QUADRATIC EQUATIONS AND THE ZERO PRODUCT PROPERTY

In this unit we will take a look at the different methods of factoring polynomials, especially binomials and trinomials. We will then use these factoring skills to solve quadratic equations using the zero product property.

Factoring Quadratic Expressions

Factoring reverses the process of multiplying two expressions.

A. Two terms
-look for a greatest common factor

1.) $27c^2 - 18c$
$9c(3c - 2)$

GCF is 9c, divide each term by 9c. The result is the expression in the parentheses.

2.) $5z(2z + 1) - 2(2z + 1)$
$(2z + 1)(5z - 2)$

GCF is (2z + 1), divide each term by (2z + 1). The result is the expression in the second quantity.

B. Three terms (leading coefficient = 1 or –1)

-Look for a greatest common factor, then:

-find two factors of the constant term, that when added together, result in the middle term.

*if the last term is **positive,** then **both** factors will have the **same sign**, that sign will be the sign of the middle term.

*if the last term is **negative,** then **one factor is positive** and the **other is negative.**

1.) $x^2 + 12x + 27$

factors of +27	sum
1, 27	28
3, 9	12

$(x + 3)(x + 9)$ or $(x + 9)(x + 3)$

2.) $x^2 - 15x - 54$

factors of –54		sum	
–1, 54	1, –54	53	–53
–2, 27	2, –27	25	–25
–3, 18	3, –18	15	–15

$(x + 3)(x - 18)$

*If the leading coefficient is negative, factor out a –1 first, then proceed to find factors of the last term that add up to the middle term.

1) $-x^2 - 5x - 6$

$-1(x^2 + 5x + 6)$

factors of +6	sum
1, 6	7
2, 3	5

$-1(x + 2)(x + 3)$

C. Difference of Squares (two terms)
- the first term will be a perfect square
-the last term will be a perfect square
-the terms will be separated with a subtraction sign

-look for a GCF
-factor using the following process

difference

1) $x^2 - 49$

perfect square   perfect square

one (–) factor, one (+) factor

$(x - 7)(x + 7)$

$\sqrt{\ }$ of first term $\sqrt{\ }$ of last term

2) $16x^2 - 81$

$(4x + 9)(4x - 9)$

D. Three terms (leading coefficient > 1) **Trial and Error**

-factor the first term

-factor the second term

-the sum of the outside product and inside product must equal the middle term

1.) $5x^2 + 14x + 8$

factors of $5x^2$	factors of 8
$5x$, x	1, 8
	2, 4

a.) $(5x + 2)(x + 4)$ **does not work**

$20x + 2x \neq 14x$

b.) $(5x + 4)(x + 2)$ **does work** therefore the factored form is $(5x + 4)(x + 2)$

$10x + 4x = 14x$

*The combination of $(5x + 4)(x + 2)$ works because the product of the outside terms $10x$ and the product of the inside terms $4x$ will result in the middle term $14x$.

*If this had not worked, the 1 and 8 would have been used to try to determine a combination that gave the correct product sum.

Again this is called **Trial and Error**, so there will be some cases where you would have to exhaust many possible combinations to find the correct combination. **Be patient, this will get easier with practice.**

Let's try another example:

Factor: $12x^2 - 3x - 9$

Factor out the GCF (3) $3(4x^2 - x - 3)$

Factor the trinomial $4x^2 - x - 3$
using trial and error

factors of $4x^2$	factors of –3
x, $4x$	1, –3
$2x$, $2x$	–1, 3

Trial and error combinations a.) $(x + 1)(4x - 3)$

$-3x + 4x = x$ does not work; we need a $(-x)$

Switch the signs on the 1
and 3. b.) $(x - 1)(4x + 3)$

$3x - 4x = -x$ this does work since we need $(-x)$

So $12x^2 - 3x - 9$ factors into $3(x - 1)(4x + 3)$

Zero Product Property

Zero Product Property

If $xy = 0$, then $x = 0$ or $y = 0$

This property is used to find zeros of a function.

A zero of a function f is any number r such that $f(r) = 0$, or the solution.

To use the zero product property
1.) set the quadratic equal to zero
2.) factor the quadratic
3.) set each factor = to zero and solve

Example #1: $x^2 + 4x - 32 = 0$

$(x+8)(x-4) = 0$	factor the trinomial
$x+8=0$ or $x-4=0$	set each factor equal to zero
$x=-8$ or $x=4$	solve each equation

The zeros or solutions to this quadratic are –8 and 4.

Example #2: $x^2 + 4x - 32 = 0$

$(x+8)(x-4) = 0$	factor the trinomial
$x+8=0$ or $x-4=0$	set each factor equal to zero
$x=-8$ or $x=4$	solve each equation

The zeros or solutions to this quadratic are –8 and 4.

Example #3: $4x^2 + 4x - 48 = 0$

$4(x^2 + x - 12) = 0$ factor out the GCF of 4

<u>product</u>	<u>sum</u>
$(+4)(-3) = -12$	$(+4)+(-3) = 1$

$4(x+4)(x-3) = 0$

$4 = 0$ $x+4=0$ $x-3=0$ set each factor equal to zero and solve

$4 \neq 0$ $x=-4$ $x=3$

The solutions to $4x^2 + 4x - 24 = 0$ are $x = -4$ and $x = 3$. We do not use the 4 because there is nothing to solve for and $4 \neq 0$.