QUADRATIC FUNCTIONS

Quadratic functions have important applications in science and engineering. For example, the parabolic path of a bouncing ball can be described by a quadratic function. This unit will introduce you to quadratic functions and how to solve quadratic equations.

Introduction to Quadratic Functions

Quadratic functions have the form $f(x) = ax^2 + bx + c$ where the highest exponent is 2.

ax^2 is the quadratic term

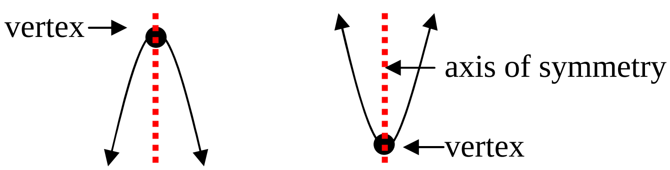
bx is the linear term

c is the constant term

Are the following functions, quadratic functions?

$f(x) = 7x^2 - 3x + 4$	yes, the highest exponent is 2
$g(x) = 3x^2 - 5$	yes, the highest exponent is 2
$h(x) = 4x^3 - 2x^2 + x - 8$	no, the highest exponent is 3
$j(x) = 6x + 7$	no, the highest exponent is 1

parabola: the graph of a quadratic function



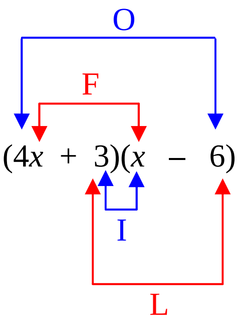
axis of symmetry: a line that divides the parabola into two parts that are mirror images of each other.

vertex: either the lowest point on the graph or the highest point on the graph.

domain of any quadratic function: the set of all real numbers.

range: all real numbers $\geq$ the minimum value of the function (when opening up) or all real numbers $\leq$ the maximum value of the function (when opening down).

If given a function, such as $f(x) = (4x + 3)(x - 6)$, and asked to express it into quadratic form, use FOIL (First Outer Inner Last) multiplication to write it in the form $ax^2 + bx + c$.



Example #1: $f(x) = (4x + 3)(x - 6)$

$$\begin{array}{cccc} \text{F} & \text{O} & \text{I} & \text{L} \end{array}$$
$$(4x \cdot x) + (4x \cdot -6) + (3 \cdot x) + (3 \cdot -6)$$

$4x^2 - 24x + 3x - 18$ combine like terms

$4x^2 - 21x - 18$

$a = 4 \quad b = -21 \quad c = -18$

Example #2: Put the following function in quadratic form:

$f(x) = (3x + 9)(4x - 6)$ Remember to FOIL the terms

$(3x \cdot 4x) + (3x \cdot -6) + (9 \cdot 4x) + (9 \cdot -6)$

$12x^2 - 18x + 36x - 54$ combine like terms

$12x^2 + 18x - 54$

$a = 12 \quad b = 18 \quad c = -54$

By examining "*a*" in $f(x) = ax^2 + bx + c$, you can identify whether the function has a maximum value (opens up) or a minimum value (opens down).

If a > 0, the graph opens up and the *y*-coordinate of the vertex is the minimum value of the function *f*.

If a < 0, the graph opens down and the *y*-coordinate of the vertex is the maximum value of the function *f*.

Example #3: a.) $f(x) = -5x + 2x^2 + 2$ b.) $g(x) = 7 - 6x - 2x^2$

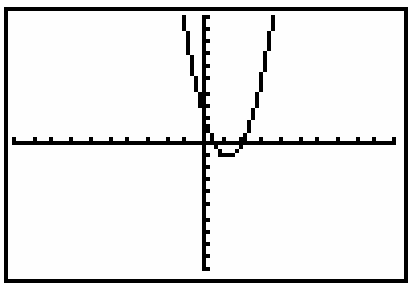
***put equations in descending order (largest exponent first)**

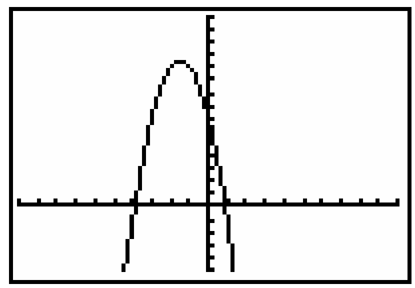
$f(x) = 2x^2 - 5x + 2$

Since "*a*" is positive, this means that the graph opens up and has a minimum at the vertex.

$g(x) = -2x^2 - 6x + 7$

Since "*a*" is negative, this means that the graph opens down and has a maximum at the vertex.





Solving Quadratic Equations

To solve quadratic equations

1) isolate the quadratic term
2) find the square root of each side

Example #1: Solve for *x*: $4x^2 + 13 = 253$

$$4x^2 + 13 = 253$$
$$-13 \quad -13$$
$$\frac{4x^2}{4} = \frac{240}{4}$$
$$x^2 = 60$$
$$\sqrt{x^2} = \sqrt{60}$$
$$x^2 = \pm\sqrt{60}$$
$$\approx \pm 7.75$$

If a number is > 0, then it has two square roots, one is positive (+) the other negative (–).

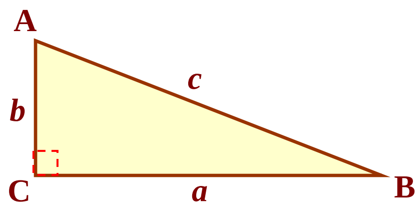
Example #2: Solve for *x*: $9(x-2)^2 = 121$

$$\frac{9(x-2)^2}{9} = \frac{121}{9}$$
$$(x-2)^2 = \frac{121}{9}$$
$$\sqrt{(x-2)^2} = \sqrt{\frac{121}{9}}$$
$$(x-2) = \pm\frac{11}{3}$$

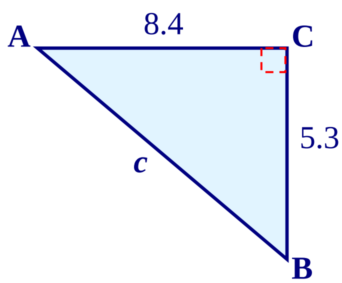
$$x = \frac{11}{3} + 2 \qquad x = -\frac{11}{3} + 2$$
$$x = \frac{17}{3} \qquad x = -\frac{5}{3}$$

The Pythagorean Theorem

If $\triangle ABC$ is a right triangle with a right angle at C, then $a^2 + b^2 = c^2$.



Example #1:



$$(5.3)^2 + (8.4)^2 = c^2$$

$$28.09 + 70.56 = c^2$$

$$98.65 = c^2$$

$$\sqrt{98.65} = \sqrt{c^2}$$

$$9.9 \approx c$$