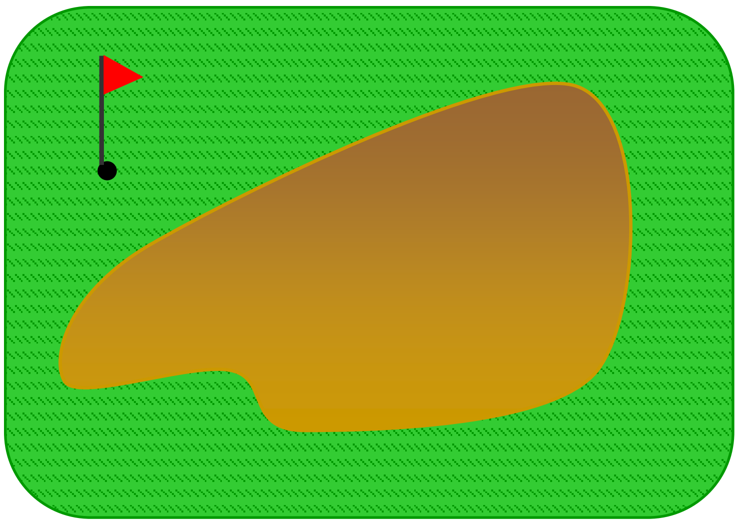
AREA

This unit is about calculating the area of many different two-dimensional shapes such as rectangles, squares, parallelograms, trapezoids, and circles. The topic of area is extended to include estimating area of irregular shapes, finding the area circle sectors, and calculating the area of composite shapes.

Estimating Area

Sometimes it is necessary to estimate the area of an object that has an irregular shape.

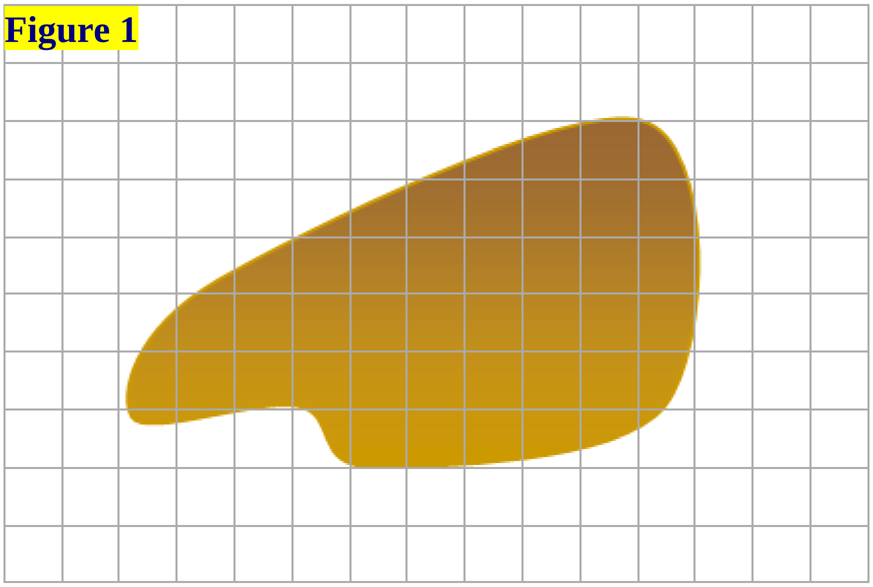
For example, the manager of a golf course must determine the area of one of the sand traps. The sand trap does not have a shape that has straight edges and square corners; therefore, it is necessary to *estimate* the area of the sand trap.



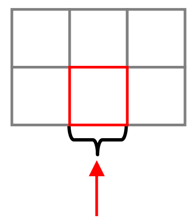
To find the area of an irregular shape like a sand trap, a simple method is to separate it into *regular and irregular* parts.

One way to get a good estimate is to transpose the shape onto a scaled grid as shown in Figure 1.

Figure 1

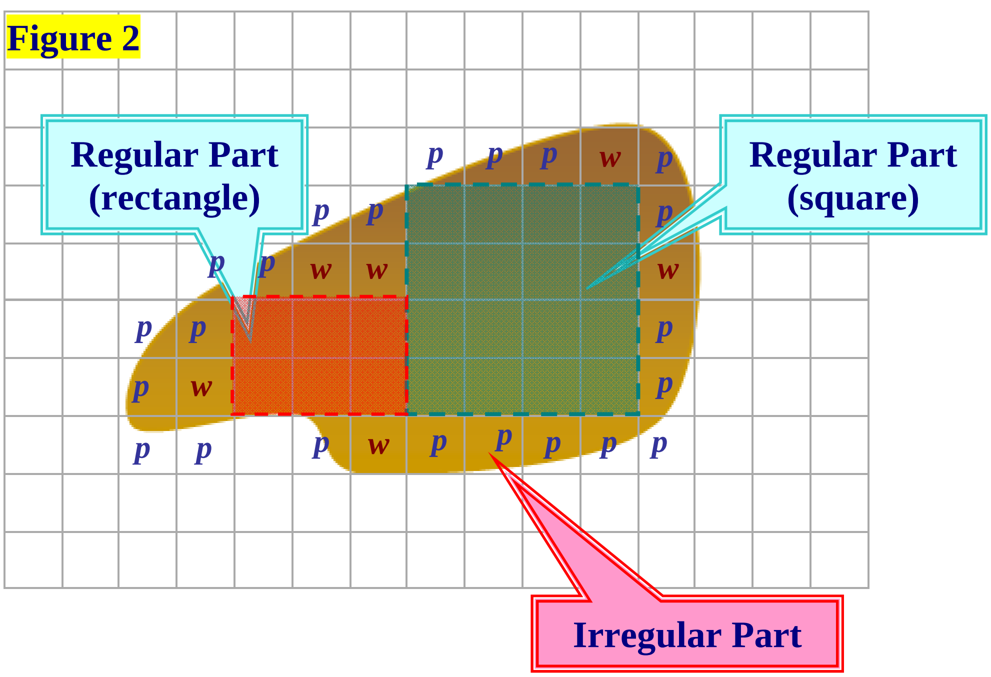


Each square represents **one** square foot, a square that is one foot wide and one foot long.



Separate the image into regular and irregular parts as shown in Figure 2.

Figure 2



In Figure 2, the rectangle and the square enclose a **regular** part. Count the number of squares enclosed in each regular part.

- Rectangle (3 by 2): 6 complete squares = 6 square feet
- Square (4 by 4): 16 complete squares = 16 square feet

Now *estimate* the area of the **irregular part** shaded in brown. Count both the whole squares (***w***) and the partial squares (***p***). If the partial square is shaded less than half, don't count it. If it is more than half, count it as a whole.

- Whole Squares (*w*): 6 whole squares = 6 square feet
- Partial Squares (*p*): 13 partial squares = 13 square feet

Add all the areas together to get an estimate of the area of the sand trap.

16 square feet	$\longrightarrow$	**Square**
6 square feet	$\longrightarrow$	**Rectangle**
6 square feet	$\longrightarrow$	**Whole Squares**
+ **13 square feet**	$\longrightarrow$	**Partial Squares**
41 square feet		

The *estimated area* of the sand trap is 41 square feet.

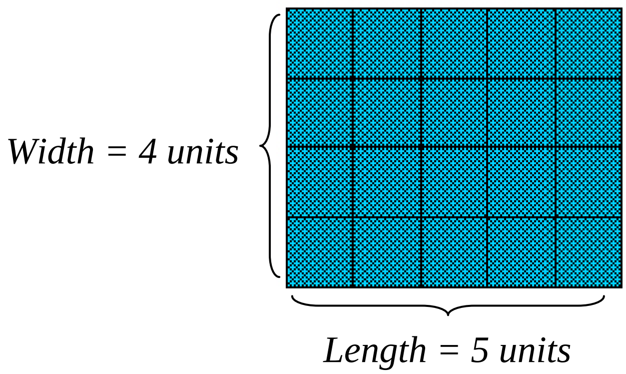
Area of a Rectangle and a Triangle

The **area of a rectangle** is the product of the length and the width.

Area is a measurement of coverage and is measured in **square units**.

$$A = lw$$

Example 1: Find the area of a rectangle that measures 5 units by 4 units.



$$A = lw$$

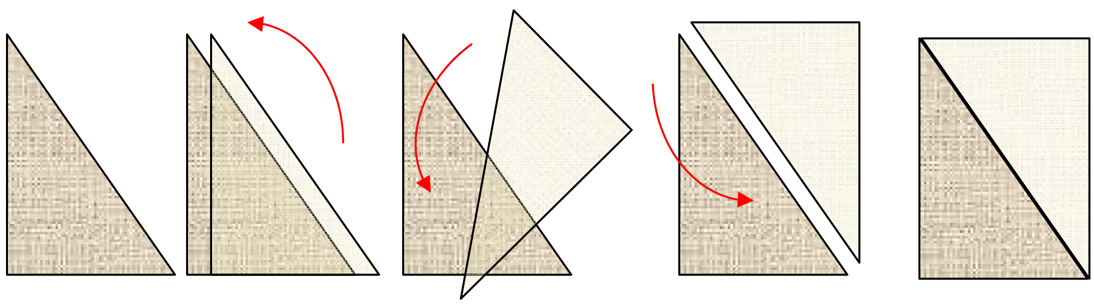
$$A = 5 \times 4$$

$$A = 20 \text{ square units}$$

The area of the rectangle is 20 square units.

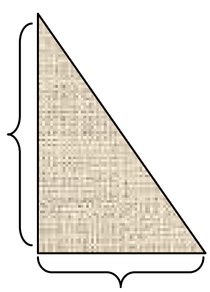
The **area of a triangle** is equal to half the area of a rectangle with the same base and height. Study the figure below and follow the arrows to see that the area is only half as much.

$$A = \frac{1}{2}bh$$



Example 2: Find the area of a triangle that measures 5 units by 6 units.

Height = 6 units



Base = 5 units

$$A = \frac{1}{2}bh$$

$$A = \frac{1}{2} \times 5 \times 6$$

$$A = \frac{1}{2} \times 30$$

$$A = 15 \text{ square units}$$

The area of the triangle is 15 square units.

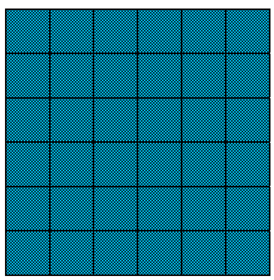
Area of a Square and a Parallelogram

Area is a measurement of coverage and is measured in **square units**.

The **area of a square** is the product of its length and width. Since squares have sides of equal length, the area of a square is the product of its length (side) and its width (side).

$$A = lw$$
$$A = s \times s$$
$$A = s^2$$

Example 1: Find the area of a square that is 6 units in length on each side.



Side = 6 units

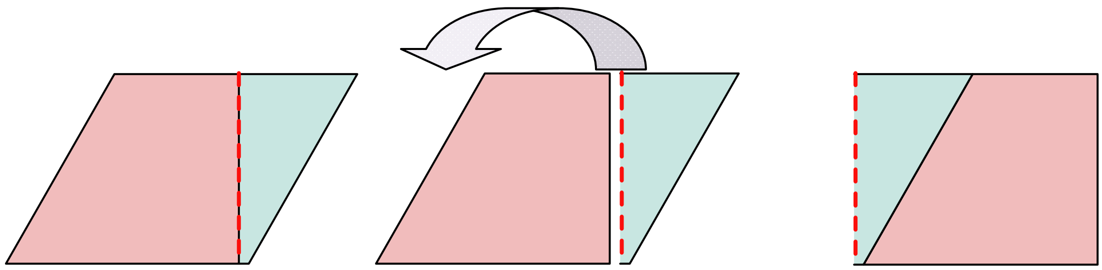
$$A = s^2$$

$$A = 6^2$$

$$A = 36 \text{ square units}$$

The area of the square is 36 square units.

The **area of a parallelogram** can be rearranged into the shape of a rectangle if the parallelogram is cut along a perpendicular height from the top to its base. Thus, a formula for the area of a parallelogram can be written based on the formula for the area of a rectangle.

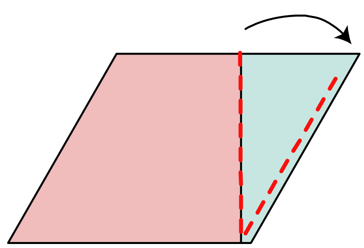


Move the triangular piece to the left side.

The **area of a parallelogram** is the product of its base and height. The height of a parallelogram is the length of a perpendicular line from the top of the parallelogram to the base.

$$A = lw$$
$$A = bh$$

*Notice that the height of a parallelogram is shorter than the length of its side. When calculating the area of a parallelogram, be sure to use the height of the parallelogram rather than the length of the side.

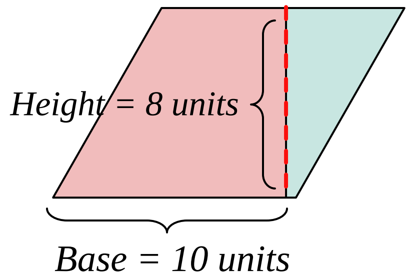


Example 2: Find the area of a parallelogram that has a base of 10 units and a height of 8 units.

$$A = bh$$

$$A = 10 \times 8$$

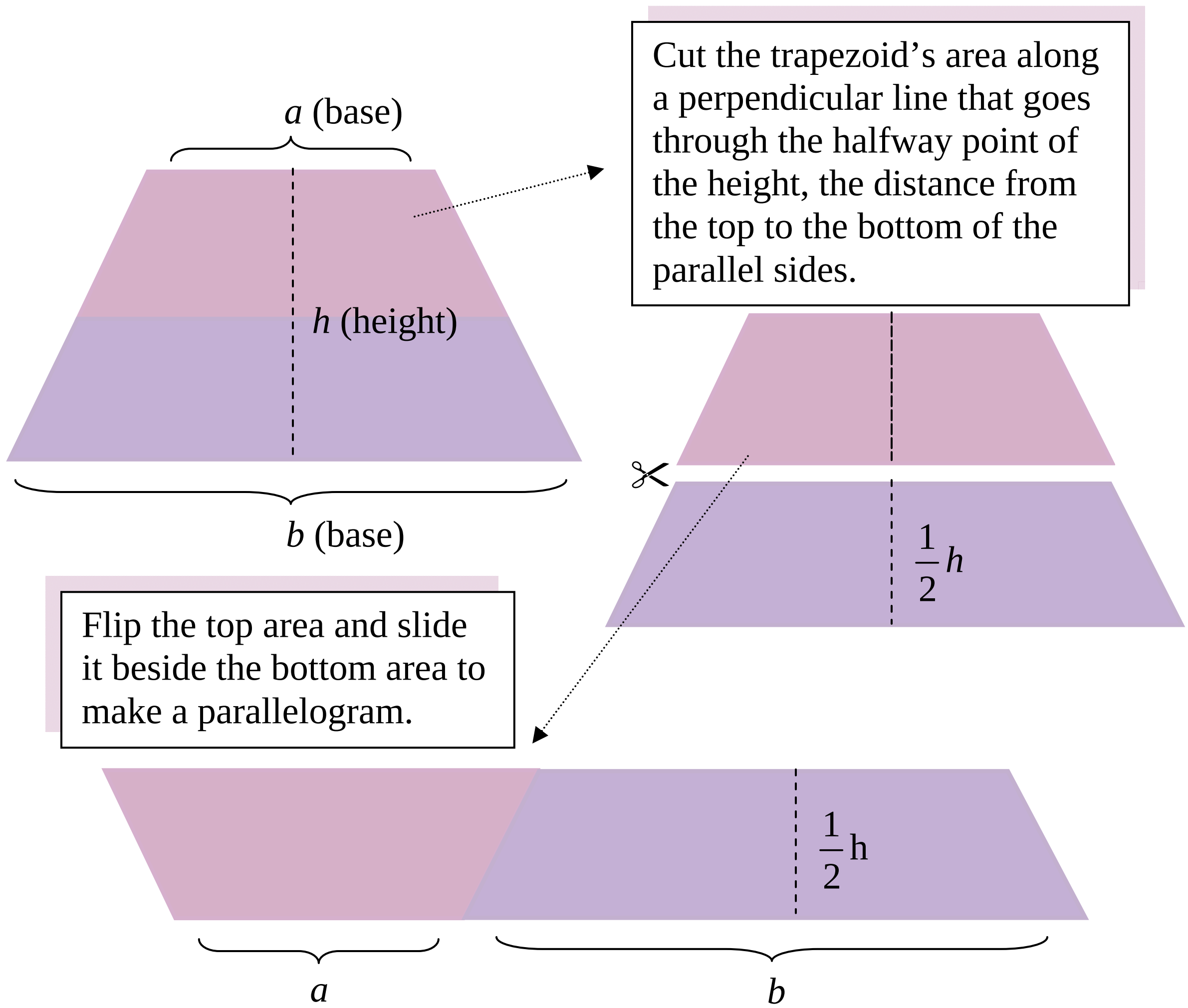
$$A = 80 \text{ square units}$$



Area of a Trapezoid

Area is a measurement of coverage and is measured in **square units**.

The **area of a trapezoid** can be rearranged into the shape of a parallelogram. Let's take a look at how this can happen.



$\text{Base} = (a + b)$ $\qquad$ $\text{Height} = \frac{1}{2}h$

Build the formula for the area of a trapezoid based on the formula for the area of a parallelogram.

$$A = bh$$

$$A = (a+b)(\frac{1}{2}h)$$

$$A = (\frac{1}{2}h)(a+b)$$ Apply the commutative property.

$$\boxed{A = \frac{1}{2}h(a+b)}$$

The area of a trapezoid equals one-half of the height times the sum of the bases.

*Note: The bases of a trapezoid are the parallel sides.

Example: Find the area of a trapezoid where the parallel sides measure 4 feet and 10 feet and the height of the trapezoid is 15 feet.

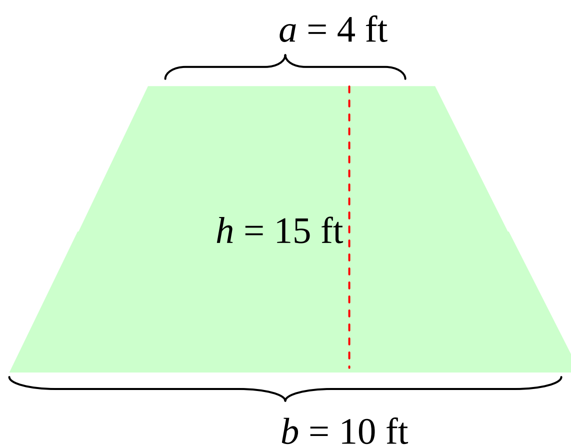
$$A = \frac{1}{2}h(a+b)$$

$$A = \frac{1}{2} \times 15 \times (4+10)$$

$$A = \frac{1}{2} \times 15 \times 14$$

$$A = \frac{1}{2} \times 210$$

$$A = 105 \text{ sq ft}$$



The area of the trapezoid is 105 square feet.

Area of a Circle

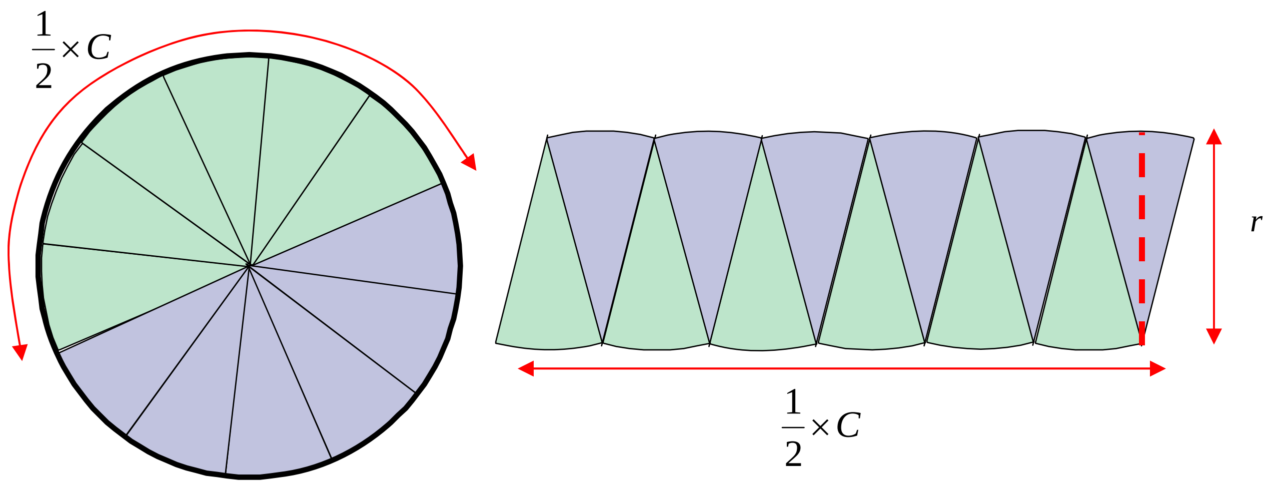
Area is a measurement of coverage and is measured in **square units**.

The area of a circle can be rearranged into a shape that approximates a parallelogram.

The length of the parallelogram is the same length as half the circle's circumference. The height of the parallelogram is the same as the radius of the circle.

Let's take a look at how this can happen.

The circle shown below is divided into 12 congruent pieces. The pieces are then laid out to make a shape that looks similar to a parallelogram.



Notice that the length of the "parallelogram" is half of the length of the circumference of the circle.

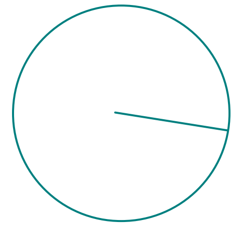
Notice that the height of the parallelogram is close to the radius of the circle.

For this theory to truly work, the circle would be divided into many, many, more pieces. When that is done, then the bottom of the parallelogram is close to a straight line and the height of the parallelogram is closer to a perpendicular line.

Now, we'll build the formula based on this theory.

Statement	Reason
$A = bh$	Formula for area of a parallelogram.
$A = (\frac{1}{2}C) \times r$	base $= \frac{1}{2}C$ height $= r$
$A = (\frac{1}{2} \times 2\pi r) \times r$	$C = 2\pi r$
$A = 1 \times \pi r \times r$	$\frac{1}{2} \times 2 = 1$
$A = \pi r \times r$	Identity Property (Any number times 1 is the number.)
$A = \pi \times (r \times r)$	Associative Property (Regrouping is allowed in multiplication.)
$A = \pi \times r^2$	$r \times r = r^2$

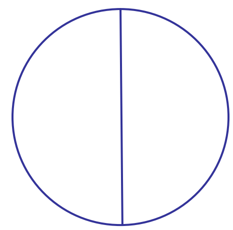
Example 1: Find the area of a circle that has a radius of five inches.



$$A = \pi r^2$$
$$A = 3.14 \times 5^2$$
$$A = 3.14 \times 25$$
$$A = 78.5 \text{ square inches}$$

The area of the circle is 78.5 square inches.

Example 2: Find the area of a circle that has a diameter of twenty feet.



*Since a diameter is given, and a diameter equals two radii, take half of 20 to determine the radius.

$$A = \pi r^2$$
$$A = 3.14 \times 10^2$$
$$A = 3.14 \times 100$$
$$A = 314 \text{ square feet}$$

The area of the circle is 314 square feet.

Formula Chart for Area

The chart below is a list of many shapes and the corresponding formulas for calculating the area of the shapes.

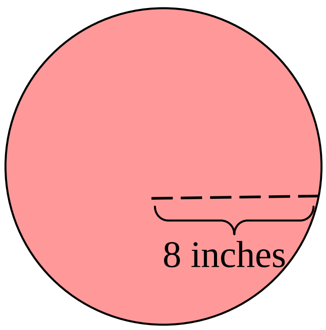
Shape	Area
Triangle	$A = \frac{1}{2}bh$
Rectangle	$A = lw$
Square	$A = s^2$
Parallelogram	$A = bh$
Trapezoid	$A = \frac{1}{2}h(a+b)$
Circle	$A = \pi r^2$

Area of a Circle Sector

To find the area of a sector of a circle, first determine the **area of the whole circle,** and then find the **fractional part** that represents the circle.

Example: Find the area of three-fourths of a circle with a radius of eight inches.

First, find the area of the whole circle.



$$A = \pi \times r^2$$

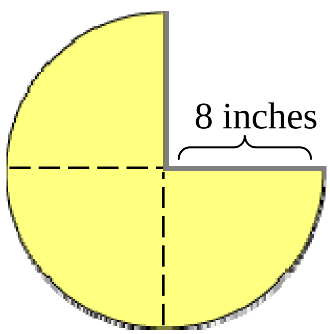
$$A = 3.14 \times 8^2$$

$$A = 3.14 \times 64$$

$$A = 200.96 \text{ sq in}$$

Then, find three-fourths of the total area.

Since the sector is 3/4 of the area of the entire circle, the area of the sector is:



$$\frac{3}{4} \text{ of } 200.96 = \frac{3}{4} \times \frac{200.96}{1}$$

$$= \frac{602.88}{4}$$

$$= 150.72 \text{ sq in}$$

The area of the circle sector that is three-fourths of the area of the whole circle is 150.72 square inches.

Area of a Composite Figure

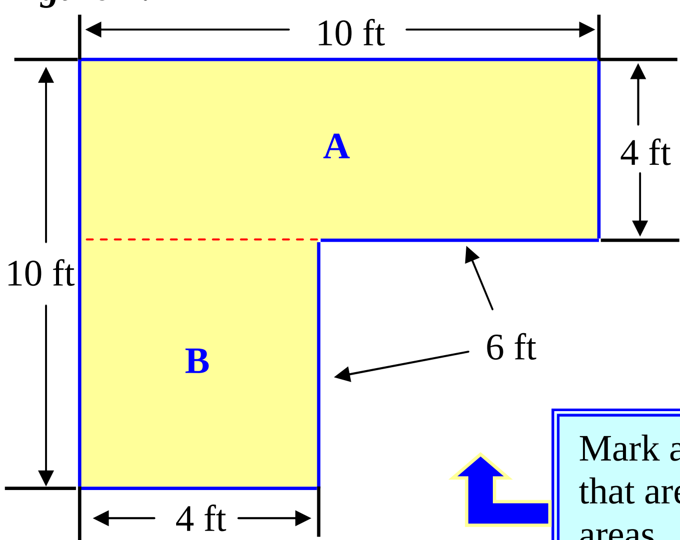
Composite figures are shapes that are made up of simpler distinct shapes.

Figure 1 is an illustration of two rectangles combined to make a composite figure.

To find the area of a composite figure, determine the simpler shapes that make up the figure. Draw lines to divide the larger figure into the smaller shapes.

The red dotted line clearly differentiates between Rectangle A and Rectangle B.

***Figure 1*:**



Mark any dimensions (lengths) that are needed to find the two areas.

Example 1: What is area of Figure 1?

First, find the area of **Rectangle A**.

- $A = L \times W$
- $A = 10 \times 4 = 40$ square feet

Next, find the area of **Rectangle B**.

- $A = L \times W$
- $A = 4 \times 6 = 24$ square feet

Finally, add the two areas.

40	Rectangle A
+24	Rectangle B
64 ft^2	Composite Area

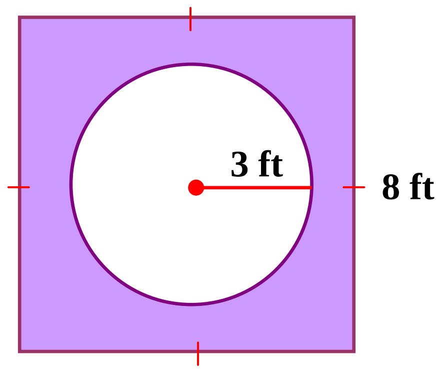
*Note: The abbreviation for square feet is ft^2.

The composite area of Figure 1 is 64 square feet.

What if the shape has a "hole" in it? In this type of problem, it is necessary to **subtract** to find the area of the shape.

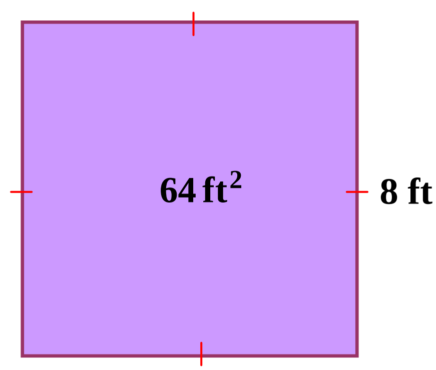
Example 2: Find the area for the region shaded outside of the circle, but within the square (purple area).

In other words, find the area of the square, but subtract away the area of the circle, the "hole".



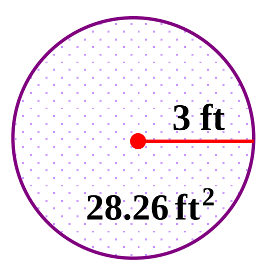
First, find the **entire area** of the **square**.

- Area (square) = s^2
- $A = 8^2 = 64 \text{ ft}^2$



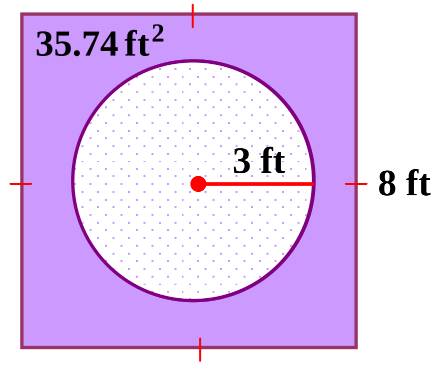
Next, find the **area** of the **circle (hole).**

- Area (circle) $= \pi \times r^2$
- $A = 3.14 \times 3^2 = 28.26 \text{ ft}^2$



Finally, **subtract** the area of the circle from the area of the square.

$$\begin{array}{r} 64.00 \\ -\underline{28.26} \\ 35.74\ \text{ft}^2 \end{array}$$



The area of the region **outside of the circle, but within the square**, (light purple region) is 35.74 square feet.