

RATIONAL EXPRESSIONS AND EQUATIONS

In this unit you will learn how to multiply, divide, add, and subtract rational expressions. You will also simplify rational expressions. The lesson concludes with solving rational equations.

Multiplying and Dividing Rational Expressions

Adding and Subtracting Rational Expressions

Solving Rational Equations

Multiplying and Dividing Rational Expressions

To simplify:

*factor the numerator and denominator

*cancel any common factors

Example #1: Simplify $\frac{x^2 - 10x + 9}{x^2 + 2x - 3}$.

$$\frac{(x-9)\cancel{(x-1)}}{(x+3)\cancel{(x-1)}} \quad \text{cancel the } (x-1) \text{ that is common}$$

$$= \frac{(x-9)}{(x+3)} \quad \text{*you **can not** cancel the 9 and the 3 because they are connected to the } x \text{ values by } (-) \text{ and } (+) \text{ signs}$$

To multiply:

*factor **all** numerators and denominators

*cancel any common factors

*multiply straight across

Example #2: Multiply $\frac{x^2 - 4x - 5}{x^2 - 3x + 2} \cdot \frac{x^2 - 4}{x^2 - 3x - 10}$.

$$\frac{(x-5)(x+1)}{(x-2)(x-1)} \cdot \frac{(x+2)(x-2)}{(x-5)(x+2)}$$

$$\frac{\cancel{(x-5)}(x+1)}{\cancel{(x-2)}(x-1)} \cdot \frac{\cancel{(x+2)}\cancel{(x-2)}}{\cancel{(x-5)}\cancel{(x+2)}}$$

cancel the $(x-5)$, $(x+2)$ and $(x-2)$ because these are all common factors

$$= \frac{(x+1)}{(x-1)}$$

To divide:

*multiply by the reciprocal, change the problem to a multiplication problem, and use the reciprocal (flip) of the rational expression you are dividing by

*factor all numerators and denominators

*cancel any common factors

*multiply straight across

Example #3: Divide $\frac{4x^3 - 9x}{2x - 7} \div \frac{3x^3 + 2x^2}{4x^2 - 14x}$.

$$\frac{4x^3 - 9x}{2x - 7} \cdot \frac{4x^2 - 14x}{3x^3 + 2x^2}$$

$$\frac{x(4x^2 - 9)}{2x - 7} \cdot \frac{2x(2x - 7)}{x^2(3x + 2)}$$

$$\frac{\cancel{x}(2x + 3)(2x - 3)}{\cancel{2x} - 7} \cdot \frac{\cancel{2x}(2x - 7)}{\cancel{x}^2(3x + 2)}$$

cancel the x^2 with the x in the first expression and the x in the second expression and the $2x - 7$'s can cancel

$$\frac{2(4x^2 - 9)}{3x + 2} = \frac{8x^2 - 18}{3x + 2}$$

multiply straight across

Let's try a couple more examples.

Example #4: Simplify $\frac{x^2 - 9}{x^2 + 5x + 6}$

a.) factor the numerator and denominator

$$\frac{(x + 3)(x - 3)}{(x + 3)(x + 2)}$$

b.) cancel any common factors

$$\frac{\cancel{(x + 3)}(x - 3)}{\cancel{(x + 3)}(x + 2)}$$

$$= \frac{x-3}{x+2}$$

Example #5: Multiply $\frac{(m-3)^2}{m^2-6m+9} \cdot \frac{m^3-9m}{m^2-9}$

a.) factor all numerators and denominators

$$\frac{(m-3)(m-3)}{(m-3)(m-3)} \cdot \frac{m(m^2-9)}{(m-3)(m+3)}$$

$$\frac{(m-3)(m-3)}{(m-3)(m-3)} \cdot \frac{m(m-3)(m+3)}{(m-3)(m+3)}$$

b.) cancel any common factors

$$\frac{\cancel{(m-3)}\cancel{(m-3)}}{\cancel{(m-3)}\cancel{(m-3)}} \cdot \frac{m\cancel{(m-3)}\cancel{(m+3)}}{\cancel{(m-3)}\cancel{(m+3)}}$$

$$= m$$

Example #6: Divide $\frac{x^2-4}{x+3} \div \frac{x^2-4x+4}{x^2+3x}$

a.) factor all numerators and denominators, change to multiplication, and multiply by the reciprocal.

$$\frac{(x+2)(x-2)}{x+3} \cdot \frac{x(x+3)}{(x-2)(x-2)}$$

b.) cancel any common factors and multiply straight across

$$\frac{(x+2)\cancel{(x-2)}}{\cancel{x+3}} \cdot \frac{x\cancel{(x+3)}}{(x-2)\cancel{(x-2)}}$$

$$= \frac{x(x+2)}{(x-2)}$$

Adding and Subtracting Rational Expressions

To add and subtract rational expressions, you must have a common denominator. If you do, just add the numerators together and keep the common denominator.

Example #1: Find the sum: $\frac{9x}{x+7} + \frac{2x}{x+7}$

$$\frac{9x}{x+7} + \frac{2x}{x+7} = \frac{11x}{x+7}$$

Example #2: Find the difference: $\frac{7x}{x-4} - \frac{4x+12}{x-4}$

After adding or subtracting the numerators, if possible, factor and cancel the common factors.

$$\begin{aligned}\frac{7x}{x-4} - \frac{4x+12}{x-4} &= \frac{7x - (4x+12)}{x-4} \\ &= \frac{7x - 4x - 12}{x-4} \\ &= \frac{3x - 12}{x-4} \\ &= \frac{3(x-4)}{\cancel{x-4}} = 3\end{aligned}$$

*If the denominators are not the same

- factor all denominators
- find all common factors
- multiply the denominators by what is missing in the common denominator

Example #3: Find the difference: $\frac{x}{x+6} - \frac{72}{x^2-36}$

$$\frac{x}{x+6} - \frac{72}{(x+6)(x-6)}$$

$$\frac{x}{x+6} \left(\frac{x-6}{x-6} \right) - \frac{72}{(x+6)(x-6)}$$

the least common denominator will be $(x+6)(x-6)$ because those are the factors that are combined between the denominators

$$\frac{x^2-6x}{(x+6)(x-6)} - \frac{72}{(x+6)(x-6)}$$

multiply the first expression by $\frac{x-6}{x-6}$ so that you will have a common denominator

$$\frac{x^2-6x-72}{(x+6)(x-6)}$$

factor the numerator

$$\frac{(x-12)\cancel{(x+6)}}{\cancel{(x+6)}(x-6)}$$

cancel common factors

$$= \frac{x-12}{x-6}$$

Let's try another example.

Example #4: Find the difference: $\frac{x}{3x-3} - \frac{x}{x^2-1}$

$$\frac{x}{3(x-1)} - \frac{x}{(x-1)(x+1)}$$

LCD will be $3(x-1)(x+1)$

$$\frac{x}{3(x-1)} \left(\frac{x+1}{x+1} \right) - \frac{x}{(x-1)(x+1)} \left(\frac{3}{3} \right)$$

$$\frac{x^2+x}{3(x-1)(x+1)} - \frac{3x}{3(x-1)(x+1)}$$

$$\frac{x^2+x-3x}{3(x-1)(x+1)}$$

factor the numerator to see if you can cancel any common factors; in this case you can not, so either of the answers are correct

$$\frac{x^2-2x}{3(x-1)(x+1)} \quad \text{OR} \quad \frac{x(x-2)}{3(x-1)(x+1)}$$

Solving Rational Equations

To solve rational equations:

- find all excluded values by factoring the denominator and setting all factors equal to zero and solving
- find the LCD and multiply all terms by the LCD to eliminate fractions
- solve the equation and check your answer

Example #1: Solve $\frac{x}{x+3} = \frac{6}{x-1}$.

$$x+3=0$$

$$x \neq -3$$

$$x-1=0$$

$$x \neq 1$$

Both of these values will produce a zero in the denominator, so they must be excluded from the solution.

The LCD will be $(x+3)(x-1)$.

$$(x+3)(x-1)\left(\frac{x}{x+3}\right) = (x+3)(x-1)\left(\frac{6}{x-1}\right) \quad \text{Multiply both sides of the equation by } (x+3)(x-1).$$

$$\cancel{(x+3)}(x-1)\left(\frac{x}{\cancel{x+3}}\right) = (x+3)\cancel{(x-1)}\left(\frac{6}{\cancel{x-1}}\right) \quad \text{Cancel common factors.}$$

Multiply and solve.

$$x(x-1)x = (x+3)6$$

$$x^2 - x = 6x + 18$$

$$x^2 - 7x - 18 = 0$$

$$(x-9)(x+2) = 0$$

$$x-9=0$$

$$x+2=0$$

$$x=9$$

and

$$x=-2$$

Check your answers by replacing x with 9 first then replacing it with -2 .

$$\frac{9}{9+3} = \frac{6}{9-1}$$

$$\frac{-2}{-2+3} = \frac{6}{-2-1}$$

$$\frac{9}{12} = \frac{6}{8} \quad \text{true} \qquad \frac{-2}{1} = \frac{6}{-3} \quad \text{true}$$

Example #2: Solve $\frac{x}{x+2} - \frac{8}{x^2-4} = \frac{2}{x-2}$.

Step #1: Factor all denominators to find the LCD and excluded values. In this case the LCD is $(x-2)(x+2)$ and the excluded values are 2 and -2 because if x is equal to either of these, the denominator will become zero.

$$\frac{x}{x+2} - \frac{8}{(x-2)(x+2)} = \frac{2}{x-2}$$

Step #2: Multiply each term by the LCD to eliminate all fractions.

$$(x-2)(x+2) \left(\frac{x}{x+2} \right) - (x-2)(x+2) \left(\frac{8}{(x-2)(x+2)} \right) = (x-2)(x+2) \left(\frac{2}{x-2} \right)$$

$$x(x-2) - 8 = 2(x+2)$$

$$x^2 - 2x - 8 = 2x + 4$$

Step #3: Combine all like terms, factor and solve.

$$x^2 - 4x - 12 = 0$$

$$(x-6)(x+2) = 0$$

$$x-6=0 \qquad x+2=0$$

$$x=6 \qquad x=-2$$

Since -2 is an excluded value, the only solution is 6.