POLYNOMIALS: LONG DIVISION, SYNTHETIC DIVISION

In this unit you will multiply and divide polynomials using long division and synthetic division. At the conclusion of the unit the Remainder and Factor Theorems will be used to solve problems.

Long Division of Polynomials

Synthetic Division

Factor Theorem

Long Division of Polynomials

To use long division:

1) Make sure that all terms are in descending order.

2) Make sure that all terms are represented, for example, if there is no x^2 term in the polynomial insert $0x^2$ for that term that is missing.

Example #1: $\dfrac{x^3+3x^2-4x-12}{x-2}$

x^2+5x+6	
$x-2\overline{)\,x^3+3x^2-4x-12}$	how many times will x go into x^3
$-(x^3-2x^2)$ ←	multiply $x-2$ by x^2 and subtract
$5x^2-4x$ ←	bring down the $-4x$,
$-(5x^2-10x)$ ←	how many times will x go into $5x^2$
$6x-12$	multiply $x-2$ by $5x$ and subtract
$6x-12$	how many times will x go into $6x$
0	

Example #2: $(x^3-7x-6)\div(x+1)$

since there is no x^2 term, replace it with $0x^2$ to hold the place value

x^2-x-6	
$x+1\overline{)\,x^3+0x^2-7x-6}$	how many times will x go into x^3
$-(x^3+x^2)$ ←	multiply $x+1$ by x^2 and subtract
$-x^2-7x$ ←	bring down the $-7x$
$-(x^2-x)$ ←	how many times will x go into $-x^2$
$-6x-6$	multiply $x+1$ by $-x$ and subtract
$-6x-6$	how many times will x go into $-6x$
0	

Synthetic Division

Another form of division is called synthetic division. Synthetic division can be used to divide a polynomial only by a linear binomial of the form *x* – *r* and only uses the coefficients of each term. When using nonlinear divisors, long division must be used.

Example #1: Use synthetic division to solve: $(x^3+3x^2-4x-12)\div(x-2)$

Step #1: Write out the coefficients of the polynomial, and then write the *r*-value, 2, of the divisor *x* – 2. **Notice that you use the opposite of the divisor sign**. Write the first coefficient, 1, below the line.

2	1	3	–4	–12
	1			

Step #2: Multiply the *r*-value, 2, by the number below the line, 1, and write the product, 2, below the next coefficient.

2	1	3	–4	–12
		2		
	1			

Step #3: Write the **sum** (not the difference) of 3 and 2, (5), below the line. Multiply 2 by the number below the line, 5, and write the product, 10, below the next coefficient.

2	1	3	–4	–12
		2	10	
	1	5		

Step #4: Write the sum of –4 and 10, (6), below the line. Multiply 2 by the number below the line, 6, and write the product, 12, below the next coefficient.

2	1	3	–4	–12
		2	10	12
	1	5	6	0

The remainder is 0, and the resulting numbers 1, 5 and 6 are the coefficients of the quotient. The quotient will start with an exponent that is one less than the dividend.

The result is x^2+5x+6.

Thus, $(x^3+3x^2-4x-12)\div(x-2)=x^2+5x+6$.

Now let’s try one that has a remainder.

Example #2: $(x^3-2x^2-22x+40)\div(x-4)$

4	1	–2	–22	40
		4	8	–56
	1	2	–14	–16

Since there is a remainder in this problem, the answer is written using a fraction with the divisor, $x-4$, as the denominator. Notice that the sign between the last term and the fraction is the same as the sign of the remainder.

Answer: $x^2+2x-14-\dfrac{16}{x-4}$

Factor Theorem

Factor Theorem: $x - r$ is a factor of the polynomial expression that defines the function P, if and only if, r is a solution of $P(x) = 0$; that is, if and only if $P(r) = 0$.

What this is saying is that a binomial is a factor of a polynomial expression, if and only if, the numerical value r, when replaced for all the x values, yields 0.

Example #1: $x + 2$ is a factor of $x^3 - 2x^2 - 5x + 6$, if and only if, $f(-2) = 0$.

To find $f(-2)$ replace all x values with –2. Notice this is the opposite of the value given for the factor $x + 2$.

$f(-2) = (-2)^3 - 2(-2)^2 - 5(-2) + 6$ replace all x values with –2

$= -8 - 8 + 10 + 6$

$= -16 + 16$

$= 0$

This means that $x + 2$ is a factor of the polynomial because $f(-2) = 0$.

Example #2: Is $x - 1$ a factor of $f(x) = x^3 - x^2 - 5x - 3$?

$f(1) = (1)^3 - (1)^2 - 5(1) - 3$ replace all x values with 1

$= 1 - 1 - 5 - 3$

$= -8$

This means that $x - 1$ is not a factor of the polynomial because $f(1) \neq 0$.