SYSTEMS OF LINEAR INEQUALITIES

This unit is a review of solving systems of linear equations and inequalities. You will solve systems of equations using three techniques: graphing, substitution, and elimination. At the conclusion of the lesson you will be graphing linear inequalities, systems of linear inequalities, and solving linear programming problems.

Graphing Systems of Equations

Substitution

Elimination

Linear Inequalities

System of Linear Inequalities

Graphing Systems of Equations

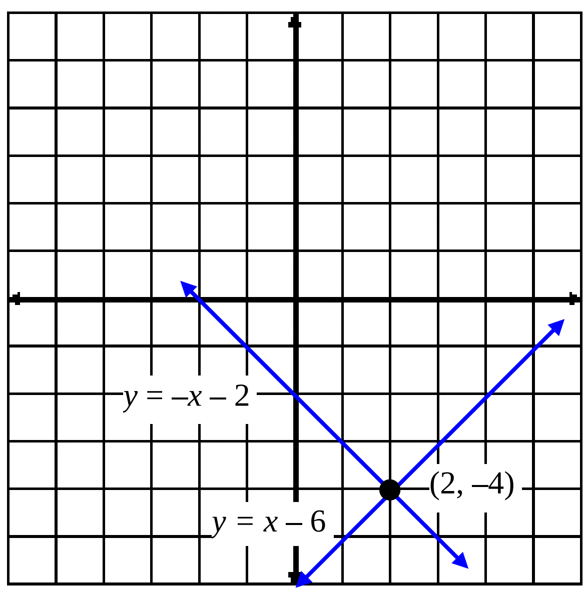
The solution of a system of two linear equations in x and y is any ordered pair, (x, y) that satisfies both equations. The solution (x, y) is also the point of intersection of the graphs.

There are 3 possible solutions.

1 solution	The lines intersect at one point.	The slopes of the lines are different.	Consistent and independent
many solutions	The lines intersect at many points.	The lines are exactly the same.	Consistent and dependent
no solution	The lines do not intersect.	The slopes are the same.	Inconsistent

To graph a line, remember to solve for y and use the slope intercept form of $y = mx + b$.

Plot the y-intercept, use the slope ratio of $\frac{rise}{run}$ to plot more points, and then connect the points using a straight edge.



Example #1: $x + y = -2$
$x - y = 6$

Solve both for y.
$y = -x - 2$
$y = x - 6$

The solution to the system is (2, –4) and the system is consistent and independent.

Example #2: $x + y = 2$

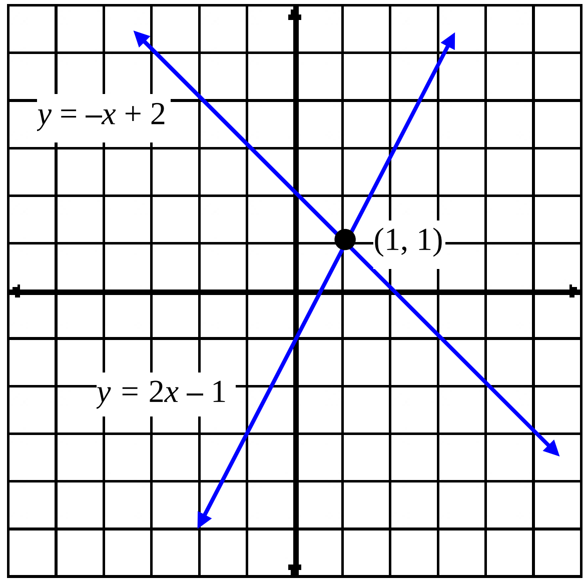
$-2x + y = -1$

Solve both for y.

$y = -x + 2$

$y = 2x - 1$

The solution to the system will be (1, 1) and the system would be consistent and independent.



Substitution

To solve a system by substitution

1.) solve one of the equations for a variable (hint: solve for a variable that has a coefficient of 1)
2.) substitute this value into the other equation to find the value of one of the variables
3.) substitute this value back into either of the equations to find the second variable.

Example #1: $4x - y = 16$ ← solve for y $y = 4x - 16$

$3x + 2y = 1$ replace the value of y in the second equation with $4x - 16$ and solve for x

$3x + 2(4x - 16) = 1$

$3x + 8x - 32 = 1$

$11x = 33$

$x = 3$ replace x in the equation solve for y above to find y

$y = 4(3) - 16$

$y = 12 - 16$

$y = -4$

The solution is (3, –4).

Example #2: $x = y - 8$

$4x - y = -2$

since the first equation is already solved for x, replace the value of x in the second equation with $y - 8$

$4(y - 8) - y = -2$

$4y - 32 - y = -2$ solve for y, using inverse operations

$3y = 30$

$y = 10$ substitute 10 for y in the first equation and solve for x

$x = 10 - 8$

$x = 2$

The solution to this system of equations is (2, 10).

Elimination

To solve a system by elimination:

1.) the coefficients of the same variable must be the same
2.) if the coefficients **are** the same either subtract the equations or add the equations to eliminate that variable, depending on the signs (same sign – subtract, different signs – add)
3.) substitute this value back into one of the equations to find the other variable

Example #1: $x + 3y = 6$
$x - y = 2$

*Since both x's have a coefficient of one they can be eliminated by subtracting the two equations

$$\begin{array}{r} x + 3y = 6 \\ \underline{-\,(x - y = 2)} \\ 4y = 4 \\ y = 1 \end{array}$$

Substitute 1 for y into either equation to solve for x.

$$\begin{array}{r} x + 3(1) = 6 \\ x + 3 = 6 \\ x = 3 \end{array}$$

The solution to this system is (3, 1).

To solve a system by elimination:

1.) the coefficients of the same variable must be the same
2.) if the coefficients **are not** the same make them the same by multiplying one or both equations by a factor to make them the same
3.) add or subtract the result to eliminate a variable
4.) substitute that value back into either equation to find the other variable

Example #2: $3x - 4y = 7$ To eliminate x, multiply the first by 2.
$-6x + y = -7$ To eliminate y, multiply the second by 4

a.) eliminating x first

$$\begin{array}{r} 2(3x - 4y = 7) = 6x - 8y = 14 \\ \underline{+\ \ -6x + y = -7} \\ -7y = 7 \\ y = -1 \end{array}$$

In situation "a", substitute –1 for y in either equation to find x.

$$\begin{array}{r} 3x - 4(-1) = 7 \\ 3x + 4 = 7 \\ 3x = 3 \\ x = 1 \end{array}$$

b.) eliminating y first

$$\begin{array}{r} 3x - 4y = 7 \\ 4(-6x + y = -7) = \underline{+\ -24x + 4y = -28} \\ -21x = -21 \\ x = 1 \end{array}$$

In situation "b", substitute 1 for x in either equation to find y.

$$\begin{array}{r} 3(1) - 4y = 7 \\ 3 - 4y = 7 \\ -4y = 4 \\ y = -1 \end{array}$$

The solution to the system is (1, –1).

Linear Inequalities

A solution to a linear inequality in two variables, *x* and *y*, is an ordered pair (*x*, *y*) that satisfies the inequality. The solution to a linear inequality is a region of the coordinate plain and is called a *half-plane* bounded by a *boundary line*.

*If a linear inequality is a < or > the boundary line will be a dashed line ---------

*If a linear inequality is a $\le$ or $\ge$ the boundary line will be a solid line ________

To graph

1.) solve the inequality for *y*
2.) plot the *y*-intercept
3.) use the slope $\frac{rise}{run}$ to plot more points
4.) determine whether to connect with a dashed line or a solid line
5.) shade a region of the coordinate plane (this is determined by either <, >, $\le$, or $\ge$).If < or $\le$ shade below the boundary line, if > or $\ge$ shade above the boundary line.

You can also choose a point to test, (0, 0) is easy if it does not lie on the line. If you plug 0 in for *x* and *y* and get a true statement shade the portion of the coordinate plane that contains that point. If you plug 0 in for *x* and *y* and get a false statement shade the portion of the coordinate plane that does not contain that point.

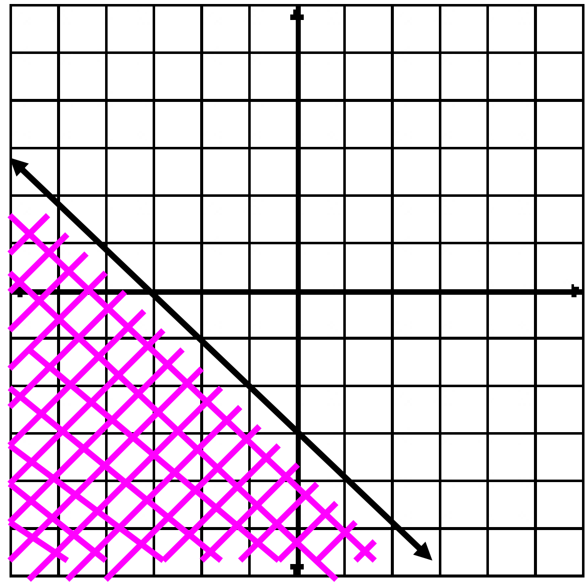
Example #1: $y \le -x - 3$

Since this is a $\le$ inequality the line is solid. Shading will occur below the boundary line.

To test the point (0, 0) replace *x* and *y* with 0.   Since we chose to shade the region that does not include (0, 0) we should get a false statement.

$0 \le -(0) - 3$
$0 \le -3$

This is a false statement therefore we chose To shade the correct half-plane.



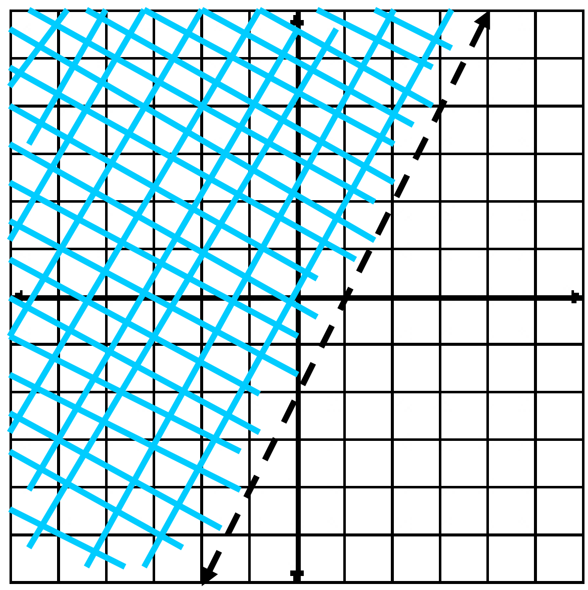
Example #2: $y > 2x - 2$

Since this is > the boundary line will be dashed. Shading will occur above the line or up and left.

To test the point (0, 0) replace *x* and *y* with 0. Since we chose to shade the region that does include (0, 0) we should get a true statement.

$0 > 2(0) - 2$
$0 > -2$

This is a true statement therefore we chose to shade the correct region, the one that includes the point (0, 0).



System of Linear Inequalities

A system of linear inequalities is a collection of linear inequalities in the same variables. The solution is any ordered pair that satisfies each of the inequalities.

To graph a system of linear inequalities

1) graph each inequality individually, decide which half-plane to shade
2) after all inequalities are graphed, the solution is the intersection of all the individual solutions.

Example #1: Graph $y < 2x$
$y \geq -x + 2$

*Notice that the < inequality has a dashed boundary while the ≥ inequality has a solid boundary.