APPLICATIONS OF MATRICES

A matrix is a system of rows and columns that is used to organize numbers or data. In this unit you will learn how to multiply a matrix by a constant, add and subtract matrices, and finally multiply two matrices.

Using Matrices to Represent Data

matrix: a system of rows and columns that is used as a tool for organizing numbers or data so that each position in the matrix has a purpose.

element: each value in a matrix, the numbers below

Example #1: $A = \begin{bmatrix} 3 & -2 & 1 \\ 2 & 1 & -4 \end{bmatrix}$ rows

columns

*Matrices are named using their dimensions (rows × columns) therefore the matrix above would be known as a 2 × 3 matrix and would look something like this:

$$A_{2\times 3}$$

Each element of a matrix has a special location. For example –2 is in the first row, second column and would be represented as a_{12}, –4 would be represented as a_{23}.

Special Matrices

row matrix: only one row $\begin{bmatrix} 2 & 0 & -7 \end{bmatrix}$ *(1 x 3 matrix)

column matrix: only one column $\begin{bmatrix} 5 \\ -4 \\ 1 \end{bmatrix}$ *(3 x 1 matrix)

square matrix: same number of rows and columns

$\begin{bmatrix} 3 & 0 \\ -6 & 4 \end{bmatrix}$ or $\begin{bmatrix} 3 & -9 & 4 \\ 0 & -6 & 3 \\ 5 & 13 & -4 \end{bmatrix}$ *(2 x 2 matrix and 3 x 3 matrix)

Two matrices are considered equal if they have the same dimensions and each element of one matrix is equal to the corresponding element of the other.

Example #2: $\begin{bmatrix} 2x + y \\ x - 3y \end{bmatrix} = \begin{bmatrix} 6 \\ 31 \end{bmatrix}$

*Since the matrices have the same dimensions and they are equal, the corresponding elements are equal. When you write the sentences that show this equality, two linear equations are formed. To solve for x and y use either substitution or elimination.

$$\begin{array}{lcccr} 2x + y = 6 & \longrightarrow & 3(2x + y = 6) & \longrightarrow & 6x + 3y = 18 \\ x - 3y = 31 & & x - 3y = 31 & & \underline{+ \quad x - 3y = 31} \\ & & & & 7x = 49 \\ & & & & x = 7 \end{array}$$

Substitute 7 for x in either of the original equations to solve for y.

$$\begin{aligned} 2(7) + y &= 6 \\ 14 + y &= 6 \\ y &= -8 \end{aligned}$$

The solution to the system of equations is (7, –8).

Scalar Multiplication

Multiplying a matrix by a constant, each element of the matrix is multiplied.

Example #1: Multiply matrix A by 3. (This is represented as 3A.)

$$A = \begin{bmatrix} 2 & 1 \\ -3 & 0 \\ 6 & -4 \end{bmatrix} \qquad 3A = \begin{bmatrix} 6 & 3 \\ -9 & 0 \\ 18 & -12 \end{bmatrix}$$

*Notice that each element of matrix A was multiplied by three.

Adding or Subtracting Matrices

Matrices must have the same dimensions.

Add or subtract the corresponding elements.

Example #1: $\begin{bmatrix} 2 & -1 & 8 \\ 4 & 7 & 9 \end{bmatrix} + \begin{bmatrix} -1 & 4 & -3 \\ 7 & 2 & -6 \end{bmatrix} = \begin{bmatrix} 2+-1 & -1+4 & 8+-3 \\ 4+7 & 7+2 & 9+-6 \end{bmatrix}$

$$= \begin{bmatrix} 1 & 3 & 5 \\ 11 & 9 & 3 \end{bmatrix}$$

If scalar multiplication and addition or subtraction occurs in a problem, do the scalar multiplication first.

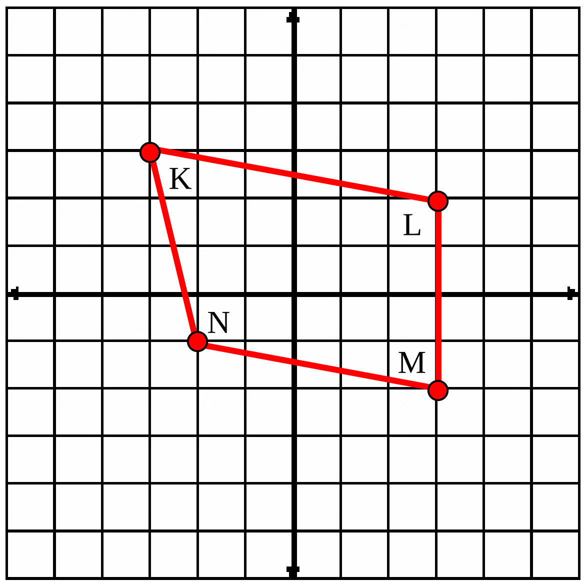
Example #2: $3\begin{bmatrix} 4 \\ 1 \\ 7 \end{bmatrix} + 2\begin{bmatrix} 3 \\ -2 \\ 6 \end{bmatrix} - 5\begin{bmatrix} -2 \\ 3 \\ 6 \end{bmatrix} = \begin{bmatrix} 12 \\ 3 \\ 21 \end{bmatrix} + \begin{bmatrix} 6 \\ -4 \\ 12 \end{bmatrix} + \begin{bmatrix} 10 \\ -15 \\ -30 \end{bmatrix} = \begin{bmatrix} 28 \\ -16 \\ 3 \end{bmatrix}$

Notice that the last matrix was multiplied by a (–5); therefore, it will change to addition because all of the negatives are within the matrix.

Geometric Transformations using Matrices

Given a polygon in the coordinate plane, you can perform transformations using matrices. Let's take a look at an example:

Example #1: Given polygon KLMN, we are going to represent it as a matrix using *x* and *y* coordinates.



K(–3, 3)   L(3, 2)   M(3, –2)   N(–2, –1)

Because each point has two coordinates and there are four points, we are going to create a 2 × 4 matrix Q.

$$Q = \begin{array}{c} \begin{array}{cccc} K & L & M & N \end{array} \\ \begin{bmatrix} -3 & 3 & 3 & -2 \\ 3 & 2 & -2 & -1 \end{bmatrix} \end{array} \begin{array}{l} \\ x-coordinates \\ y-coordinates \end{array}$$

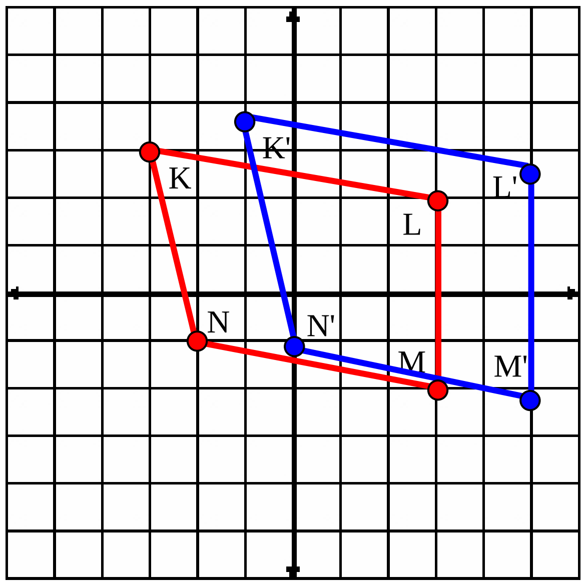
If we perform addition on this matrix, we will be able to move the original polygon within the coordinate plane. Let's take a look at what happens if we add the following matrix to matrix Q. Our result will be $K'L'M'N'$.

$$\begin{array}{c} \begin{array}{cccc} K & L & M & N \end{array} \\ \begin{bmatrix} -3 & 3 & 3 & -2 \\ 3 & 2 & -2 & -1 \end{bmatrix} \end{array} + \begin{array}{c} (new\ matrix) \\ \begin{bmatrix} 2 & 2 & 2 & 2 \\ 1 & 1 & 1 & 1 \end{bmatrix} \end{array} = \begin{array}{c} \begin{array}{cccc} K' & L' & M' & N' \end{array} \\ \begin{bmatrix} -1 & 5 & 5 & 0 \\ 4 & 3 & -1 & 0 \end{bmatrix} \end{array}$$

The new coordinates of our polygon will be:

K'(–1, 4)   L'(5, 3)   M'(5, –1)   N'(0, 0)

If we graph these four points, you will notice how the polygon shifted right 2 units and up 1 unit. This is because we added 2 to each of the *x*-coordinates and 1 to each of the *y*-coordinates



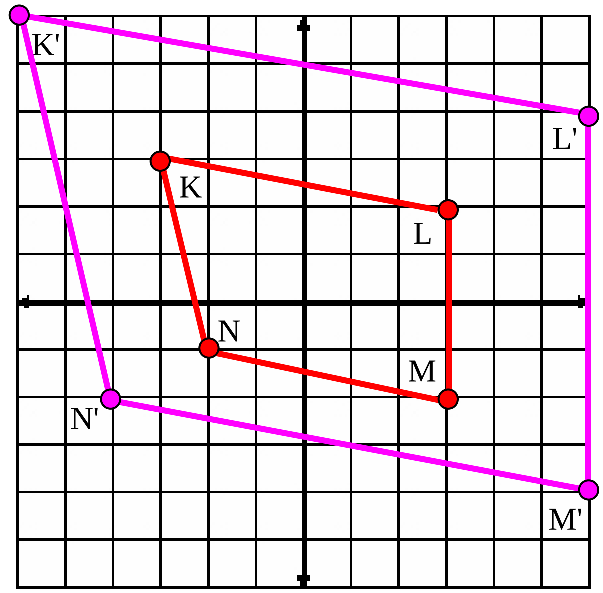
If we perform scalar multiplication to matrix Q, the product represents either an enlarged image or a reduced image. Let's apply scalar multiplication to our matrix Q that represents quadrilateral KLMN and let K'L'M'N' represent the image.

$$Q = \begin{array}{c} \begin{matrix} K & L & M & N \end{matrix} \\ \begin{bmatrix} -3 & 3 & 3 & -2 \\ 3 & 2 & -2 & -1 \end{bmatrix} \end{array} \qquad 2Q = \begin{array}{c} \begin{matrix} K' & L' & M' & N' \end{matrix} \\ \begin{bmatrix} -6 & 6 & 6 & -4 \\ 6 & 4 & -4 & -2 \end{bmatrix} \end{array}$$

The new coordinates of our polygon will be:

K'(–6, 6)   L'(6, 4)   M'(6, –4)   N'(–4, –2)

If we graph these four points, you will notice that the polygon was enlarged by a scale factor of 2.


K'
L'
K
L
N
M
N'
M'

Matrix Multiplication

Matrix multiplication involves multiplication and addition.

To multiply any two matrices, the number of columns in the first matrix must be the same as the number of rows in the second matrix.

Example #1: $\begin{bmatrix} 5 & 4 & 2 \end{bmatrix} \cdot \begin{bmatrix} 6 \\ 1 \\ 3 \end{bmatrix} = \begin{bmatrix} (5 \cdot 6) + (4 \cdot 1) + (2 \cdot 3) \end{bmatrix} = \begin{bmatrix} 40 \end{bmatrix}$

Notice that a 1 × 3 matrix multiplied by a 3 × 1 matrix results in a 1 × 1 matrix. To multiply any two matrices, the *inner dimensions* must be the same; then the *outer dimensions* become the dimensions of the resulting product matrix.

Outer dimensions   Product dimensions

$2 \times 3 \quad\quad 3 \times 4 \quad = \quad 2 \times 4$

Inner dimensions

Matrix Multiplication

If matrix A has dimensions $m \times n$ and matrix B has dimensions $n \times r$, then the product AB has dimensions $m \times r$.

Example #2:

$$\begin{bmatrix} 5 & 3 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 4 & 2 & -1 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} (5 \cdot 4) + (3 \cdot 0) & (5 \cdot 2) + (3 \cdot 1) & (5 \cdot -1) + (3 \cdot 3) \\ (0 \cdot 4) + (1 \cdot 0) & (0 \cdot 2) + (1 \cdot 1) & (0 \cdot -1) + (1 \cdot 3) \end{bmatrix} = \begin{bmatrix} 20 & 13 & 4 \\ 0 & 1 & 3 \end{bmatrix}$$

* Notice that you are taking the first row [5   3] and multiplying each column, then picking up the second row and multiplying each column.