QUADRATIC EQUATIONS

In this unit you will find solutions of quadratic equations by completing the square and you will use this technique to write quadratic equations in vertex form. The unit will conclude with an explanation of imaginary numbers that will be used in the next unit.

Completing the Square

Vertex Form of a Quadratic Function

Completing the Square

When a quadratic equation does not contain a perfect square, you can create a perfect square in the equation by *completing the square*. **Completing the square** is a process by which you can force a quadratic expression to factor.

1.) make sure the quadratic term and the linear term are the only terms on one side of the equation (move the constant term to the other side)

2.) the coefficient of the quadratic term must be one, if not this will be addressed in the next section

3.) take one-half of the linear term and square it

4.) add this number to both sides of the equation

5.) factor the perfect square trinomial

6.) solve the equation

Example #1: Complete the quadratic expression into a perfect square.

$$x^2 - 20x$$

$$x^2 - 20x + 100 \qquad \frac{1}{2}(20) = 10, \quad 10^2 = 100$$

$$(x-10)^2$$

The completed perfect square is $x^2 - 20x + 100$ *or* $(x-10)^2$.

Example #2: Solve for *x* by completing the square.

$$x^2 + 6x - 16 = 0$$
$$x^2 + 6x = 16$$
$$x^2 + 6x \underline{+9} = 16 \underline{+9}$$

$$\frac{1}{2}(6) = 3 \rightarrow 3^2 = 9$$

$$(x+3)^2 = 25$$
$$\sqrt{(x+3)^2} = \sqrt{25}$$
$$x + 3 = \pm 5$$

$x = 5 - 3$ and $x = -5 - 3$

$x = 2$ and $x = -8$

Example #3: Solve for *x* by completing the square.

$$x^2 - 10x + 24 = 0$$
$$x^2 - 10x = -24$$
$$x^2 - 10x \underline{+25} = -24 \underline{+25}$$
$$(x-5)^2 = 1$$
$$\sqrt{(x-5)^2} = \sqrt{1}$$
$$x - 5 = \pm 1$$

$x = 1 + 5$ and $x = -1 + 5$

$x = 6$ and $x = 4$

*If the coefficient of the quadratic term is not 1, you must **divide all terms** by the coefficient to make it one.

Example #4: Solve for x by completing the square.

$3x^2 - 6x = 5$ divide all terms by 3

$$\frac{3x^2}{3} - \frac{6x}{3} = \frac{5}{3}$$

$$x^2 - 2x \underline{+1} = \frac{5}{3} \underline{+1}$$

$$(x-1)^2 = \frac{8}{3}$$

$$\sqrt{(x-1)^2} = \sqrt{\frac{8}{3}}$$

$$x - 1 = \pm\sqrt{\frac{8}{3}}$$

$$x = \sqrt{\frac{8}{3}} + 1 \quad \text{and} \quad x = -\sqrt{\frac{8}{3}} + 1$$

$$x \approx 2.63 \quad \text{and} \quad x \approx -.63$$

Vertex Form of a Quadratic Function

$y = a(x-h)^2 + k$ where the vertex is located at (h, k) and the axis of symmetry is $x = h$

To write a quadratic in vertex form, complete the square first, using quadratic and linear terms only, if the coefficient of the quadratic term is 1.

Example #1: Write the given quadratic in vertex form, and then state the coordinates of the function's vertex and the axis of symmetry.

$g(x) = x^2 + 6x + 5$

$x^2 + 6x = -5$ subtract 5 from each side to work only with the quadratic and linear terms

$x^2 + 6x \underline{+9} = -5 \underline{+9}$ take half of the linear term (6) and square it to get 9, then add this to both sides

$(x+3)^2 = 4$ factor $x^2 + 6x + 9$ into $(x+3)^2$

$g(x) = (x+3)^2 - 4$ Subtract 4

$y = a(x-h)^2 + k$ Rewrite the quadratic function into vertex form.

$g(x) = (x-(-3))^2 + (-4)$

The vertex of this quadratic function is located at (–3, –4) and the axis of symmetry is $x = -3$.

*If the leading coefficient is not one, factor the coefficient out of the quadratic and linear terms only.

Example #2: Write the given quadratic in vertex form, and then state the coordinates of the function's vertex and the axis of symmetry.

$f(x) = 2x^2 + 12x + 13$ subtract 13 from both sides

$-13 = 2x^2 + 12x$

$-13 = 2(x^2 + 6x)$ factor a 2 out of the quadratic and linear terms

$-13 + 18 = 2(x^2 + 6x + 9)$ complete the square

This is a little different because when you take one-half of 6 and then square it, you must multiply it by 2, and then add it to the other side. Eighteen is added because there has been a 2 factored out of some number to get 9.

$-13 + 18 = 2(x + 3)^2$

$5 = 2(x + 3)^2$

$f(x) = 2(x + 3)^2 - 5$ Put into vertex form.

$f(x) = 2(x - (-3))^2 + (-5)$

The vertex of this quadratic function is located at (–3, –5) and the axis of symmetry is $x = -3$.