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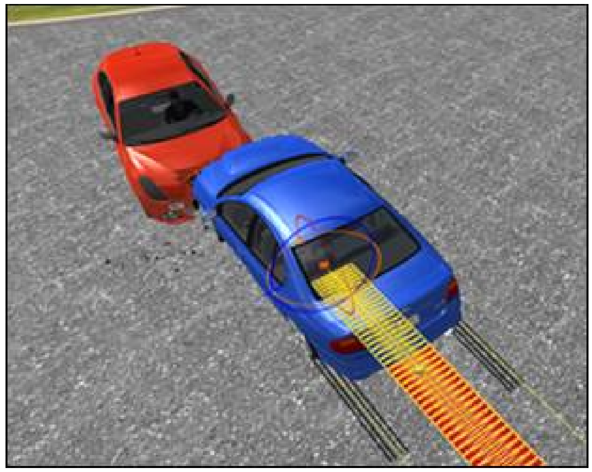


LINEAR MOMENTUM

Unit Overview

During this unit, we will learn how velocity and mass work together to provide an object with momentum. Momentum also affects the rate at which objects are able to speed up, or slow down. Depending on an objects momentum also determines what happens during various types of collisions. These scenarios will be explored during this unit.

Introduction to Momentum

In the previous unit we learned about the law of conservation of energy, and how energy is not created or destroyed, only transferred from one form to another. Besides energy, there are other quantities that are conserved, such as linear momentum, angular momentum, as well as electric charge. Therefore it is important to remember that momentum is conserved in a problem. If a collision take place where two objects that both have momentum hit each other, the total amount of momentum of the system will never will change. The velocities and masses of the two objects may change, but their collective amount of momentum doesn't change.

Momentum is defined as the product of an object's mass and velocity. Momentum is represented by the symbol **p**. Therefore the equation that we will be using for momentum is:

$$\mathbf{P = mv} \qquad \text{(momentum = mass X velocity)}$$

The units for momentum are simply kg*m/s since we are multiplying mass and velocity.

Because velocity is a vector, momentum is also a vector, so it is important to think about an object's direction of motion (velocity is negative for an object travelling west or south).

When thinking about objects with momentum, it is fair to say that a faster moving truck contains more momentum than a slower identical truck. Likewise a truck that has more mass than a car traveling at the same speed will have more momentum. Momentum depends on mass and velocity; therefore two objects of different masses may have the same momentum as long as their velocities are proportional. The more momentum an object has, the harder it is to change its direction or slow it down.

Any object with mass that is moving, has momentum. There are a lot of times when an additional force acts on an object to change its momentum. This force acting over a time period to change an object's momentum is called ***impulse***. Impulse defines the rate of change of momentum in that a large force, will result in a large change of momentum. Inversely the longer it takes to change momentum, the less force acts on the object. There are many examples of how impulse affects momentum. In other words, the impulse tells us that we can get the same change in momentum with a large force acting for a short time, or a small force acting for a longer time. This relationship between impulse and momentum is called the ***impulse-momentum theorem.***

Examples of impulse:

- This is why you should bend your knees when you land
- why airbags work
- why landing on a pillow hurts less than landing on concrete.

All of the above examples show how an increase in time, before coming to a stop, will decrease the force of impact.

The equation for impulse:

Impulse = force X time

A force is required to change the momentum of an object; therefore *the rate of change of momentum of an object is equal to the net force applied to it.* According to Newton's 2nd law (formally learned as F=ma) we can also apply this to momentum by deriving the equation;

F = Δp/t (force = momentum / time) ** remember the delta (**Δ**) means change in …. Final – initial.

Another way to write this equation in order to show all variables involved is written below:

Ft = Δmv (force X time = change in mass X velocity)

In the following video the physics tutor defines momentum, force, and impulse. Momentum has a magnitude and direction, and is equal to the mass times the velocity. Force is defined as the change in momentum over the change in time, while impulse is equal to the force times the amount of time the force is applied.

QuickTime **Momentum and Impulse Defined**

Examples Problems
1. A 54458 gram dog is running at a speed of 12 m/s. a. **What is the dogs initial momentum?** b. **If the dog then comes to rest in 8 seconds, how much force did the dog use to stop?**

a. **GIVEN**
mass = 54.458 Kg (mass must always be in kg)
initial velocity = 12 m/s
final velocity = 0 m/s

SOLVING FOR
initial momentum = ?

EQUATION
P = mv
P = 54.458(12) = **653.5 kg*m/s**

b. **GIVEN**
momentum from part a = 653.5 kg*m/s
time = 8 seconds
final momentum = 0 kg*m/s (because the dog is now at rest, so velocity = 0, therefore momentum is 0)

SOLVING FOR
force = ?

EQUATION
$Ft = \Delta mv$
F(8) = 0 - 653.5
F = -81. 7 N **** knowing that force is negative indicates that it is acting in the opposite direction of motion of the object, slowing it down**

2. A 670 kg sports car starts at a speed of 10 m/s and accelerates at a rate of 2 m/s^2 for 7 seconds. What is its momentum at the end of the 7 seconds?

GIVEN
Mass = 670 kg
Initial velocity = 10 m/s
Acceleration = 2 m/s^2
Time = 7 seconds

SOLVING FOR
momentum = ?

** Before we can solve for momentum at the end of the 7 seconds, we need to first find the velocity at the end of the 7 seconds since momentum = mv. So don't forget your kinematics equations!!

EQUATIONS
Vf = Vo +at
Vf = 10 +2(7)
Vf = 24 m/s
P = mv
P = 670(24)
P = 16080 kg*m/s

3. A constant force of 1200 N is acting on a car for 40 seconds to slow it down; the car has a mass of 4800 Kg, what is the change in velocity of the car?

GIVEN
Force = -1200 N (in order to slow it down, force is acting in an opposite direction of movement, therefore force is negative)
Time = 40 s
Mass = 4800 kg

SOLVING FOR
velocity = ?

EQUATION
Ft = mv
(-1200)(40) = (4800) (v)
-48000 = 4800v
v = -10 m/s

** Since we are finding the change in velocity – we do not need to worry about the initial – final. Whatever number we derive from the equation will indicate how much the car has slowed down.

**Velocity slowed by 10 m/s

Collisions

Outside forces can be applied to objects to cause a change in momentum. Sometimes these forces come from collisions between two or more objects, each containing their own momenta. As we learned in the beginning of the unit, momentum is conserved during a collision just like energy was conserved in the previous unit. During a collision, velocity or mass of the total system will change in order to have the same total momentum before and after the collision of all objects involved. Because it is conserved, we yield the equation:

$$p_{before} = p_{after} \text{ or}$$

$$m_A \vec{\mathbf{v}}_A + m_B \vec{\mathbf{v}}_B = m_A \vec{\mathbf{v}}'_A + m_B \vec{\mathbf{v}}'_B$$

Where Ma is the mass of object a, Va is the velocity of object a. Likewise Mb is the mass of object b and Va is the velocity of object b. It is important to remember that everything on the left side of the equal sign is before the collision takes place, and everything on the right side is after the collision occurs.

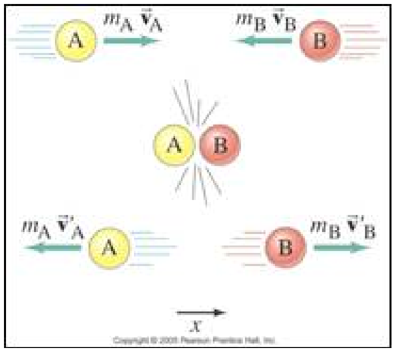
There are three types of collisions that we are going to explore in this unit, they are:

1. elastic
2. inelastic
3. "explosions"

In all cases of collisions, momentum is conserved. A big difference between elastic and inelastic collisions is that kinetic energy is also conserved in elastic collisions, but not during inelastic collisions.

Elastic Collisions

During an elastic collision the total amount of kinetic energy is the same before the collision, as well as after the collision. Just like with momentum. During these collisions, two objects will collide and then bounce back in opposite directions. As seen in the picture below:



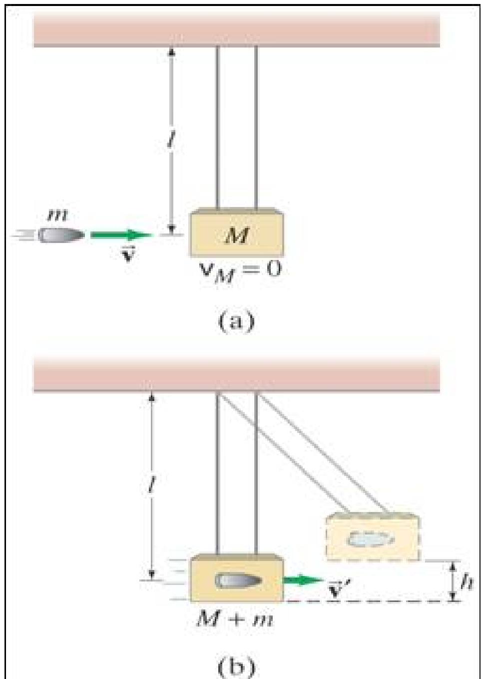
Don't forget that the equation for kinetic energy is **KE = ½ mv^2**

Inelastic Collisions

With inelastic collisions, some of the initial kinetic energy is lost to thermal or potential energy. It may also be gained during explosions, as there is the addition of chemical or nuclear energy. Therefore KE is not conserved.

A completely inelastic collision is one where the objects stick together afterwards, so there is only one final velocity.

The picture below shows an inelastic collision where the bullet enters the box, and now they move together after the collision as a system. When this happens we will add the masses of the bullet and the box to find the final velocity after the collision, which is why kinetic energy is not conserved.

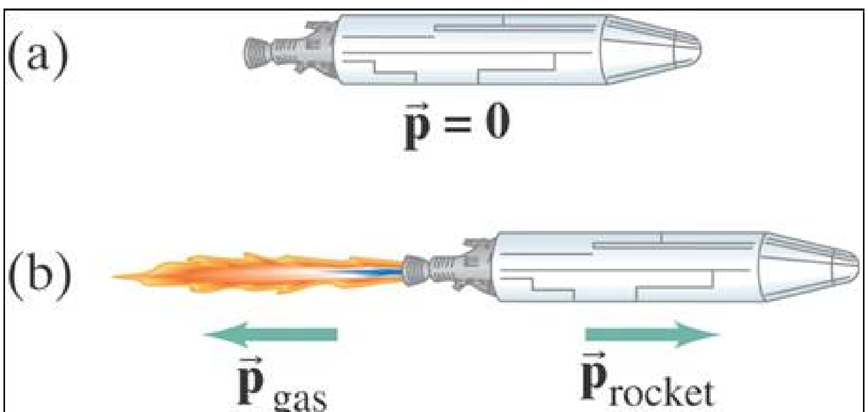


"Explosions"

An "explosion" is a situation where there is an internal force acting on a system to make it move.

For Example:

- **Gun Powder**
- **Firecracker**
- **Rocket Propulsion**
- **Momentum conservation works for a rocket as long as we consider the rocket and its fuel to be one system, and account for the mass loss of the rocket.**



In most linear explosion problems the object will contain two parts after the explosion traveling in opposite directions. It is important to remember then the direction of one of the particles will be negative, therefore giving it a negative velocity.

In an explosion problem, momentum and kinetic energy are conserved only after the explosion, because they were at rest before the internal combustion. Therefore;

Pa = - Pb

- ***In this case Pa = particle one after the explosion and Pb = particle two after the explosion.***

** the momentum of the systems are equal in magnitude but opposite in direction** (Newton's Third Law)

Elastic Examples:

1. A ball with a mass of 883 grams is moving west at a speed of 3.4 m/s where it hits another ball with a mass of 900 grams moving east at 5 m/s. If the first ball moves east after the collision with a speed of 2.7 m/s, how fast and in what direction does the second ball move?

GIVEN	SOLVING FOR
Mass a = .883 kg Mass b = .900 kg Va initial= -3.4 m/s Vb initial = 5 Va after = 2.7 m/s	Vb after the collision = ?

EQUATION – because we know all variables except one, we can use our momentum equation.

$p_{before} = p_{after}$

(ma)(va) + (mb)(vb) = (ma)(va) + (mb)(vb)

(.883)(-3.4) + (.900)(5) = (.883)(2.7) + (.900)(Vb)
-3.0 +4.5 = 2.38 + .900Vb
1.5 = 2.38 + .900Vb
-.88 = .900Vb
Vb = - .98 m/s

Since velocity is negative, we know that it is moving west after the collision.

Inelastic Example

1. A 6 kg fish swimming at 1 m/sec swallows a 2 kg fish that is swimming towards it at 2 m/sec. Find the velocity of the fish immediately after "lunch"

GIVEN
Ma = 6 kg
Mb = 2 kg
Va = 1 m/s
Vb = -2 m/s ** since it is swimming towards it, it must be moving in the opposite direction so velocity is negative, or moving west

SOLVING FOR
Velocity = ?

EQUATION

$p_{before} = p_{after}$

Since this is an inelastic problem, where the two items stick together after the collision, they will have one momentum as a system after the collision, but we will add their masses.

(Ma)(Va) +(Mb)(Vb) = (Ma +Mb)(Vab)
(6)(1) + (2)(-2) = (6 +2)(Vab)
6 + -4 = 8(Vab)
2 = 8(Vab)
Vab = .25 m/s East

"Explosion" Example

1. A cart with a mass of 3 kg is sitting next to a cart with a mass of 2 kg. Between them is a compressed spring. When the spring is released, the 3 kg cart moves at a speed of 5 m/s. How fast did the 2 kg cart move?

GIVEN
Mass a = 3 kg
Mass b = 2 kg
Vabefore = 0
Vb before = 0
Va after = - 5 m/s

SOLVING FOR
Vb after =?

** I made the velocity of object a negative because we can assume that that one of them will be moving west, and the other is moving east when the spring is released. It really doesn't matter if you make Va negative, or Vb negative, just as long as you note that they will be moving in opposite directions.

Because both cars were at rest before the spring was released, they did not have any momentum, so we only need to worry about what happened after the spring was released.

EQUATION Pa = -Pb (Ma)(Va) = -(Mb)(Vb) (3)(-5) = -(2)(Vb) -15 = -2Vb **Vb = 7.5 m/s** …
This makes sense that Vb is a positive number because it would be moving east, since object a moved west after the collision.

A Look Ahead

In the next unit, you will perform a laboratory experiment to observe momentum and the affects of different types of collisions based on an object's mass and velocity. We will mostly be looking at elastic and inelastic collisions in one dimension. You will be able to observe how momentum and kinetic energy are conserved for elastic collisions, and how momentum is conserved for inelastic collisions.

Below are additional educational resources and activities for this unit.

Unit 15 Resource 1

Unit 15 Resource 2