GRAPHING TECHNIQUES FOR FUNCTIONS

In this unit you will review graphing techniques and how these techniques can be used to graph functions. You will learn how to identify the domain and range of a function and how to restrict the domain and range of a relation to create a function. Finally you will examine how to employ the TI-83+ graphing calculator to graph and analyze critical values of functions.

Review of Graphing

Identifying the Domain & Range from a Graphed Function or Relation

The Vertical Line Test

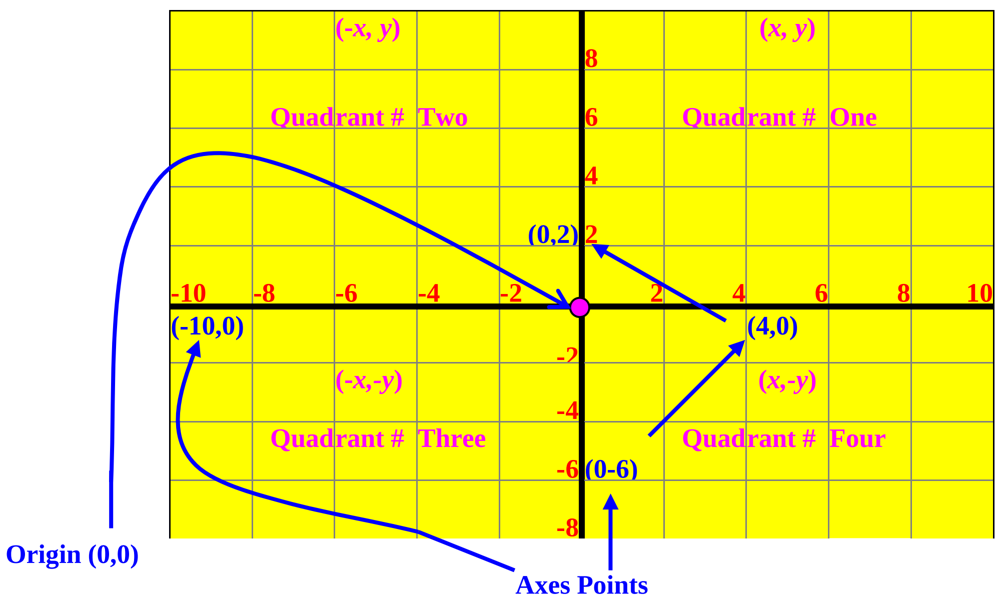
Review of Graphing

Since a function is a relation of two sets, A and B, and written as a set of ordered pairs (i.e. $f(x)=\{(6,3);(-6,3);(0,5);(2,2)\}$), it is common, useful, and often necessary to graph the set of ordered pairs in order to better understand the nature of a function.

The Coordinate Axes

The traditional graph is called the "xy-plane". An overview of this plane, its partitions, and special components will be useful in establishing the graphing terminology to be used in this course.

a.) The xy-plane is formed by setting two real number lines at right angles to one another. The 'x' number line or 'x-axis', comprises the horizontal component of the graph. The 'y' number line or 'y-axis', comprises the vertical component of the graph. The x and y axes divide the plane into four 'quadrants'. The 'quadrants', 'quadrant points', 'axis points', and origin of the graph are labeled below.



Graphing Functions

As mentioned previously, this course will focus on graphing functions of variable domain (such as $y=3x+8$ or $y=x^3-3x+1$). To examine equations of this type as functions, we will reintroduce the notation from the a previous unit:

$$y=3x+8 \;\Rightarrow\; f(x)=3x+8$$
$$y=x^3-3x+1 \;\Rightarrow\; g(x)=x^3-3x+1$$

From this we give the, 'Common Notational Form' of graphed functions.

$$y = f(x)$$

This implies that, "y graphs a function of x".

Example #1: Graph $f(x) = 3x + 2$

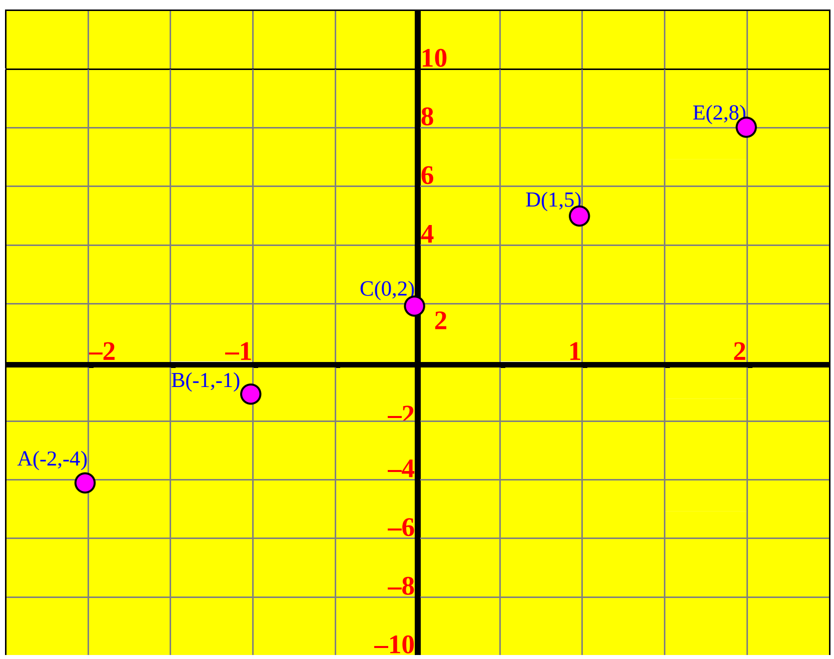
Perhaps one common method to graph this equation in your previous math courses was to use a table of values to identify points to be graphed.

For the equation $y = 3x + 2$, the table might look like the following:

x	–2	–1	0	1	2
y	–4	–1	2	5	8

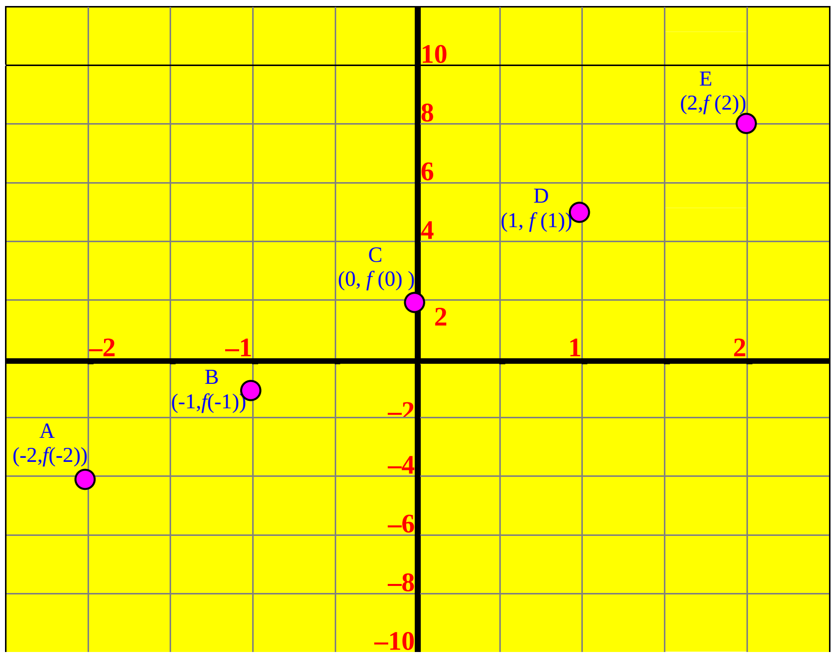
This table yields the following set of points to be graphed:

$$\{A(-2,-4); B(-1,-1); C(0,2); D(1,5); E(2,8)\}$$



As a function, the above equation, table, and graph now appears in the following manner:

x	-2	-1	0	1	2
$f(x)$	$f(-2)=-4$	$f(-1)=-1$	$f(0)=2$	$f(1)=5$	$f(2)=8$



Notice in the previous example that the second value in each ordered pair can now be denoted as the set:

$$f_r = \{ f(-2); f(-1); f(0); f(1); f(2) \}$$

Therefore for any function, the range, ‘ f_r ’, will always be the y values on the graph.

Also in the above example, the function values found in the table of values would be used to provide a general outline or shape of the variable valued function to be graphed – in this case, a line. Once the general shape of a function is determined, the points can be connected to indicate that the graph takes on all unrestricted values* for x in the domain of $f(x)$.

*The notion of restricted values for the domain and/or range of a function will be discussed shortly.

Identifying the Domain and Range from a Graph

Not all functions are as smooth or as easily graphed as the previous example. In fact, even though not all relations are functions, it is sometimes not easy to determine which relations are functions until they have been graphed. In addition, the identification of a function's domain and range is sometimes quite difficult until the function is graphed. Consider the following equation:

$$y = \sqrt{x}$$

If we use a table of values to find points for this relation, we may begin in the following manner:

Table #1

x	-2	-1	0	1	2
y					

Immediately we are faced with a dilemma for negative values of x. If we let $x = -2$, then $y = \sqrt{-2}$, which by our laws of exponents and radicals from Algebra II is equal to $y = \sqrt{-1 \times 2} = \sqrt{-1} \times \sqrt{2} = i\sqrt{2}$, which is an imaginary number. Since the xy-plane consists of two 'real' number lines placed at right angles to one another, we cannot graph the point $(-2,\ i\sqrt{2})$ on this plane. (Later we will discuss how to graph imaginary numbers). For all negative numbers, $x \le 0$, our equation, $y = \sqrt{x}$, returns imaginary numbers. Therefore we cannot consider any negative values for the variable x in our equation. This directs us to 'restrict' our domain to positive values or zero. Using interval notation, we can state the restriction on the domain in the following manner:

For $y = \sqrt{x}$, the domain of the variable x (denoted, D_x) is

$$D_x = [0, \infty)$$

Now that the domain for our equation is established, we turn our attention back to the table of values to interpret the range:

Table #2

x	0	1	2	3	4
y	0	1	$\sqrt{2}$	$\sqrt{3}$	2

A problem still exists with table #2. Recall that for any positive number, $x \geq 0$, there are two values for $\sqrt{x}$ (i.e., $\sqrt{4} = 2$ and $-2 = \pm 2$ since $2^2 = 4$ and $(-2)^2 = 4$). To reflect this property for $\sqrt{x}$, our table of values should read:

Table #3

x	0	1	1	2	2	3	3	4	4
y	0	1	-1	$\sqrt{2}$	$-\sqrt{2}$	$\sqrt{3}$	$-\sqrt{3}$	2	-2

It is now apparent (from the definition of function) that our equation, $y = \sqrt{x}$, is not a function since every positive x is associated with more than one y. Does this imply that the relation, $y = \sqrt{x}$, cannot serve as, or be analyzed as a function? The answer is 'no', if we restrict the range.

Remember that a relation is a function, **"…if for every element of the domain, there is one and only one element in the range"**. If we limit or 'restrict' the range elements in $y = \sqrt{x}$ to take on only those values that prevent any x in the domain from being paired with more than one y in the range, then we can define a function for $y = \sqrt{x}$. One such restriction would be to select only the negative values of y found in table #3 as in the following:

Table #4

x	0	1	2	3	4
y	0	-1	$-\sqrt{2}$	$-\sqrt{3}$	-2

We can now rewrite $y = \sqrt{x}$ as a function based on the following restrictions:

$$f(x) = \sqrt{x} \text{ for } D_x = [0, \infty) \text{ and } R_y = (-\infty, 0]$$

Note: **Table #2** above now identifies $y = \sqrt{x}$ to also be a function for all positive range values and based on the following restrictions:

$$f(x) = \sqrt{x} \text{ for } D_x = [0, \infty) \text{ and } R_y = [0, \infty)$$

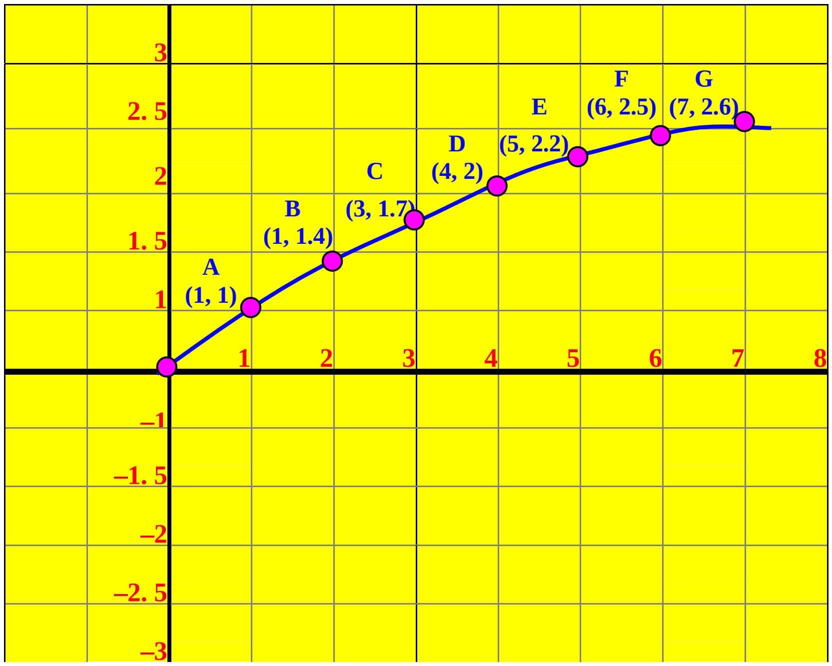
Notice that the only change is for the values in the range.

It now becomes useful to graph $y = \sqrt{x}$ for **Table #2, Table #3,** and **Table #4** and connect points.

Graph for Table #2

$y = f(x) = \sqrt{x}$ for $D_x = [0, \infty)$ and $R_y = [0, \infty)$

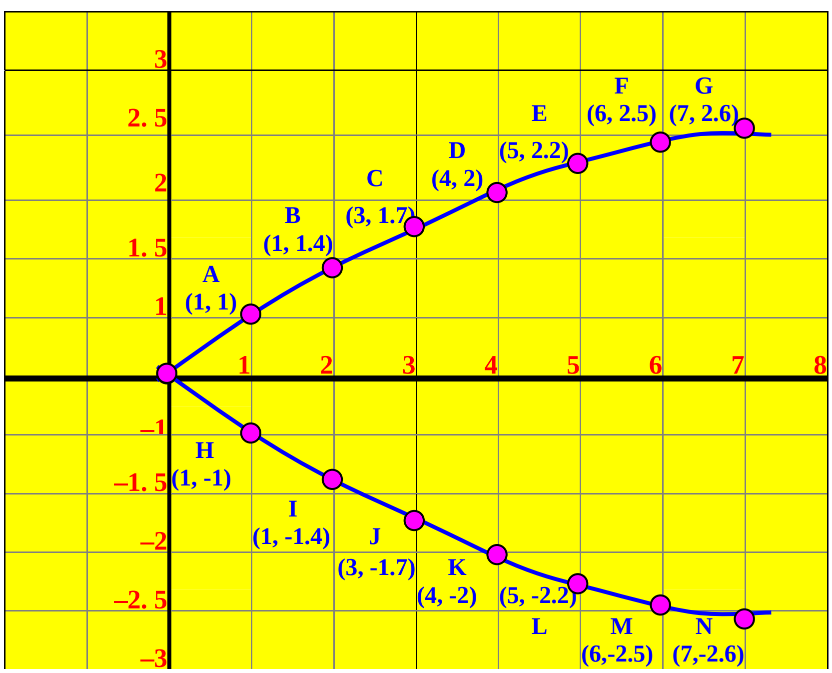
(Additional points are calculated for $x = 5,\ 6,\ 7$)



Graph for Table #3

$y \neq f(x) = \sqrt{x}$; for $D_x = [0, \infty)$ and $R_y = (-\infty, \infty)$

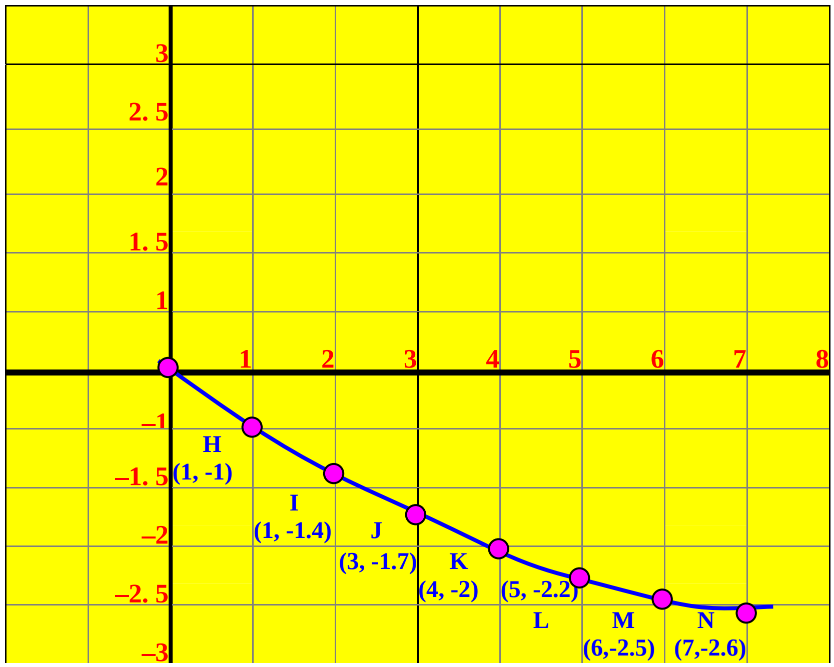
(Additional points are calculated for $x = 5,\ 6,\ 7$)



Graph for Table #4

$y = f(x) = \sqrt{x}$; for $D_x = [0, \infty)$ and $R_y = (-\infty, 0]$

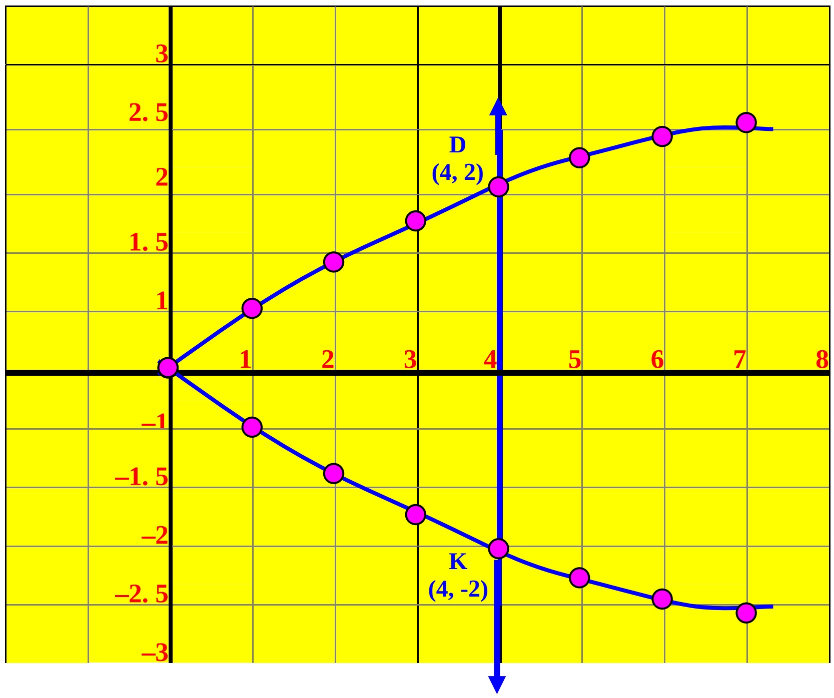
(Additional points are calculated for $x = 5,\ 6,\ 7$)



By comparing the three graphs of $y = \sqrt{x}$, a method for determining if a relation is a function is suggested. This method is called **The Vertical Line Test.**

The Vertical Line Test

Lets reexamine the graph of **Table #3** in the link to "Identifying the Domain and Range in a Graph" for the relation $y = \sqrt{x}$ and focus only on points $D(4,2)$ and $K(4,-2)$



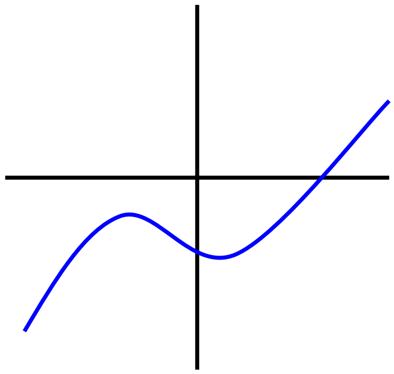
We know from our definition of function that **Table #3** does not designate a function since for any $x \in D_x$, there is more than one $y \in R_y$. For points $D(4,2)$ and $K(4,-2)$ above (in fact for multiple points in **Table #3**), this fact is not only seen in the table but on the graph as well. From Geometry, we know that 2 points may determine a straight line. If we connect points $D(4,2)$ and $K(4,-2)$ to form $\overleftrightarrow{DK}$, we see that this **vertical line** intersects the graph of the relation at two points. In fact, a vertical line can be drawn for any x in the domain of $y = \sqrt{x}$ except for $x = 0$. This provides a graphical representation to test if a relation is a function and can be stated in the following manner.

The Vertical Line Test

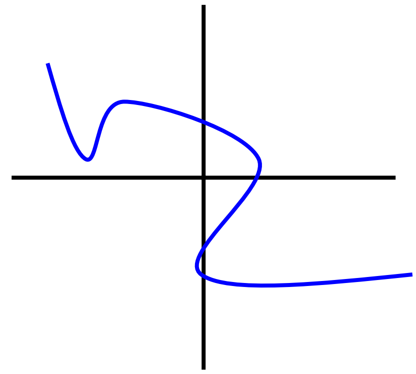
Given a relation, $R \in \mathbb{R}$ and any $h \in R_d$, the relation $R = F$ is a function $F \in \mathbb{R}$ if the vertical line, $x = h$, intersects the graph of R in only one point for every $h \in R_d$.

Examples: Which of the following graphs are functions?

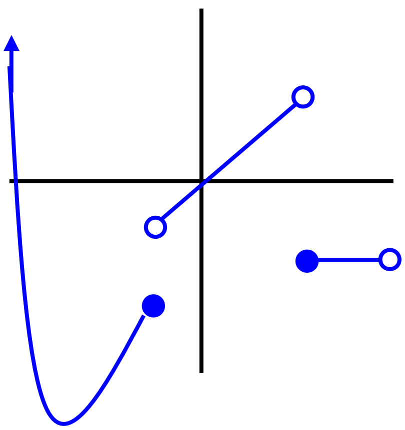
A.)



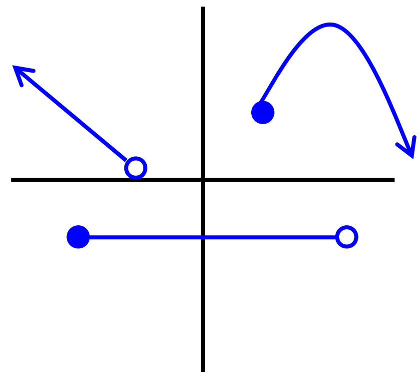
D.)



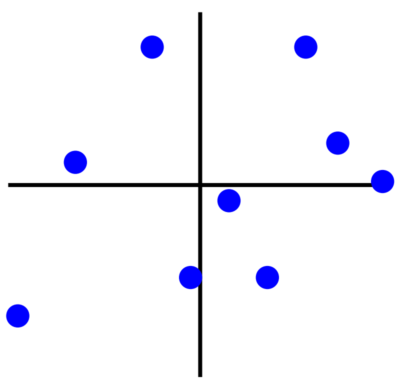
B.)



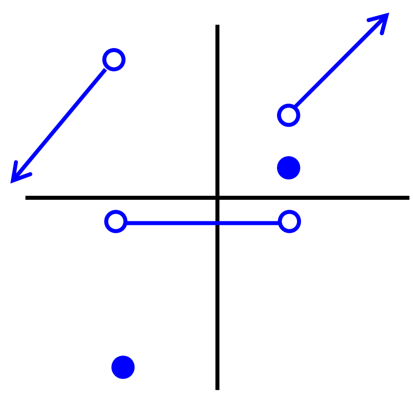
E.)



C.)



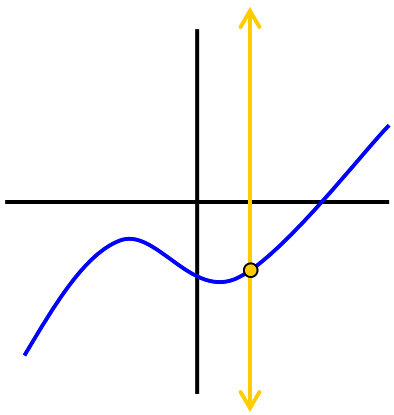
F.)



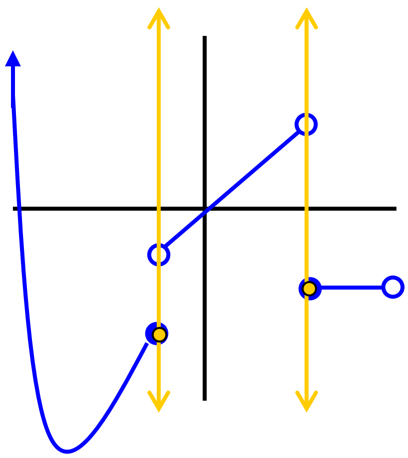
.

Solutions: Choices, **A, B, C** & **F** are all functions and choices **D**, and **E** are not.

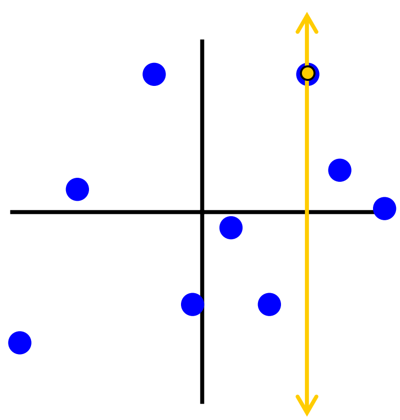
A.)



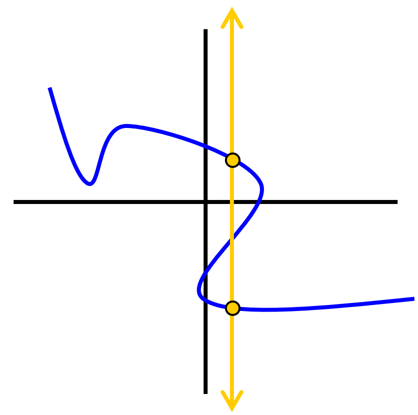
B.)



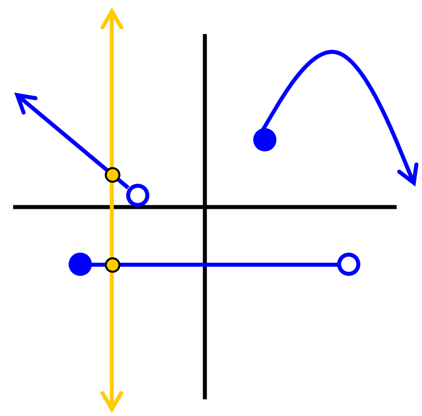
C.)



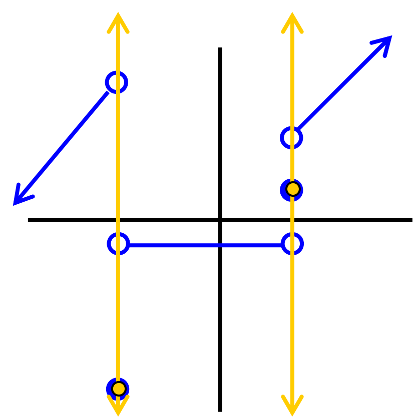
D.)



E.)



F.)



The vertical *gold (lighter)* lines are drawn to show the "Vertical Line Test". Note that some of the graphed lines have open dots to represent endpoints that are not part of the function. The *gold (lighter)* points represent the points of intersection between the "vertical lines" and the graphs.