

POLYNOMIALS

Polynomial functions are important in real-world situations. For example a polynomial function can model the relationship between the dimensions of a rectangular box and its volume. Polynomial functions can also model the volume of irregularly shaped buildings. In this lesson you will identify, evaluate, add, subtract, and classify polynomials.

Introduction to Polynomial Functions

Evaluating Polynomial Functions

Adding and Subtracting Polynomials

Products and Factors of Polynomials

Introduction to Polynomial Functions

monomial: a numeral, a variable, or a product of a numeral and one or more variables.

Examples: 5, x , $-3xy$

constant: a monomial with no variable, basically any number that has a constant value.

Examples: 6, -9

coefficient: the numerical factor of a monomial.

Examples: in $3xy$ the coefficient is 3, in $\frac{x}{6}$ (also written as $\frac{1}{6}x$) the coefficient is $\frac{1}{6}$

degree of a monomial: the sum of the exponents of the variables.

Example: the monomial x^2yz (also written as $x^2y^1z^1$) has a degree of 4

polynomial: a monomial or a sum of terms that are monomials (there can not be any variable exponents or any variables in the denominator of a term).

Binomials and trinomials are polynomials used frequently in algebra.

binomial: a polynomial with two terms.

Example: $3x + 5$

trinomial: a polynomial with three terms.

Example: $-4x^2 + 6x - 8$

degree of a polynomial: the largest monomial degree of the polynomial.

Example: $9x^4 + 3x^3 + 4x^2 - x + 1$ has a degree of 4 because the highest exponent is 4.

Example #1: Evaluate the polynomial for $x = 3$.

$$-x^2 - 7x + 11$$

$$-(-3)^2 - 7(-3) + 11$$

$$-9 + 21 + 11$$

$$23$$

Replace x with (-3) and simplify.

The polynomial evaluates to 23 when $x = 3$.

Evaluating Polynomial Functions

To evaluate a polynomial function for a specific value of x , replace all x variables with the value given.

Example #1: Evaluate $x^3 - 2x^2 + 4x - 6$ for $x = -2$.

$$(-2)^3 - 2(-2)^2 + 4(-2) - 6 \quad \text{replace all } x\text{'s with } -2$$

$$(-8) - 2(4) + 4(-2) - 6 \quad \text{simplify}$$

$$-8 - 8 - 8 - 6$$

$$-30$$

The value of $x^3 - 2x^2 + 4x - 6$ for $x = -2$ is -30 .

Adding and Subtracting Polynomials

To add or subtract polynomials combine like terms; this means add or subtract the coefficients of any terms that have the same variable and the same exponent. Make sure the like terms are rewritten next to each other; this will make it easier to add or subtract.

When **subtracting polynomials**, make sure to **change all the signs** of the **second quantity** before you rewrite the problem.

Example #1: $(6x^3 + 3x^2 - 4) + (10 - 3x - 5x^2 + 3x^3)$

$$6x^3 + 2x^3 + 3x^2 - 5x^2 - 3x - 4 + 10$$

arrange the terms in descending powers
collect like terms

$$8x^3 - 2x^2 - 3x + 6$$

Example #2: $(5x^2 - 6x - 11) - (-8x^3 + x^2 + 2)$

$$5x^2 - 6x - 11 + 8x^3 - x^2 - 2$$

*notice that all the signs have changed on the second quantity.

$$8x^3 + 5x^2 - x^2 - 6x - 11 - 2$$

$$8x^3 + 4x^2 - 6x - 13$$

Products and Factors of Polynomials

To write a function in standard form means to multiply all terms together and write them in descending order (largest to smallest).

Example #1: $f(x) = (x-1)(x+4)(x-3)$

$$f(x) = (x^2 + 3x - 4)(x-3) \quad \text{FOIL the first two quantities}$$

$$f(x) = x^2(x-3) + 3x(x-3) - 4(x-3) \quad \text{use the distributive property to multiply}$$

$$f(x) = x^3 - 3x^2 + 3x^2 - 9x - 4x + 12 \quad \text{combine like terms}$$

$$f(x) = x^3 - 13x + 12$$

Factoring Polynomials

Just as a quadratic expression is factored by writing it as a product of two factors, a polynomial expression of a degree greater than 2 is factored by writing it as a product of more than two factors.

Example #2: $x^3 - 10x^2 + 16x$ factor out the GCF of x

$$x(x^2 - 10x + 16) \quad \text{factor the trinomial into binomials}$$

$$x(x-8)(x-2)$$

Example #3: $x^3 + 6x^2 - 5x - 50$ this polynomial can be factored in pairs

$$(x^3 + 6x^2) + (-5x - 50) \quad \text{group the terms in pairs}$$

$$x^2(x+6) - 5(x+6) \quad \text{factor out the GCF of each pair to produce a quantity that is common}$$

$$(x+6)(x^2 - 5) \quad \text{factor } (x+6) \text{ from each term}$$

Factoring the Sum and Difference of Cubes

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

Example #4: Factor $x^3 + 27$.

$$(x + 3)(x^2 - 3x + 9)$$

The x in the first quantity comes from $\sqrt[3]{x^3}$.

The 3 in the first quantity comes from $\sqrt[3]{27}$.

The x^2 in the second quantity comes from squaring x in the first quantity.

The $3x$ in the second quantity comes from multiplying both terms in the first quantity.

The 9 in the second quantity comes from squaring the 3 in the first quantity.

The signs for the sum of cubes will always be $(+, - +)$.

Example #5: Factor $x^3 - 125$.

$$(x - 5)(x^2 + 5x + 25)$$

The signs for the difference of cubes will always be $(-, + +)$.