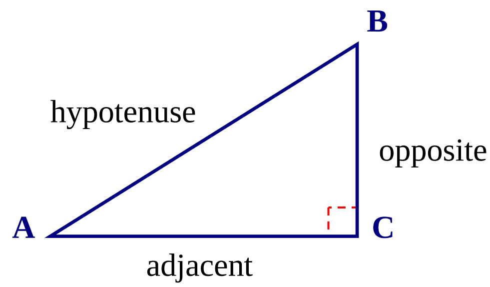
RIGHT TRIANGLE TRIGONOMETRY

Trigonometry is commonly described as the study of the relationship between the angles and sides of a triangle. In this unit you will find the trig functions of acute angles and solve right triangles by using trigonometric functions. The unit will conclude by finding coterminal and reference angles of theta, a Greek letter used to represent an angle, and angles of rotation in standard form.

Right Triangle Trigonometry

Trigonometry Functions

In this right triangle we are going to identity the six (6) trig functions associated with angle A. Notice that the sides of the triangle are labeled in relationship to angle A



$$\text{sine } \measuredangle \text{ A} = \frac{\textit{opposite}}{\textit{hypotenuse}} \qquad \text{cosecant } \measuredangle \text{ A} = \frac{\textit{hypotenuse}}{\textit{opposite}}$$

$$\text{cosine } \measuredangle \text{ A} = \frac{\textit{adjacent}}{\textit{hypotenuse}} \qquad \text{secant } \measuredangle \text{ A} = \frac{\textit{hypotenuse}}{\textit{adjacent}}$$

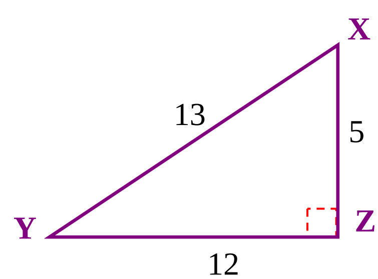
$$\text{tangent } \measuredangle \text{ A} = \frac{\textit{opposite}}{\textit{adjacent}} \qquad \text{cotangent } \measuredangle \text{ A} = \frac{\textit{adjacent}}{\textit{opposite}}$$

The above trigonometric functions are abbreviated as **sin**, **cos**, **tan**, **csc**, **sec**, and **cot**. Let's take a look at an example of how to identify the six (6) trig ratios of angle X in the following triangle. We will round our answers to the nearest ten thousandth.

$$\sin \measuredangle \text{ X} = \frac{12}{13} \approx .9231 \qquad \csc \measuredangle \text{ X} = \frac{13}{12} \approx 1.0833$$

$$\cos \measuredangle \text{ X} = \frac{5}{13} \approx .3846 \qquad \sec \measuredangle \text{ X} = \frac{13}{5} \approx 2.6$$

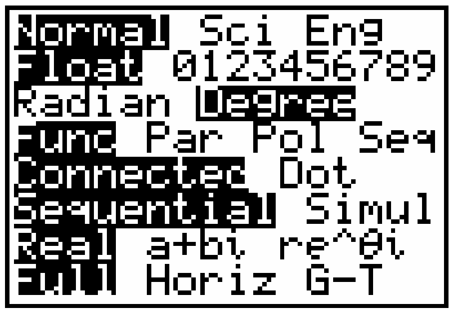
$$\tan \measuredangle \text{ X} = \frac{12}{5} \approx 2.4 \qquad \cot \measuredangle \text{ X} = \frac{5}{12} \approx .4167$$



Finding unknown information

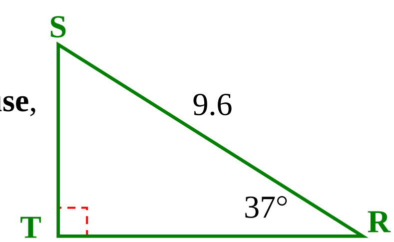
A given angle measure and the trig functions on your calculator can be used to find unknown lengths of the sides of a triangle.

***It is very important that your calculator is in Degree Mode. To do this, press the MODE button on your calculator to get the screen below. Make sure that the Degree is highlighted. To do this, move the cursor down by using the down arrow until you reach Radian, and then move the cursor right until the Degree is highlighted and press ENTER.**



Example #1: Find the length of *RT* and *ST* in the triangle below if angle *R* measures 37 degrees.

a.) Find the length of *RT*.
-Since *RT* is the **adjacent** side to angle *R* and the length of *SR* is known which is the **hypotenuse**, you can use the trig ratio that uses adjacent and hypotenuse which is **cos**.



$\cos 37 = \dfrac{RT}{9.6}$ -multiply both sides by 9.6

$(9.6)(\cos 37) = \left(\dfrac{RT}{9.6}\right)(9.6)$ -enter the left side in your calculator and round your answer to the nearest tenth.

$7.7 \approx RT$

b.) Find the length of *ST*.
-Since *ST* is the **opposite** side to angle *R* and the length of *SR* is known which is the **hypotenuse**, you can use the trig ratio that uses opposite and hypotenuse which is **sin**.

$\sin 37 = \dfrac{ST}{9.6}$ -multiply both sides by 9.6

$(9.6)(\sin 37) = \left(\dfrac{ST}{9.6}\right)(9.6)$ -enter the left side in your calculator and round your answer to the nearest tenth.

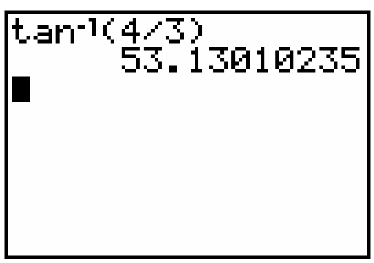
$5.8 \approx ST$

Inverse Trig Functions

When the trig ratio of an angle is known, you can find the measure of the angle by using the inverse of trig ratio. Follow the example below.

Example #2: If $\tan \measuredangle A = \dfrac{4}{3}$, to find the measure of $\measuredangle A$ use your calculator.

Press $\boxed{2^{nd}}$ $\boxed{\text{tan}}$ $(4 \div 3)$ $\boxed{\text{ENTER}}$. Your screen should look like the figure below.

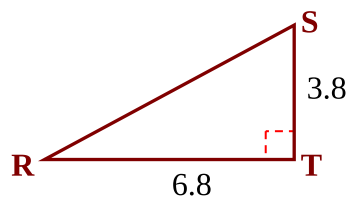


This tells us that the measure of $\measuredangle A$ rounded to the nearest whole degree is 53.

We will use this information to answer the following questions in the next example.

Example #3: Given the triangle at the right, find

a.) $m\measuredangle R$, b.) $m\measuredangle S$, and c.) the length of RS.



a.) $m\measuredangle R$ can be found by using the tan trig ratio because the opposite and adjacent sides to $\measuredangle R$ are known.

$$\tan \measuredangle R = \frac{3.8}{6.8}$$

*use the inverse tan on your calculator to find the measure of $\measuredangle R$

$$m\measuredangle R = \tan^{-1}\left(\frac{3.8}{6.8}\right)$$

$$m\measuredangle R \approx 29 \text{ degrees}$$

b.) $m\measuredangle S$ can be found by using the tan trig ratio because the opposite and adjacent sides to $\measuredangle$S are known.

$$\tan \measuredangle S = \frac{6.8}{3.8}$$

*use the inverse tan on your calculator to find the measure of $\measuredangle S$

$$m\measuredangle S = \tan^{-1}\left(\frac{6.8}{3.8}\right)$$

$$m\measuredangle S \approx 61 \text{ degrees}$$

***After you found the measure of $\measuredangle R$, you could have found the measure of $\measuredangle$S by subtracting the value of angle *R* from 90 degrees because the two unknown angles (R and S) are complimentary in a right triangle.**

c.) Find the length of *RS*.

-The length of *RS* can be found using different trig functions; however, for our purposes we are going to use the Pythagorean Theorem.

$$a^2 + b^2 = c^2$$

a and *b* represent the sides or legs of the triangle and *c* represents the hypotenuse:

$$(6.8)^2 + (3.8)^2 = (RS)^2$$

$$46.24 + 14.44 = (RS)^2$$

$$60.68 = (RS)^2$$

$$\sqrt{60.68} = RS$$

$$7.8 \approx \text{RS}$$

Angles of Rotation

trigonometry definition of an angle: a ray that is rotated around it's endpoint

*Each position of the rotated angle, relative to it's starting point, creates an angle of rotation, θ, theta. This Greek letter is commonly used to name an angle of rotation.

Definitions

initial side: the initial position of the angle

terminal side: the final position of the angle

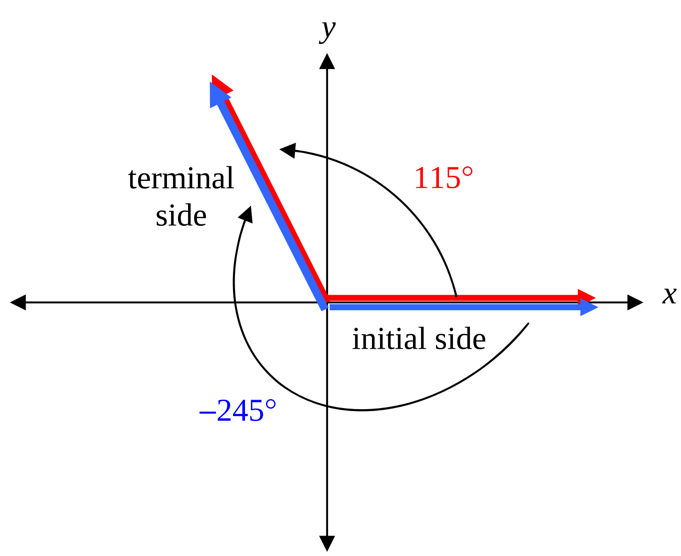
standard position: the initial side lies along the positive *x*-axis and the endpoint is at the origin.

*There are two types of measures of the angles of rotation

-If the direction of rotation is **counter clockwise**, the angle of rotation will have a **positive** measure.

-If the direction of rotation is **clockwise**, the angle of rotation will have a **negative** measure.

Let's take a look at a figure that represents the definitions given above.



*Notice that the **red** angle, which rotates **counter clockwise**, is **positive** while the **blue** angle, which rotates **clockwise**, is **negative**.

*Notice also that both angles share a terminal side. These angles are called **coterminal angles**.

Coterminal Angles

If given an angle and asked to find the coterminal angle θ such that $-360° < \theta < 360°$, you will add 360 to the given angle until your answer is not within the given range, and then you will subtract 360 from the given angle until your answer is not within the given range. You will only use answers that are within $-360° < \theta < 360°$ range.

Example #1: Find the coterminal angle(s) of $-210°$.

1.) $\theta = -210 + 360 = 150$

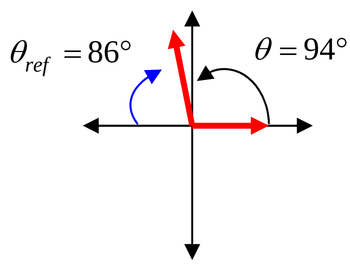
$\theta = 150 + 360 = 510$

2.) $\theta = -210 - 360 = -570$

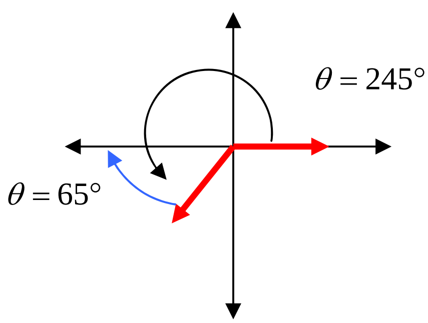
There are two answers that must be discarded, the 510 and the –570 are not within the range of $-360° < \theta < 360°$. Therefore the coterminal angle of $-210°$ is $150°$.

Reference Angles

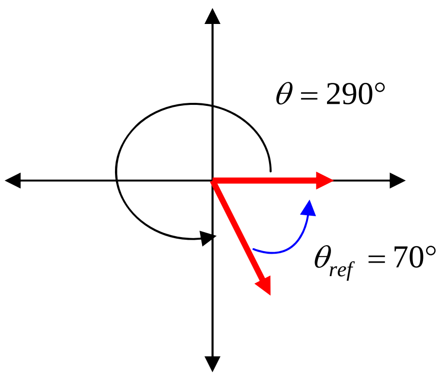
For an angle θ in standard position, the **reference angle** is the positive acute angle formed by the terminal side of θ and the nearest part of the *x*-axis. Follow along with the examples below.



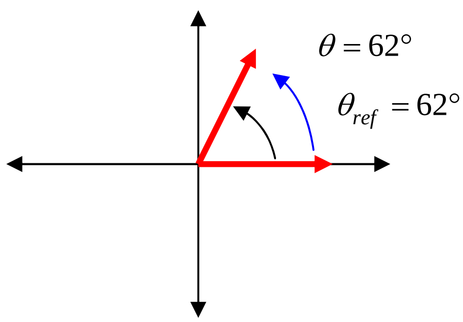
In this example θ is equal to $94°$. Its reference angle will be the measure of the angle from the *x*-axis to the terminal side of θ. In this case, if you subtract 94 from 180, the result will be the reference angle $86°$.



In this example θ is equal to $245°$. Its reference angle will be the measure of the angle from the *x*-axis to the terminal side of θ. In this case, if you subtract 180 from 245, the result will be the reference angle $65°$.



In this example θ is equal to $290°$. Its reference angle will be the measure of the angle from the *x*-axis to the terminal side of θ. In this case if you subtract 290 from 360 the result will be the reference angle.



In this example θ is equal to $62°$. Its reference angle will be the measure of the angle from the *x*-axis to the terminal side of θ. In this case it is the same angle; therefore, the reference angle is equal to $62°$.

Based on the examples above we can make a few conclusions. If the terminal side of an angle lies within certain quadrants, you can subtract certain numbers to determine the reference angle.

Quadrant I - The reference angle is the angle itself.

Quadrant II - The reference angle is: $|180-\theta|$.

Quadrant III - The reference angle is: $|\theta-180|$.

Quadrant IV - The reference angle is: $|360-\theta|$.

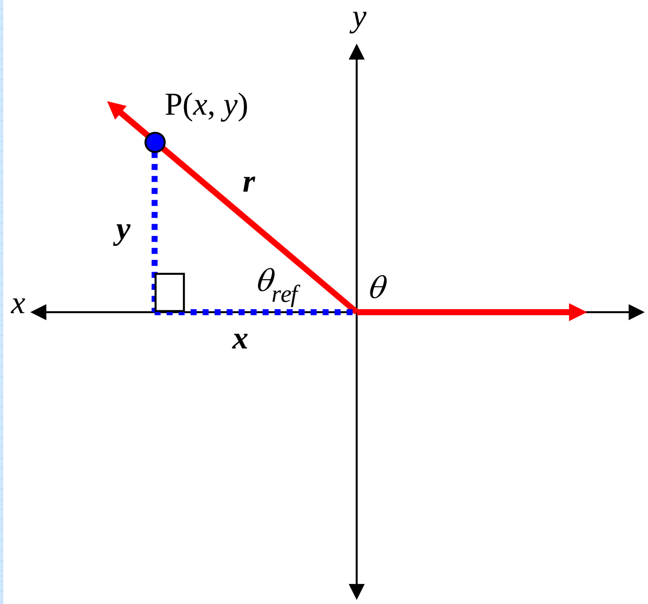
Trigonometric Functions of "theta"

Let P(x, y) be a point on the terminal side of θ in standard position. The distance from the origin to P is given by $r = \sqrt{x^2 + y^2}$.

$\sin\ \theta = \dfrac{y}{r}$ $\qquad$ $\csc\ \theta = \dfrac{r}{y}$

$\cos\ \theta = \dfrac{x}{r}$ $\qquad$ $\sec\ \theta = \dfrac{r}{x}$

$\tan\ \theta = \dfrac{y}{x}$ $\qquad$ $\cot\ \theta = \dfrac{x}{y}$



Example #2: Let P(–2, 1) be a point on the terminal side of θ in standard position. Find the exact value of the six (6) trigonometric functions of θ.

1.) Find r:

$$r = \sqrt{x^2 + y^2} \qquad \text{P}(-2,\ 1) \quad x = -2, y = 1$$

$$r = \sqrt{(-2)^2 + (1)^2}$$

$$r = \sqrt{5}$$

2.) use the values from above to determine the exact values of the six (6) trigonometric functions of θ.

$$\sin\ \theta = \frac{y}{r} = \frac{1}{\sqrt{5}} = \frac{1}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{5}}{5} \qquad \csc\ \theta = \frac{r}{y} = \frac{\sqrt{5}}{1} = \sqrt{5}$$

$\cos\,\theta\ =\ \frac{x}{r}=\frac{-2}{\sqrt{5}}=\frac{-2}{\sqrt{5}}\cdot\frac{\sqrt{5}}{\sqrt{5}}=-\frac{2\sqrt{5}}{5}$ $\sec\,\theta\ =\ \frac{r}{x}=\frac{\sqrt{5}}{-2}=-\frac{\sqrt{5}}{2}$

$\tan\,\theta=\frac{y}{x}=\frac{1}{-2}=-\frac{1}{2}$ $\cot\,\theta=\frac{x}{y}=\frac{-2}{1}=-2$

If you know what quadrant contains the terminal side of θ and the exact value of one of the trigonometric functions of θ, you can find the values of the other trigonometric functions.

Example #3: The terminal side of θ lies in Quadrant IV, and $\cos\,\theta\ =\ \frac{5}{13}$. Find the value of $\sin\,\theta$.

1.) If $\cos\,\theta\ =\ \frac{5}{13}$, this means that $x = 5$ and $r = 13$.

2.) If we want to find $\sin\,\theta$, then we are going to have to find y.

3.) If we are given x and r, we can find y by using the following:

$r=\sqrt{x^2+y^2}$

$13=\sqrt{5^2+y^2}$ –solve by squaring both sides

$(13)^2=(\sqrt{25+y^2})^2$

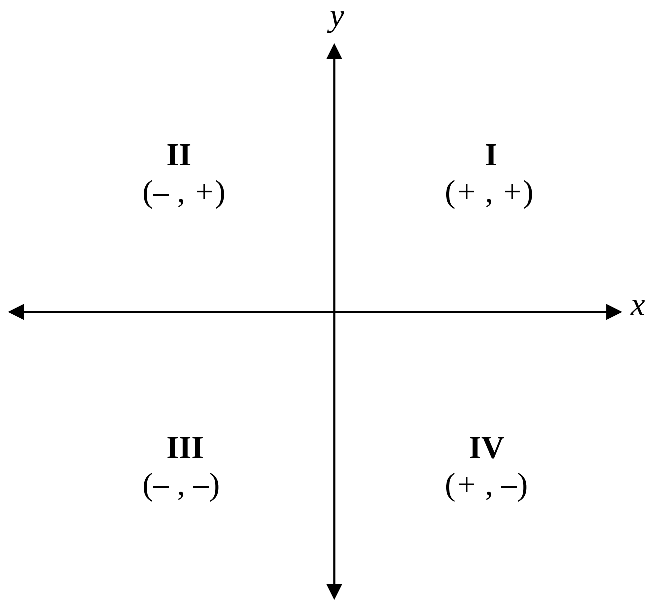
$169=25+y^2$

$144=y^2$

$\sqrt{144}=\sqrt{y^2}$

$\pm 12=y$

4.) Recall the signs of the ordered pairs in the four quadrants.



5.) If $\sin\,\theta = \frac{y}{r}$ and we know that $y = \pm 12$, $r = 13$, and we are in Quadrant IV, we must use the –12 because y in Quadrant IV is negative. Therefore:

$$\sin\theta = \frac{-12}{13}$$

Example #4: The terminal side of θ is in Quadrant III and cos $\theta = \frac{-1}{2}$, find tan θ.

1.) If cos θ = $\frac{-1}{2}$, we know that x = –1 and r = 2.

2.) If we want to find tan θ, we need to find y.

3.) If we are given x and r, we can find y by using the following:

$$r = \sqrt{x^2 + y^2}$$

$$2 = \sqrt{(-1)^2 + y^2}$$

$$(2)^2 = (\sqrt{1 + y^2})^2$$

$$4 = 1 + y^2$$

$$3 = y^2$$

$$\pm\sqrt{3} = y$$

4.) if tan θ = $\frac{y}{r}$ and we know that x = –1, $y = \pm\sqrt{3}$, and we are in Quadrant III, we need to decide if y is + or –. In this case it is negative so:

$$\tan\theta = \frac{-\sqrt{3}}{-1} = \sqrt{3}$$