REVIEW OF CONIC SECTIONS

This unit is a review of the conic sections and their equations. You will also review the distance formula and the midpoint formula.

Conic Sections

Distance Formula and Midpoint Formula

Conic Sections

Conic section	Equation	Vertex or center	Direction of opening
parabola	$y = a(x-h)^2 + k$ $x = a(y-k)^2 + h$	(h, k)	Up or down Left or right
circle	$(x-h)^2 + (y-k)^2 = r^2$	center (h, k) radius = $\sqrt{r^2}$	
ellipse	$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$	center (h, k)	
hyperbola	$\frac{(x-h)^2}{a^2} - \frac{(y-h)^2}{b^2} = 1$ $\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$	center (h, k) center (h, k)	Left and right Up and down

Equations of Conic Sections

A general equation for all conic sections is shown below.

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$$

If $A = C$, then the equation is a **circle**.

If A and C are different signs, then it is a **hyperbola**.

If A and C have the same sign, then it is an **ellipse**.

If A or $C = 0$ or (if there is only one squared term), then it is a **parabola**.

*Notice that A is the coefficient of x^2 and C is the coefficient of y^2.

Identify each of the following.

Example #1:

$x^2 - 2 = 2 - y^2$	Set the equation = to 0 and compare *A* and *C*.
$x^2 + y^2 - 4 = 0$	Since *A* = 1 and *C* = 1 and they are equal, this is a **circle**.

Example #2:

$r^2 - 3s = 46$	Set the equation = to 0 and compare *A* and *C*.
$r^2 - 3s - 46 = 0$	Since *C* is equal to 0, this is a **parabola**.

Example #3:

$4x^2 - 5 = y^2 + 2$	Set the equation = to 0 and compare *A* and *C*.
$4x^2 - y^2 - 7 = 0$	Since *A* = 4 and *C* = –1 and they have different signs, this is a **hyperbola**.

Example #4:

$2x^2 - 3 = 6 - 5y^2$	Set the equation = to 0 and compare *A* and *C*.
$2x^2 + 5y^2 - 9 = 0$	Since *A* = 2 and C = 5 and they have the same sign, this is an **ellipse**.

Distance Formula and Midpoint Formula

The **distance** between two points on a coordinate plane $A(x_1, y_1)$ and $B(x_2, y_2)$ is given by the following formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

The **midpoint** of a line segment with endpoints $A(x_1, y_1)$ and $B(x_2, y_2)$ can be found by using the following formula:

$$\mathrm{M} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Example #1: Given the points (–2, 5) and (4, –1), find the distance between the points.

$$d = \sqrt{(4-(-2))^2 + (-1-5)^2}$$

$$d = \sqrt{(6)^2 + (-6)^2}$$

$$d = \sqrt{36+36}$$

$$d = \sqrt{72}$$

$$d = \sqrt{36} \cdot \sqrt{2}$$

$$d = 6\sqrt{2}$$

The distance between points (–2, 5) and (4, –1) is $6\sqrt{2}$.

Example #2: Given the points (–2, 5) and (4, –1), find the midpoint of the line segment.

$$M = \left(\frac{-2+4}{2}, \frac{5+(-1)}{2}\right)$$

$$M = \left(\frac{2}{2}, \frac{4}{2}\right)$$

$$M = (1, 2)$$

The midpoint of the line segment between points (–2, 5) and (4, –1) is (1,2).

Example #3: Find the center, circumference, and area of the circle whose diameter has the endpoints P(–3, 6) and Q(7, –4).

The center is the midpoint of the diameter.

$$M = \left(\frac{-3+7}{2}, \frac{6+(-4)}{2}\right)$$

$$M = \left(\frac{4}{2}, \frac{2}{2}\right)$$

$$M = (2,1)$$

Find the length of the radius of the circle. This length is the distance between the center and either of the endpoints of the diameter.

We'll use the $M(2,1)$ and $P(-3,6)$ to find the length of the radius.

$$d = \sqrt{(2-(-3))^2 + (1-6)^2}$$

$$d = \sqrt{(5)^2 + (-5)^2}$$

$$d = \sqrt{25+25}$$

$$d = \sqrt{50}$$

$$d = r = 5\sqrt{2}$$

Next find the circumference and area of the circle.

$$C = 2\pi r \qquad A = \pi r^2$$

$$C = 2\pi(5\sqrt{2}) \qquad A = \pi(5\sqrt{2})^2$$

$$C = 10\pi\sqrt{2} \qquad A = 50\pi$$

The center of the circle whose diameter has the endpoints P(–3, 6) and Q(7, –4) is point $M(2,1)$; and the circle has a circumference of $10\pi\sqrt{2}$ and an area of 50π .

Example #4: Write the equation of a circle whose center is at (5, –2) and whose radius is 3.

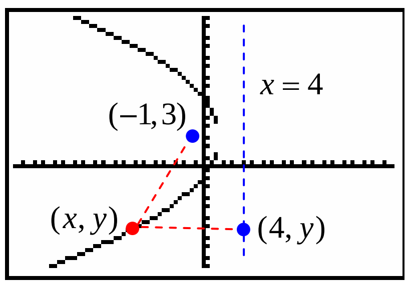
The equation of a circle is given by $(x-h)^2+(y-k)^2=r^2$, so you want to replace the *h* and *k* with the given center and the *r* with 3.

$$(x-5)^2+(y-(-2))^2=3^2$$

$$(x-5)^2+(y+2)^2=9$$ -This is the equation of the circle.

Example #5: Write the equation of a parabola that has a focus at (–1, 3) and a directrix of $x = 4$.

By the definition of a parabola, the distance between some point (*x*, *y*) and the focus is **equal** to the distance between the same point (*x*, *y*) and the directrix.



d between (*x*, *y*) and (–1, 3) = *d* between (*x*, *y*) and (4, *y*)

$$\sqrt{(x-(-1))^2+(y-3)^2}=\sqrt{(x-4)^2+(y-y)^2}$$

$$\sqrt{(x+1)^2+(y-3)^2}=\sqrt{(x-4)^2+0}$$ -square both sides

$$(x+1)^2+(y-3)^2=(x-4)^2$$ -expand the quantities containing the variable *x*

$$x^2+2x+1+(y-3)^2=x^2-8x+16$$ -solve for *x*

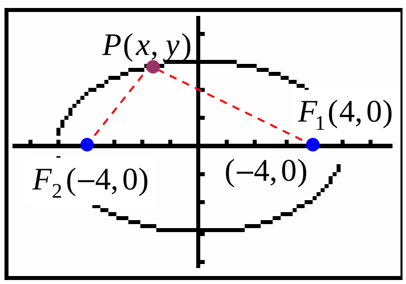
$$-15+(y-3)^2=-10x$$ -divide both sides by –10

$$\frac{3}{2}-\frac{1}{10}(y-3)^2 = x$$ -rearrange the terms so it is in the form $x = a(y-k)^2 + h$

$$x = \frac{-1}{10}(y-3)^2 + \frac{3}{2}$$ -standard form of the parabola

Example #6: Write the equation of an ellipse with $F_1(4, 0)$ and $F_2(-4, 0)$ and a constant 10.

By the definition of an ellipse, the distance from some point $P(x, y)$ and two fixed points F_1 and F_2 is a constant sum.



$$\overline{PF_2} + \overline{PF_1} = 10$$

distance between (x, y) and (4, 0) + distance between (x, y) and (–4, 0) = 10

$$\sqrt{(x-4)^2+(y-0)^2} + \sqrt{(x+4)^2+(y-0)^2} = 10$$ -isolate one radical

$$\left[\sqrt{(x-4)^2+y^2}\right]^2 = \left[10-\sqrt{(x+4)^2+y^2}\right]^2$$ -square both sides

$$(x-4)^2+y^2 = 100-20\sqrt{(x+4)^2+y^2}+(x+4)^2+y^2$$

$$x^2-8x+16+y^2 = 100-20\sqrt{(x+4)^2+y^2}+x^2+8x+16+y^2$$

$$-16x-100 = -20\sqrt{(x+4)^2+y^2}$$ -divide both sides by 4

$$\left[-4x-25\right]^2=\left[-5\sqrt{(x+4)^2+y^2}\right]^2$$ -square both sides

$$(-4x-25)(-4x-25)=25(x^2+8x+16+y^2)$$

$$16x^2+200x+625=25x^2+200x+400+25y^2$$ -put this in standard form

$$225=9x^2+25y^2$$

$$1=\frac{x^2}{25}+\frac{y^2}{9}$$ -standard form of the ellipse

Example #7: Write the equation of a hyperbola with $F_1(0,\ 2\sqrt{5})$ and $F_2(0,$ $-2\sqrt{5})$ and a constant difference of 4.

By the definition of a hyperbola the distances from some point *P*(*x*, *y*) and two fixed points F_1 and F_2 is a constant difference.

distance between (*x*, *y*) and (0, $2\sqrt{5}$) – distance between (*x*, *y*) and (0 $-2\sqrt{5}$) = 4

$$\sqrt{(x-0)^2+(y+2\sqrt{5})^2}-\sqrt{(x-0)^2+(y-2\sqrt{5})^2}=4$$ -isolate a radical

$$\sqrt{x^2+(y+2\sqrt{5})^2}=4+\sqrt{x^2+(y-2\sqrt{5})^2}$$ -square both sides

$$x^2+y^2+4y\sqrt{5}+20=16+8\sqrt{x^2+(y-2\sqrt{5})^2}+x^2+y^2-4y\sqrt{5}+20$$

$$8y\sqrt{5}-16=8\sqrt{x^2+(y-2\sqrt{5})^2}$$ -÷ both sides by 8

$$y\sqrt{5}-2=\sqrt{x^2+(y-2\sqrt{5})^2}$$ -square both sides

$$(y\sqrt{5}-2)(y\sqrt{5}-2)=\sqrt{x^2+(y-2\sqrt{5})^2}$$

$$5y^2-4y\sqrt{5}+4=x^2+y^2-4y\sqrt{5}+20$$

$$4y^2-x^2=16$$ -divide by 16

$$\frac{y^2}{4}-\frac{x^2}{16}=1$$ -standard form of the hyperbola