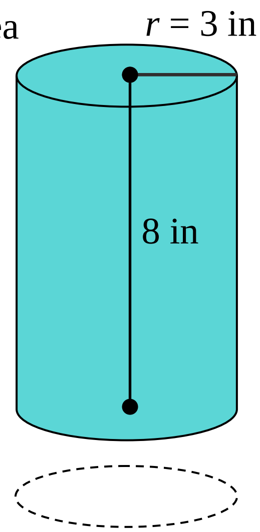
Example: Find the surface area of a cylinder that has a radius of three inches and a height of eight inches.

Step 1: Write the formula for the surface area of a cylinder.

$$T = 2\pi r^2 + 2\pi rh, \text{ where } \pi = 3.14$$

Step 2: Substitute the given information in the formula and simplify.

$$T = 2\pi r^2 + 2\pi rh$$
$$T = 2(3.14)(3^2) + 2(3.14)(3)(8)$$
$$T = 2(3.14)(9) + 2(3.14)(3)(8)$$
$$T = 56.52 + 150.72$$
$$T = 207.24$$



The surface area of the cylinder is 207.24 square inches.

Surface Area of a Cone

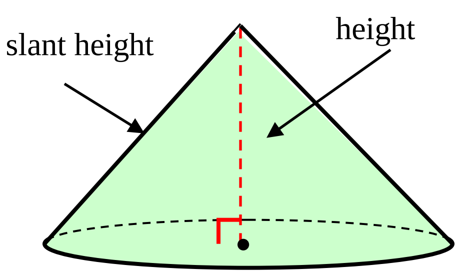
cone - A cone is a three-dimensional figure that has a circular base and one vertex. The lateral face is a circle sector.

base – The base of a cone is a circle.

height – The height of a cone is a segment that has an endpoint at the vertex and is perpendicular to the base.

slant height – The slant height of a right cone is the length of any segment that joins the vertex to the edge of the base.

Cone



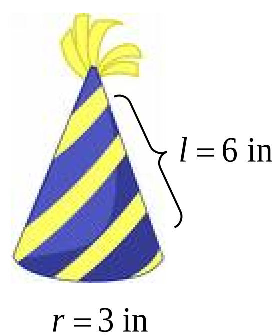
lateral surface area – The lateral surface area of a cone is the area of the curved surface.

To find the lateral surface of a cone, use the following formula:

$$LA = \pi r l$$

*Note: The development of this formula is left to study in a more advanced mathematics course.

Example 2: Find the lateral surface area of a party hat that has a radius of three inches and a slant height of six inches.

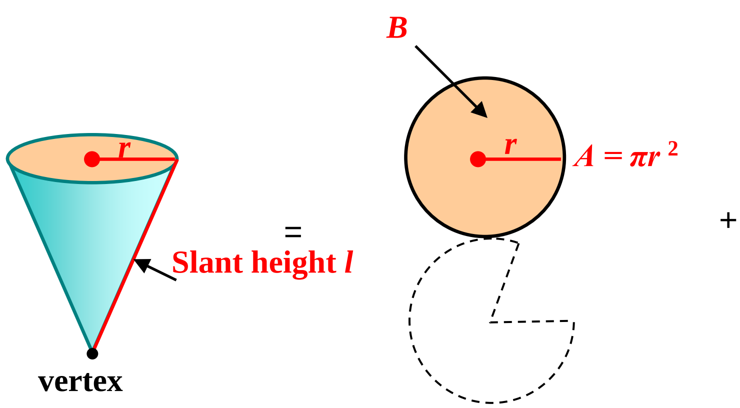


$LA = \pi rl$	Formula for Lateral Area of a Cone
$LA = \pi(3)(6)$	Substitution ($r = 3,\ l = 6$)
$LA = 18\pi$	Simplify
$LA = 56.52$	Simplify

The lateral area of the party hat (cone) is 56.52 square inches.

surface area – The surface area of a cone is the sum of the lateral area and the base area.

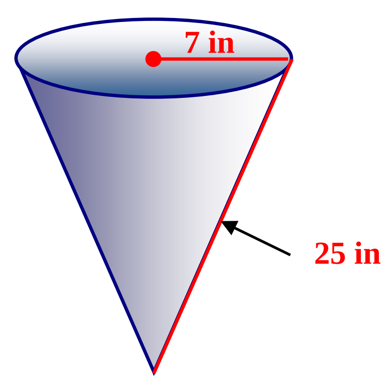
Surface area	=	Base Area	+	Lateral Area
SA	=	B	+	πrl



The surface area of a cone is the sum of the area of its base and its lateral area.

$$SA = \pi r^2 + \pi rl$$

Example 3: Find the surface area of a cone with a slant height of 25 inches and a radius of 7 inches. Round the answer to the nearest square inch.



$SA = \pi r^2 + \pi rl$	Formula for Surface Area of a Cone
$SA = \pi(7)^2 + \pi(7)(25)$	Substitution
$SA = 49\pi + 175\pi$	Simplify
$SA = 224\pi$	Simplify
$SA = 703.36$	Simplify

The surface area of the cone is approximately 703 square inches.

Surface Area of a Sphere

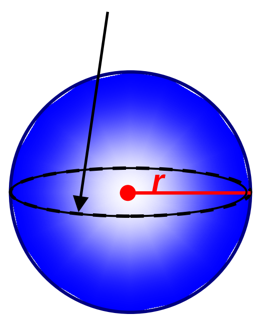
A **sphere** is a three-dimensional figure with all points the same distance from a fixed point, the center.

A **hemisphere** is half of a sphere that is created by a plane that intersects a sphere through its center.

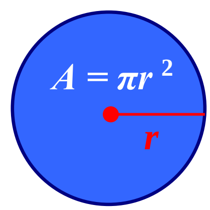
The edge of a hemisphere is a **great circle**.

The surface area of a sphere is four times the **base area**, the area of the circle created by the great circle.

Great Circle



Base Area

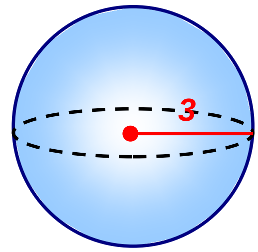


Surface area	=	4	×	Base Area
T	=	4	×	πr^2

The surface area (T) of a sphere is:

$$\boldsymbol{T = 4\pi r^2}$$

Example: Find the surface area of a sphere with a radius of two inches round to the nearest square inch.



Step 1: Write the formula for the surface area of a sphere.

$$T = 4\pi r^2, \text{ where } \pi = 3.14$$

Step 2: Substitute the given information in the formula and simplify.

$$T = 4\pi r^2$$
$$T = 4(3.14)(3^2)$$
$$T = 4(3.14)(9)$$
$$T = 113.04$$

The surface area of the sphere is 113.04 square inches.