RADICAL EXPRESSIONS AND EQUATIONS

In this unit you will learn to simplify and manipulate radical expressions so that you can solve radical equations and inequalities.

Domain of a Square Root Function

Simplifying Radical Expressions

Product and Quotient Properties of Radicals

Domain of a Square Root Function

The domain of a function is the set of all real-number values of *x*; therefore, the domain of a square root function, $f(x) = \sqrt{x}$, does not include negative numbers.

To find the domain:

1.) set the numbers under the radical sign $\geq$ to 0
2.) solve the inequality
3.) the result will be your domain

Example #1: Find the domain of $h(x) = \sqrt{-4x+7}$

$$-4x + 7 \geq 0$$

$$-4x \geq -7$$

$$x \leq \frac{7}{4}$$

*this is the domain which means that all of your *x*-values must be less than or equal to $\frac{7}{4}$.

Example #2: Find the domain of $g(x) = \sqrt{5x+18}$

$$5x + 18 \geq 0$$

$$5x \geq -18$$

$$x \geq \frac{-18}{5}$$

The domain of $g(x)$ is all *x*-values greater than or equal to $\frac{-18}{5}$.

Simplifying Radical Expressions

In the expression $\sqrt[n]{a^p}$ the $\sqrt{\ }$ is called the **radical**, n is the **index**, a is the **radicand** and p is the **power**.

To simplify:

-divide p by n, this is the exponent of the variable outside the radical sign.

-if there is a remainder, this is the new exponent of the variable under the radical sign.

You may want to remember that when you have an even index, the answer is either positive or negative. Such as $\sqrt{4} = \pm 2$. When this is required, we use an absolute value symbol. In this unit you may want to only consider the positive square root.

For example, $\sqrt{32} = \sqrt{16} \cdot \sqrt{2} = |4|\sqrt{2}$. However, for our purposes, we will not be concerned with the absolute value notation on even roots. This will become more significant in later courses.

Example #1: Evaluate $\frac{2}{3}\sqrt[3]{-27} - 5$.

$$\frac{2}{3}\sqrt[3]{-27} - 5$$

$$\frac{2}{3}(-3) - 5 \qquad \text{*the cubed-root of } -27 = -3$$

$$-2 - 5 = -7$$

Example #2: Express $3\sqrt[4]{80}$ as a simplified radical.

1.) factor 80 into 16 $\times$ 5 $\qquad 3 \cdot \sqrt[4]{16} \cdot \sqrt[4]{5}$

2.) find the 4th root of 16 $\qquad 3 \cdot 2 \cdot \sqrt[4]{5}$

3.) simplify $\qquad 6\sqrt[4]{5}$

Example #3: Simplify $\sqrt{x^3y^4}$.

This is a square root and the index is a 2. When an index is not written in a radical, it is understood to be an index of 2.

Divide each of the exponents by the index 2. This will be the new exponent of the variable outside of the radical.

$$xy^2\sqrt{x}$$

Since there was a remainder of 1 when the exponent 3 was divided by the index 2, there is still an x inside the radical sign.

Example #4: Simplify $\sqrt[3]{125x^6yz^5}$.

Since the index is odd, we do not have to worry about absolute value signs.

Take the cubed root of 125 and divide each of the exponents by 3. Any remainders will stay inside the radical sign.

$$5x^2z\sqrt[3]{yz^2}$$

*If the radicand is **not** a perfect root, then we will factor it into perfect roots, if possible.

Example #5: In $\sqrt{50a^3b^4}$, 50 is not a perfect square root; but it can be factored using a perfect square root.

1.) factor 50 into 25 × 2 $\sqrt{25}\cdot\sqrt{2a^3b^4}$

2.) find the square root of 25 $5\sqrt{2a^3b^4}$

3.) simplify the variables $5ab^2\sqrt{2a}$

Example #6: In $\sqrt[3]{250r^7s^2t^3}$, 25 is not a perfect cubed root, but it can be factored using a perfect cubed root.

1.) factor 250 into 125 × 2 $\sqrt[3]{125}\cdot\sqrt[3]{2r^7s^2t^3}$

2.) find the cubed root of 125 $5\sqrt[3]{2r^7s^2t^3}$

3.) simplify the variables $5r^2t\sqrt[3]{2rs^2}$

Product and Quotient Properties of Radicals

If a term has a rational exponent, it can be rewritten in radical form in the following way.

$2^{\frac{4}{5}}$ can be rewritten as $\sqrt[5]{2^4}$.

You can see that the **numerator** became the **exponent** of the radicand and the **denominator** became the **index**.

This process can be reversed.

$\sqrt[3]{13^2}$ can be written using a rational exponent as $13^{\frac{2}{3}}$.

Product Property of Radicals

The product property states that you can multiply two radicals together if they have the same index.

$$\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$$

Quotient Property of Radicals

The quotient property of radicals states that you can divide radicals if they have the same index.

$$\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$$

Example #1: Find the product of $\sqrt{5ab^5} \cdot \sqrt{12ab^6}$.

$\sqrt{60a^2b^{11}}$	Does 60 contain a perfect square factor? yes
$\sqrt{4}\sqrt{15a^2b^{11}}$	find the square root of 4 and simplify the variable exponents.
$2ab^5\sqrt{15b}$	

Example #2: Find the quotient of $\dfrac{9\sqrt[3]{48x^8}}{\sqrt[3]{2x^3}}$.

$$\frac{9\sqrt[3]{48x^8}}{\sqrt[3]{2x^3}} = 9\sqrt[3]{\frac{48x^8}{2x^3}}$$

Divide $\sqrt[3]{48x^8}$ by $\sqrt[3]{2x^3}$.

$9\sqrt[3]{24x^5}$	Does 24 contain a perfect cubed factor? yes, simplify.
$9\sqrt[3]{8}\cdot\sqrt[3]{3x^5}$	find the cubed root of 8 and simplify the variable exponent.
$9\cdot 2\cdot x\sqrt[3]{3x^2}$	multiply 9 and 2.
$18x\sqrt[3]{3x^2}$	