

CURVE FITTING AND QUADRATIC INEQUALITIES

In this unit you will find a quadratic function that exactly fits three data points. You will also write, solve and graph a quadratic inequality in one and two variables.

Curve Fitting

Quadratic Inequalities

Graphing Quadratic Inequalities on a Coordinate Plane

Curve Fitting

It is possible to find a quadratic function that fits a set of data points. To find a , b , and c in the function $f(x) = ax^2 + bx + c$:

1. write a system of three linear equations using the given points
2. set up a matrix equation and solve

Example #1: Given the three points $(-3, 16)$, $(2, 6)$, and $(1, -4)$, we are going to set up a system of equations and then transfer the equations into a matrix equation.

Point	Substitution	Equation
$(-3, 16)$	$a(-3)^2 + b(-3) + c = 16$	$9a - 3b + c = 16$
$(2, 6)$	$a(2)^2 + b(2) + c = 6$	$4a + 2b + c = 6$
$(1, -4)$	$a(1)^2 + b(1) + c = -4$	$a + b + c = -4$

$$\begin{bmatrix} 9 & -3 & 1 \\ 4 & 2 & 1 \\ 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 16 \\ 6 \\ -4 \end{bmatrix}$$

Multiply both sides of the equation by the inverse of the coefficient matrix..

To find the inverse of a 3×3 matrix using a graphing calculator:

1.) Enter the coefficient matrix $\begin{bmatrix} 9 & -3 & 1 \\ 4 & 2 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ into matrix A.

$\boxed{2\text{nd}}$, $\boxed{x^{-1}}$, scroll over to Edit, $\boxed{\text{ENTER}}$. Change the dimensions to 3×3 and enter the numbers, pressing $\boxed{\text{ENTER}}$ after each number. Press $\boxed{2\text{nd}}$, $\boxed{\text{MODE}}$ when finished.

2.) Find the inverse of matrix A.

$\boxed{2\text{nd}}$, $\boxed{x^{-1}}$, $\boxed{\text{ENTER}}$. You should have $[A]$ on your screen. Now press $\boxed{x^{-1}}$ and $\boxed{\text{ENTER}}$. If you see very large decimals, change everything to a fraction. $\boxed{\text{MATH}}$, $\boxed{\text{Frac}}$, $\boxed{\text{ENTER}}$, $\boxed{\text{ENTER}}$.

3.) Enter your constant matrix $\begin{bmatrix} 16 \\ 6 \\ -4 \end{bmatrix}$ into matrix C.

$\boxed{2\text{nd}}$, $\boxed{x^{-1}}$, scroll over to Edit, and go down to 3: $[C]$, $\boxed{\text{ENTER}}$. Change the dimensions to 3×1 and enter the numbers. Press $\boxed{2\text{nd}}$, $\boxed{\text{MODE}}$ when finished.

4.) To solve the equation you have created, you must multiply both sides by the inverse of the coefficient matrix that you entered into A. In a previous unit, we found that when multiplying a matrix by its inverse, the result is the identity matrix, so we are only interested in multiplying the inverse of the coefficient matrix $[A]$ and the constant matrix $[C]$.

$$\begin{bmatrix} .05 & .2 & -.25 \\ -.15 & .4 & -.25 \\ .1 & -.6 & 1.5 \end{bmatrix} \cdot \begin{bmatrix} 9 & -3 & 1 \\ 4 & 2 & 1 \\ 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} .05 & .2 & -.25 \\ -.15 & .4 & -.25 \\ .1 & -.6 & 1.5 \end{bmatrix} \cdot \begin{bmatrix} 16 \\ 6 \\ -4 \end{bmatrix}$$

Inverse of
coefficient
matrix

Inverse of
coefficient
matrix

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ -8 \end{bmatrix} \quad \text{OR} \quad \begin{bmatrix} a \\ b \\ c \end{bmatrix} = [A]^{-1}[C] = \begin{bmatrix} 3 \\ 1 \\ -8 \end{bmatrix}$$

Identity

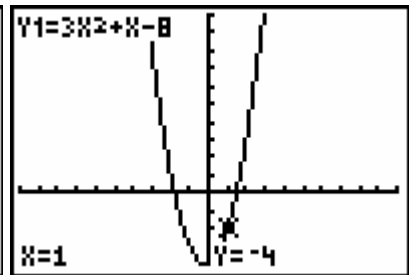
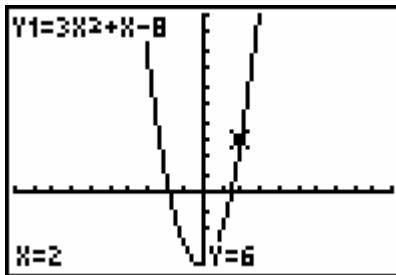
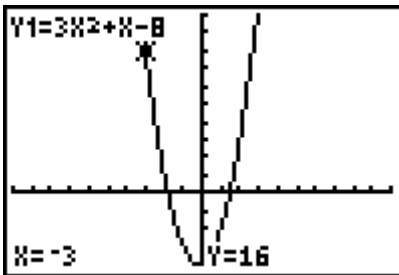
$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ -8 \end{bmatrix}$$

The solution is $a = 3$, $b = 1$ and $c = -8$. Write the quadratic function.

$$f(x) = ax^2 + bx + c$$

$f(x) = 3x^2 + x - 8$ is the function of a quadratic that contains the points $(-3, 16)$, $(2, 6)$, $(1, -4)$.

See the three graphing calculator windows below to verify.



Quadratic Inequalities

To solve a quadratic inequality:

- 1.) factor the quadratic
- 2.) the solutions represent the critical points on the x -axis and divide the axis into sections that must be tested.
- 3.) test each section to determine what area to shade.


-If the inequality is $<$ or \leq , then the solution will be connected with the word “**and**”.

-If the inequality is $>$ or \geq , then the solution will be connected with the word “**or**”.

$$x^2 + 3x - 4 < 0$$

$$(x + 4)(x - 1) < 0$$

$$x = -4 \text{ and } x = 1$$

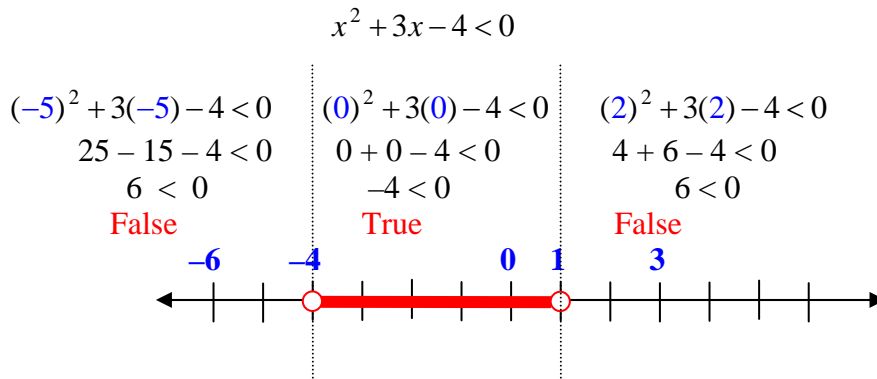


this means that we are looking for the intersection “and” and each critical point will have an open circle

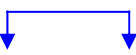
set each of these equal to zero to find the critical points

← critical points that are graphed

Choose test points from **each section** of the graph to determine which area needs to be shaded. We will test -5 , 0 , and 2 which represent each region of the number line. The shading will occur where the TRUE statement is.



Based on the results of the testing, the solution to this quadratic will be $\{x / -4 < x < 1\}$

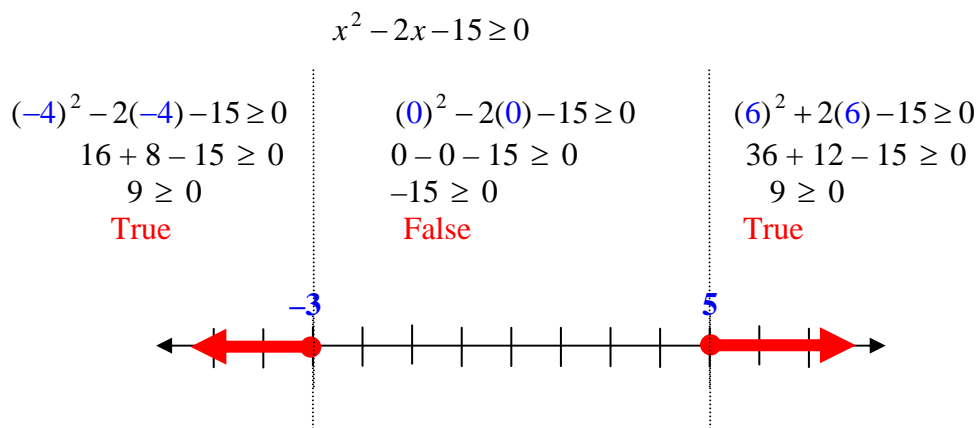


Example #2: $x^2 - 2x - 15 \geq 0$ closed circles on critical points

$$(x - 5)(x + 3) \geq 0$$

$x = 5$ $x = -3$ ← critical points

Choose points from each section to test for validity. We will test -4 , 0 , and 6 which represent each region of the number line.



As you can see the testing would result in true statements less than -3 and greater than 5 .

Therefore the solution to this quadratic inequality would be $\{x/x \leq -3 \text{ or } x \geq 5\}$

Graphing Quadratic Inequalities

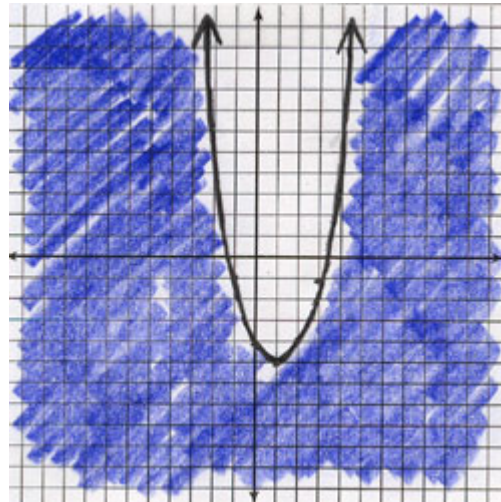
- 1.) the quadratic must be in the form of $y = a(x-h)^2 + k$ where the vertex is located at (h, k)
- 2.) graph the vertex and set up a table to find other points on the curve, choose *two* x -values greater and *two* x -values less than the x -value of the vertex
- 3.) replace the chosen points into the inequality to find y -values
- 4.) graph each point and connect them
- 5.) shade the appropriate region

Example #1: $y \leq (x-1)^2 - 5$

solid curve and shading occurs on the **outside** of the graph because the inequality is \leq .

the vertex is located at $(1, -5)$

x	y
-1	-1
0	-4
1	-5 (vertex)
2	-4
3	-1

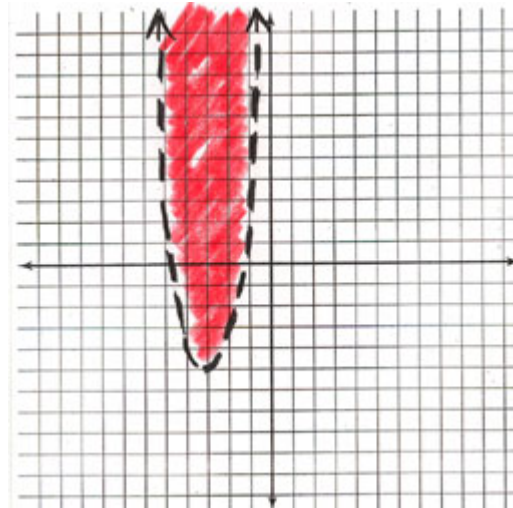


Example #2: $y > 2(x+3)^2 - 5$

dashed curve and the shading occurs **inside** the graph because of the $>$ sign

the vertex is located at $(-3, -5)$

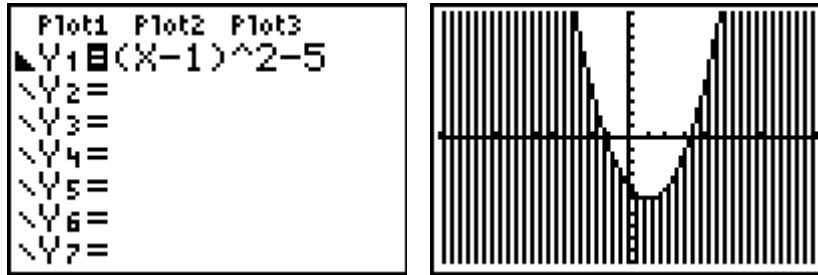
x	y
-5	3
-4	-3
-3	-5 (vertex)
-2	-3
-1	3



Let's take a look at the graphs above using the graphing calculator.

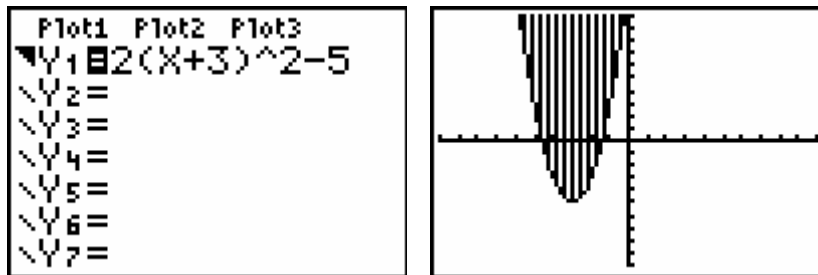
To graph $y \leq (x-1)^2 - 5$, enter $\boxed{Y=}(x-1)^2-5$.

Use the left arrow key and scroll to the left of Y1. Once there, press $\boxed{\text{ENTER}}$ repeatedly until you see a blinking triangle that indicates shading *below* the line (\blacktriangledown). Then select $\boxed{\text{GRAPH}}$.



To graph $y > 2(x+3)^2 - 5$, enter $\boxed{Y=}2(x+3)^2-5$.

Use the left arrow key and scroll to the left of Y1. Once there, press $\boxed{\text{ENTER}}$ repeatedly until you see a blinking triangle that indicates shading *above* the line (\blacktriangledown). Then select $\boxed{\text{GRAPH}}$.



*You will still need to determine if the line is solid or dotted by your knowledge of inequalities.