# FRACTIONS: ADDITION AND SUBTRACTION

This unit is about addition of fractions, subtraction of fractions, and estimating with fractions. Mastery of fractions is important for success at any level of mathematics. The unit also includes a helpful problem-solving strategy of modeling word problems by making a picture. The unit concludes with taking a look at stocks, and how profit or loss is calculated.

Rounding Fractions and Mixed Numbers

Addition of Fractions

Subtraction of Fractions

Problem Solving: Making a Drawing

Stocks

# Rounding Fractions and Mixed Numbers

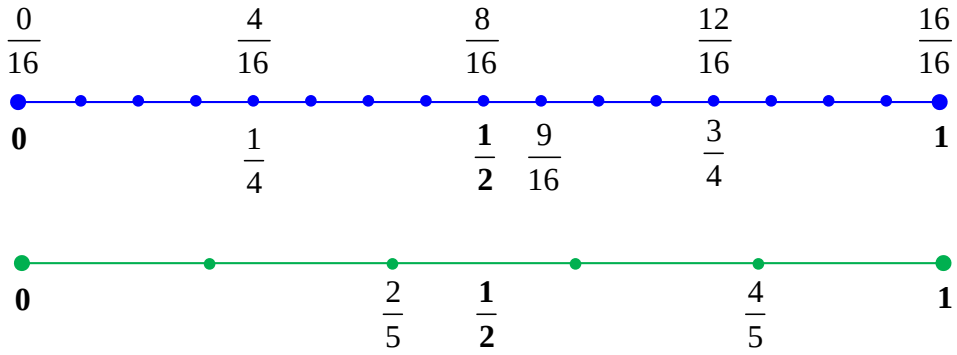
## Rounding Fractions to 0, 1/2, or 1

When rounding fractions to the **nearest half**, the results are a good estimate of the actual answer. Draw a number line to help visualize the fraction's location relative to 0, 1/2, or 1.

Two number lines are provided below to help with rounding fractions. Each of the fractions presented in the example are positioned on the number to help with the rounding.

- The first number line shows equal divisions between 0 and 1 in sixteenths. The fractions for this problem are also positioned below the number line to give a point of reference.
- The second number line shows equal division between 0 and 1 in fifths.
- When rounding to the nearest half, the possible results are 0, 1/2, or 1.

Study the number lines carefully, and then refer to the number line to round the fractions in the given examples.



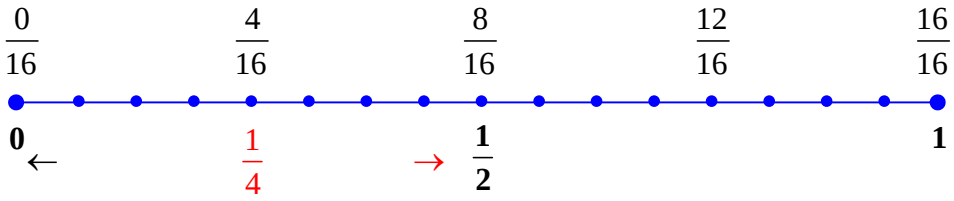

*Note: The symbol for approximately equal to is ≈.

*Example 1*: Round each of the following fractions to the nearest 1/2.

$$\frac{1}{4},\ \frac{3}{16},\ \frac{9}{16}$$

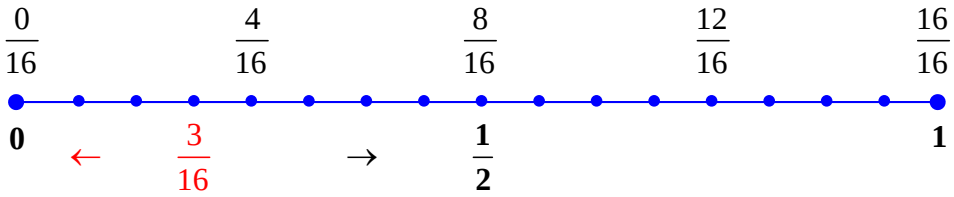
*Note: When the number line is divided into sixteenths, the exact halfway mark between 0 and 1 is 8/16.

a.) To round 1/4, notice that its location on the number line is halfway between 0 and 1/2. When the fraction is exactly halfway between two divisions on the number, round up.



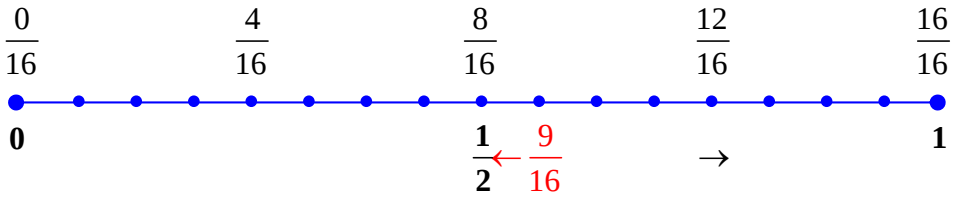

In this case, 1/4 rounds up to 1/2. $\frac{1}{4} \approx \frac{1}{2}$

b.) To round 3/16, notice that its location on the number line is closer to 0 than 1/2.




In this case, 3/16 rounds down to 0. $\frac{3}{16} \approx 0$

c.) To round 9/16, notice that its location on the number line is closer to 1/2 than 1.



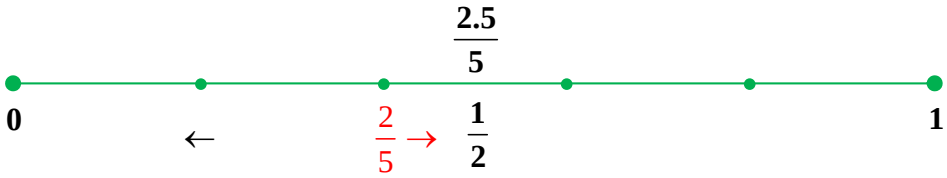

In this case, 9/16 rounds down to 1/2. $\frac{9}{16} \approx \frac{1}{2}$

*Example 2*: Round each of the following fractions to the nearest 1/2.

$$\frac{2}{5}, \frac{4}{5}$$

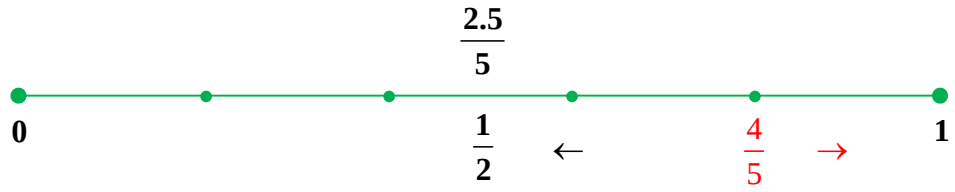
*Note: When the number line is divided into fifths, the exact halfway mark between 0 and 1 is 2.5/5 because half of 5 is 2.5.

a.) To round 2/5, notice that its location on the number line is closer to 1/2 than 0.




In this case 2/5 rounds up to 1/2. $\frac{2}{5} \approx \frac{1}{2}$

b.) To round 4/5, notice that its location on the number line is closer to 1 than 1/2.




In this case 4/5 rounds up to 1. $\frac{4}{5} \approx 1$

## Rounding Mixed Numbers to the Nearest Whole Number

When rounding mixed numbers, round to the **nearest whole number**. This results in a good estimate of the actual answer. Find the halfway point between the two whole numbers by dividing the denominator of the mixed number by two, and then compare the numerator of the fraction to this halfway point.

*Example 3*: Round the following mixed numbers to the nearest whole number.

$$1\frac{3}{4},\ 12\frac{1}{6},\ 7\frac{5}{9}$$

a.) To round 1 3/4 to the nearest whole number, find half of the denominator of the fraction, and write the equivalent fraction to 1/2. Compare this fraction to the given fraction, and then round.

*Step 1*: Find half of the denominator. $\frac{1}{2}$ of $4 = 2$, therefore $\frac{2}{4} = \frac{1}{2}$.

Thus, the halfway point is $1\frac{2}{4}$.

*Step 2*: Compare the numerators of the given mixed number and the mixed number that represents the halfway point.

$$1\frac{3}{4} \quad ? \quad 1\frac{2}{4} \qquad\qquad 1\frac{3}{4} \quad > \quad 1\frac{2}{4}$$

*Step 3*: Since 3 is greater than 2, round up to the nearest whole number.

*It is helpful to draw a partial number line to visualize the location of the mixed number relative to the two whole numbers that the mixed number falls between.

$$1 \text{----------} 1\frac{2}{4} \text{----------} 2$$

$$\uparrow$$

$$1\frac{3}{4}$$

In this case, 1 3/4 is closer to 2 than 1, so 1 3/4 rounds up to 2.

$$1\frac{3}{4} \approx 2$$

b.) To round 12 1/6 to the nearest whole number, find half of the denominator of the fraction, and write the equivalent fraction to 1/2. Compare this fraction to the given fraction, and then round.

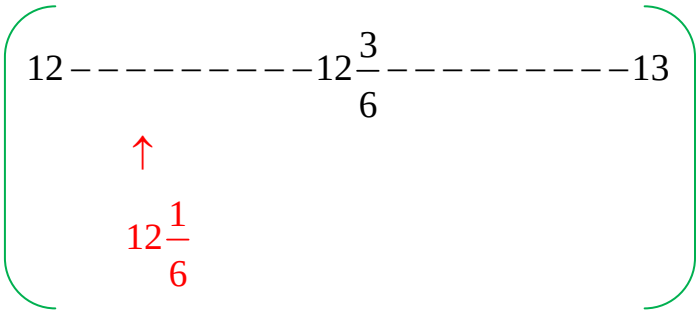
*Step 1*: Find half of the denominator. $\frac{1}{2}$ of 6 = 3, therefore $\frac{3}{6}=\frac{1}{2}$.

Thus, the halfway point is $12\frac{3}{6}$.

*Step 2*: Compare the numerators of the given mixed number and the mixed number that represents the halfway point.

$12\frac{1}{6}$ ? $12\frac{3}{6}$ $\qquad$ $12\frac{1}{6} < 12\frac{3}{6}$

*Step 3*: Since 1 is less than 3, round down to the nearest whole number.




In this case, 12 1/6 rounds down to 12. $\qquad 12\frac{1}{6}\approx 12$

c.) To round 7 5/9 to the nearest whole number, find half of the denominator of the fraction, and write the equivalent fraction to 1/2. Compare this fraction to the given fraction, and then round.

*Step 1*: Find half of the denominator. $\frac{1}{2}$ of 9 = 4.5

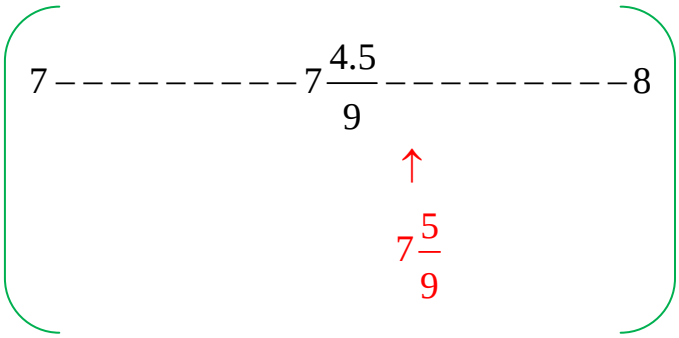
Thus, the halfway point is $7\frac{4.5}{9}$.

$$2\overline{)9.0}\quad 4.5,\ 8,\ 10,\ 10$$

*Step 2*: Compare the numerators of the given mixed number and the mixed number that represents the halfway point.

$$7\frac{5}{9} \quad ? \quad 7\frac{4.5}{9} \qquad\qquad 7\frac{5}{9} \quad > \quad 7\frac{4.5}{9}$$

*Step 3*: Since 5 is slightly greater than 4.5, round up to the nearest whole number.




In this case, 7 5/9 rounds up to 8. $7\frac{5}{9} \approx 8$

## Addition of Fractions

To add fractions, identify the numerators and the denominators, and then make the adjustments necessary to complete the computation.

*numerator* → $\frac{3}{4}$ ← *denominator*

When adding fractions and the denominators are the same, just add the numerators. If the denominators are different, write the fractions as equivalent fractions with the same denominator, and then add the numerators.

## Same Denominators

*Example* 1: Add 4/9 + 2/9.

Add the numerators of the fractions.

$$\begin{array}{r} \frac{4}{9} \\ +\frac{2}{9} \\ \hline \frac{6}{9} = \frac{2}{3} \end{array}$$

*To simplify (reduce) the answer, divide the numerator and denominator by three.

$$\frac{6}{9} \div \frac{3}{3} = \frac{2}{3}$$

*Click on the log below to play a game.*

*Example* 2: Add 2 5/8 + 9 7/8.

Add the whole numbers and add the numerators of the fractions.

$$2\frac{5}{8}$$
$$+9\frac{7}{8}$$
$$11\frac{12}{8}=12\frac{1}{2}$$

*Let's take a closer look at how to convert an improper fraction to a mixed number in simplest form. To find the equivalent mixed number, divide the numerator of the improper fraction by the denominator.

$$\frac{12}{8}=8\overline{)12}\;\begin{matrix}1\\ \\ 8\\ \hline 4\end{matrix}=1\frac{4}{8}=1\frac{1}{2}\quad \textit{therefore}\quad 11\frac{12}{8}=11+1\frac{1}{2}=12\frac{1}{2}$$

*To simplify (reduce) 4/8, divide the numerator and denominator by four.

$$\frac{4}{8}\div\frac{4}{4}=\frac{1}{2}$$

*Example* 3: Add 3/4 + 8.

$$\begin{array}{r} \frac{3}{4} \\ +8 \\ \hline 8\frac{3}{4} \end{array}$$

Just bring down the 8 since 8 + 0 = 8 and bring down the $\frac{3}{4}$ since $\frac{3}{4}+0=\frac{3}{4}$.

*Example* 4: Add 5 1/6 + 3 5/6.

Add the whole numbers and add the numerators of the fractions.

$$\begin{array}{r} 5\frac{1}{6} \\ +3\frac{5}{6} \\ \hline 8\frac{6}{6}=9 \end{array}$$

*To simplify the answer, rewrite $\frac{6}{6}$ as one, and then combine the one with the eight.

$$8\frac{6}{6}=8+1=9$$

## Different Denominators

In the previous four examples, the denominators were the same. Now let's take a look at adding fractions with different denominators.

*Example* 5: Add 7/8 + 5/16.

First, find the least common denominator (LCD) for 8 and 16.

List the multiples of 16: 16 = {16, 32, 48, 64, 80…}
List the multiples of 8: 8 = {8, 16, 24, 32, 40…}

*A good "rule of thumb" is to list the multiples of the larger denominator first because the LCD will occur sooner in the multiples of the larger number.

The "least" multiple that is common to both sets is 16; that is, the LCD = 16.

$$\begin{array}{r} \frac{7}{8} = \frac{14}{16} \\ +\frac{5}{16} = \frac{5}{16} \\ \hline \frac{19}{16} = 1\frac{3}{16} \end{array}$$

$\frac{7}{8} \times \frac{2}{2} = \frac{14}{16}$

*or* say 8 divides into 16, two times, 7 × 2 = 14.

*To find the equivalent mixed number, divide the numerator of the improper fraction by the denominator, and then express the quotient and the remainder as a mixed number.

$$\frac{19}{16} = 16\overline{)19}\begin{array}{l} 1 \\ 16 \\ \hline 3 \end{array} = 1\frac{3}{16}$$

*Example* 6: Add 3 7/9 + 2 5/6.

First, find the least common denominator (LCD) for 9 and 6.

List the multiples of 9: 9 = {9, 18, 27, 36, 45…}
List the multiples of 6: 6 = {6, 12, 18, 24, 30…}

The "least" multiple that is common to both sets is 18; that is, the LCD = 18.

When changing the fraction part of the mixed number to the same denominator, just rewrite the whole number with the fraction.

$$\begin{array}{r} 3\frac{7}{9} = 3\frac{14}{18} \\ +2\frac{5}{6} = 2\frac{15}{18} \\ \hline 5\frac{29}{18} = 6\frac{11}{18} \end{array} \qquad \left( \begin{array}{l} \frac{7}{9}\times\frac{2}{2} = \frac{14}{18} \\ \\ \frac{5}{6}\times\frac{3}{3} = \frac{15}{18} \end{array} \right)$$

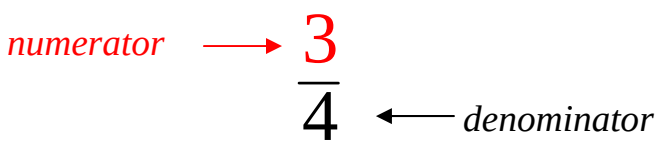
*To find the equivalent mixed number, divide the numerator of the improper fraction by the denominator, and then express the quotient and the remainder as a mixed number. Finally, combine the two whole numbers.

$$\frac{29}{18} = \begin{array}{r} 1 \\ 18\overline{)29} \\ \underline{18} \\ 11 \end{array} = 1\frac{11}{18}$$

$$5\frac{29}{18} = 5 + 1\frac{11}{18} = 6\frac{11}{18}$$

## Subtraction of Fractions

To subtract fractions, identify the numerators and the denominators, and then make the adjustments necessary to complete the operation.




When subtracting fractions and the denominators are the same, subtract the numerators. If the denominators are not the same, write the fractions as equivalent fractions with the same denominator, and then subtract the numerators. In some instances, borrowing will be necessary.

*Example* 1: Subtract 11/12 – 5/12.

Subtract the numerators of the fractions.

$$\begin{array}{r} \frac{11}{12} \\ -\frac{5}{12} \\ \hline \frac{6}{12} = \frac{1}{2} \end{array}$$

*To simplify (reduce) 6/12, divide the numerator and denominator by six.

$$\frac{6}{12} \div \frac{6}{6} = \frac{1}{2}$$

*Click on the log below to play a game.*

*Example* 2: Subtract 7/8 – 5/6.

First, find the least common denominator (LCD) for 8 and 6.

List the multiples of 8: 8 = {8, 16, 24, 32, 40…}
List the multiples of 6: 6 = {6, 12, 18, 24, 30…}

*A good "rule of thumb" is to list the multiples of the larger denominator first because the LCD will occur sooner in the multiples of the larger number.

The "least" multiple that is common to both sets is 24; that is, the LCD = 24.

$$\begin{array}{r} \frac{7}{8}=\frac{21}{24} \\ -\frac{5}{6}=\frac{20}{24} \\ \hline \frac{1}{24} \end{array}$$

$$\frac{7}{8}\times\frac{3}{3}=\frac{21}{24}$$

$$\frac{5}{6}\times\frac{4}{4}=\frac{20}{24}$$

*Example 3*: Subtract 8 14/15 – 2 3/5.

First, find the least common denominator (LCD) for 15 and 5.

List the multiples of 15: 15 = {15, 30, 45, 60, 75…}
List the multiples of 5: 5 = {5, 10, 15, 20, 25…}

The "least" multiple that is common to both sets is 15; that is, the LCD = 15.

When changing the fraction part of the mixed number to the same denominator, just rewrite the whole number with the fraction.

$$8\frac{14}{15} = 8\frac{14}{15}$$
$$-2\ \frac{3}{5} = 2\frac{9}{15} \quad \left(\frac{3}{5}\times\frac{3}{3}=\frac{9}{15}\right)$$
$$6\frac{5}{15} = 6\frac{1}{3}$$

*To simplify (reduce) 5/15, divide the numerator and denominator by five.

$$\frac{5}{15}\div\frac{5}{5}=\frac{1}{3}$$

*Example 4*: Subtract 7 – 2 4/13.

To subtract a mixed number from a whole number, borrowing is needed. In this problem, **borrow** a one from seven and write the one in thirteenths (13/13).

$$\begin{array}{r} \cancel{7}^{\,6} = 6\frac{13}{13} \quad \left(7 = 6+1 = 6+\frac{13}{13} = 6\frac{13}{13}\right) \\ -2\ \frac{4}{13} = 2\frac{4}{13} \\ \hline 4\frac{9}{13} \end{array}$$

*Example 5*: Subtract 9 3/8 – 4 7/8.

In this problem, not only will we need to **borrow** a one and change it to eighths; but, we will also have to combine the eighths that are equal to one (8/8) with the eighths that are part of the mixed number (3/8).

$$\begin{array}{r} \cancel{9}^{\,8}\,\frac{3}{8} = 8\frac{11}{8} \\ -4\ \ \frac{7}{8} = 4\ \frac{7}{8} \\ \hline 4\frac{4}{8} = 4\frac{1}{2} \end{array}$$

$$\begin{aligned} 9\frac{3}{8} &= 8+1+\frac{3}{8} \\ &= 8+\frac{8}{8}+\frac{3}{8} \\ &= 8\ \ \frac{11}{8} \end{aligned}$$

A shortcut to find 11 in $\frac{11}{8}$ is to add the 3 + 8 in $\frac{3}{8}$.

*Example 6*: Subtract 5 5/12 – 2 11/18.

In this problem, we must first find the LCD and rewrite the mixed numbers. From there, we will need to borrow as shown in the previous problem.

First, find the least common denominator (LCD) for 12 and 18.

List the multiples of 18: 18 = {18, 36, 54, 72, 90…}
List the multiples of 12: 12 = {12, 24, 36, 48, 60…}

The "least" multiple that is common to both sets is 36; that is, the LCD = 36.

$$5\frac{5}{12} = 5\frac{15}{36} = 4\frac{51}{36}$$
$$-2\frac{11}{18} = 2\frac{22}{36} = 2\frac{22}{36}$$
$$2\frac{29}{36}$$

Use the shortcut and add 15 + 36.

$$\frac{15+36}{36} = \frac{51}{36}$$

## Problem Solving: Making a Drawing

Sometimes it helps to visualize a word problem by make a drawing first.

*Example*: Chad left his house and drove 1 mile west, then 3 miles north, then 3 miles east, and then 3 miles south. About how many miles east of his house was he when he stopped driving?

*Step 1*: Explore the problem.

- What facts are given?
  The driving path is:
  - 1 mile west
  - 3 miles north
  - 3 miles east
  - 3 miles south

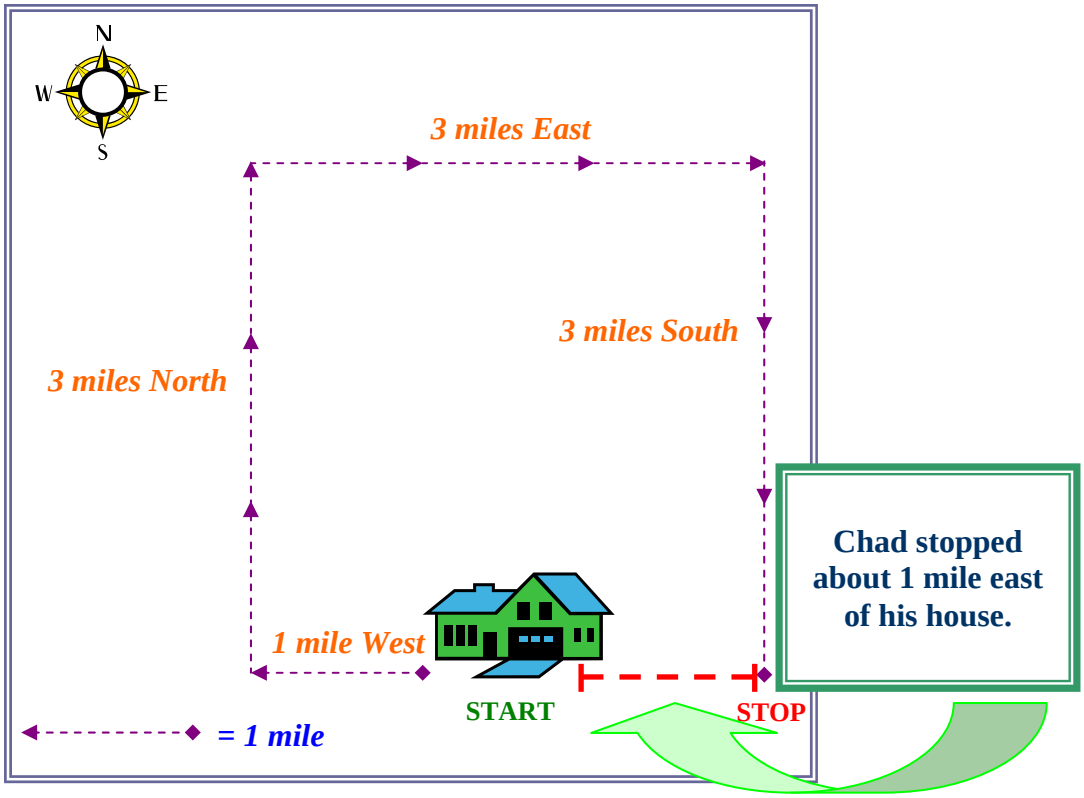
- What must be determined?
  - About how many miles east of his house did Chad stop driving?

*Step 2*: Plan the problem.

- Draw a map that shows Chad's house (START), the driving path, and the stopping point.

*Step 3*: Solve the problem.

- Draw a map of Chad's driving path.



# Stocks

Julian wanted to invest some of his money in stocks. Investing in stocks can be risky so Julian studied the stock market. The stock table below shows some stock trades. The table displays the summary of the day's transactions. Stock tables break down what is happening at each company listed on the stock exchanges, including stocks, bonds, money market funds, and mutual funds.

Here are brief descriptions of the titles of the columns in the following table.


- **Stock**: Name of the company
- **Symbol**: The stock symbol in the New York Stock Exchange (NYSE).
- **Open**: The last price paid for this stock at the end of the previous day or the price of the stock at the beginning of the day.
- **Last**: The last price paid for this stock at the end of the day or the closing price for the day.
- **High**: The highest price paid for the stock during the past year.
- **Low**: The lowest price paid for the stock during the past year.

| Stock | Symbol | Open | Last | High | Low |
|---|---|---|---|---|---|
| **Dell Inc.** | **DELL** | 40.16 | 40.17 | 40.30 | 39.81 |
| **International Business Machines** | **IBM** | 92.35 | 92.37 | 92.80 | 92.09 |
| **Apple Computer, Inc.** | **AAPL** | 39.64 | 39.60 | 40.26 | 39.10 |
| **General Electric Company** | **GE** | 35.85 | 36.02 | 36.10 | 35.80 |
| **Alltel Corporation** | **AT** | 56.42 | 56.59 | 56.67 | 56.33 |
| **Verizon Communications** | **VZ** | 36.21 | 36.21 | 36.30 | 36.11 |
| **Yahoo! Inc.** | **YHOO** | 32.43 | 31.73 | 32.56 | 31.60 |
| **NDC Health Corp.** | **NDC** | 15.00 | 14.89 | 15.21 | 14.80 |
| **Qwest Communications Intl Inc.** | **Q** | 3.88 | 3.85 | 3.96 | 3.80 |
| **Allstate Corporation** | **ALL** | 54.20 | 54.43 | 54.90 | 54.03 |

## Finding Profit or Loss:

*Profit* or *Loss* is found by comparing the quote of the share when it is bought with the quote when it is sold.

> *Example*: What profit or loss would Julian have if he bought 100 shares of NDC Stock at 31.73 and sold all of the shares for 32.56?
>
> *Step 1*: Find the difference between the purchase price and the selling price. Since Julian is selling the stock at a higher price than when he bought the stock, subtract the purchase price from the selling price to determine the profit on one share.

| | |
|---|---|
| 32.56 | Stock price for one share when Julian sold. |
| $\underline{-\ 31.73}$ | Stock price for one share when Julian purchased the stock. |
| 0.83 | Julian's profit on one share of NDC stock. |

The difference of one share is a positive number.

*Step 2*: Multiply the difference by the *number of shares bought*.

$$0.83 \times 100 = \$83.00$$

Julian made a PROFIT of $83.00

*Note: In this example, Julian made a profit. This is not always the case. Sometimes investors decide to sell their shares even though the value of the stock is less than when they purchased it. Their decisions are based on how they think the stock will perform in the future.