LINEAR INEQUALITIES

Solving and graphing linear inequalities are introduced in this unit. You will learn how to solve and graph a system of linear inequalities and fuse the system of linear inequalities to solve linear programming problems.

Linear Inequalities

System of Linear Inequalities

Linear Programming

Linear Inequalities

A solution to a linear inequality in two variables, x and y, is an ordered pair (x, y) that satisfies the inequality. The solution to a linear inequality is a region of the coordinate plain and is called a *half-plane* bounded by a *boundary line*.

*If a linear inequality is a < or >, the boundary line will be a dashed line. - - - - - - - - -

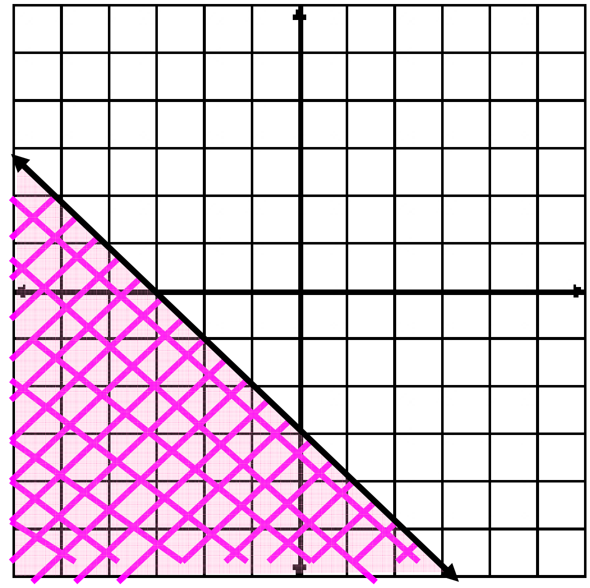
*If a linear inequality is a $\leq$ or $\geq$, the boundary line will be a solid line. ________

To graph a linear inequality:

1.) solve the inequality for y
2.) plot the y-intercept
3.) use the slope $\dfrac{rise}{run}$ to plot more points
4.) determine whether to connect with a dashed line or a solid line
5.) shade a region of the coordinate plane (this is determined by either <, >, $\leq$, or $\geq$). If < or $\leq$, shade below the boundary line; if > or $\geq$, shade above the boundary line.

You can also choose a point to test, such as (0, 0), as it is easy to test if it does not lie on the line. If you plug 0 in for *x* and *y* and get a true statement, shade the portion of the coordinate plane that contains that point. If you plug 0 in for *x* and *y* and get a false statement, shade the portion of the coordinate plane that does not contain that point.

Example #1: $y \le -x - 3$



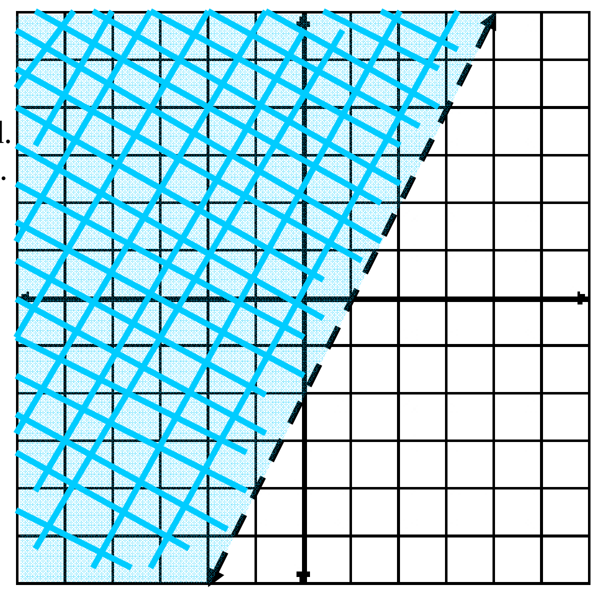
Since this is a $\le$ inequality, the line is solid. Shading will occur below the boundary line.

To test the point (0, 0), replace x and y with 0. Since we chose to shade the region that does not include (0, 0), we should get a false statement.

$0 \le -(0) - 3$
$0 \le -3$

This is a false statement; therefore, we chose to shade the correct half-plane.

Example #2: $y > 2x - 2$



Since this is >, the boundary line will be dashed. Shading will occur above the line or up and left.

To test the point (0, 0), replace x and y with 0. Since we chose to shade the region that does include (0, 0), we should get a true statement.

$0 > 2(0) - 2$
$0 > -2$

This is a true statement; therefore, we chose to shade the correct region, the one that includes the point (0, 0).

System of Linear Inequalities

A system of linear inequalities is a collection of linear inequalities in the same variables. The solution is any ordered pair that satisfies each of the inequalities.

To graph a system of linear inequalities

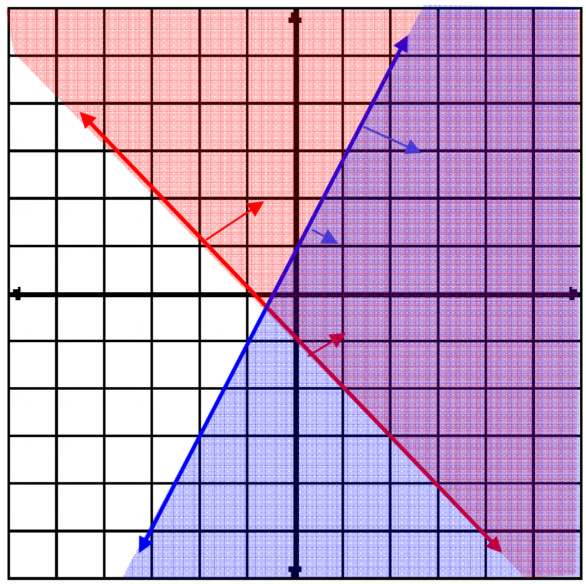
1.) graph each inequality individually, decide which half-plane to shade
2.) after all inequalities are graphed, the solution is the intersection of all the individual solutions.

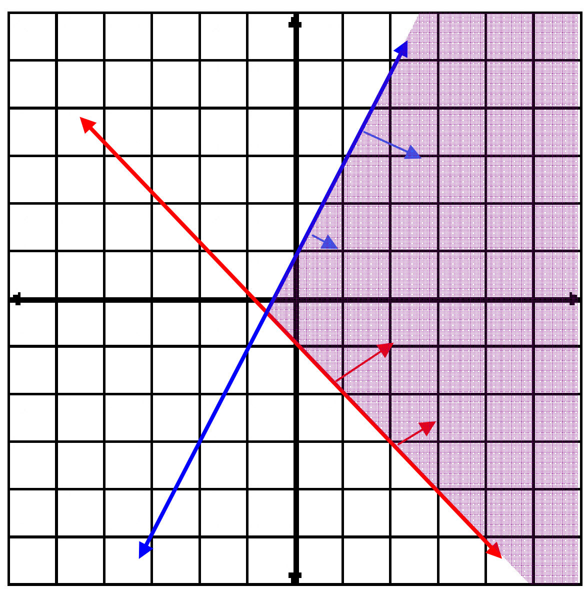
Example #1: $y \geq -x-1$

$y \leq 2x+1$

The intersection of both graphs occurs in the darkened area because of the shading.

Shading occurs above the red Line and to the right and below the blue line.





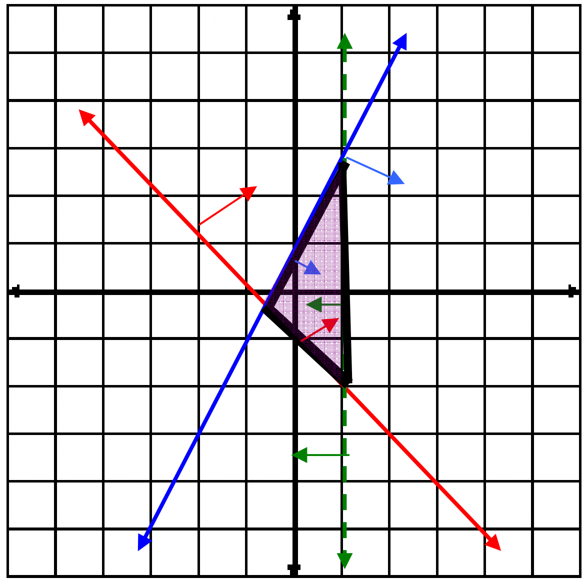
The solution to the system of Inequalities, $y \geq -x-1$ and $y \leq 2x+1$, is the intersection of both graphs.

Now let's add a third line, $x < 1$, to the system of inequalities and examine the intersection of all three lines.

Example #2: $y \geq -x - 1$

$y \leq 2x + 1$

$x < 1$



The intersection of all three graphs occurs in the darkened triangular shape because of the shading.

Shading occurs above the red line, to the right and below of the blue line and to the left of the green line.

To determine a system of inequalities from a graph:

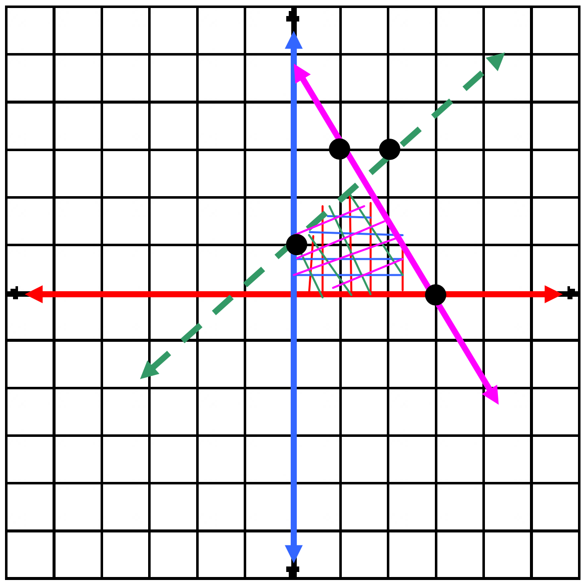
1.) find the equations for the boundary lines:

Determine the slope $\frac{y_2 - y_1}{x_2 - x_1}$, and then use the point-slope form of $y - y_1 = m(x - x_1)$ to find the equation of the line.

2.) make sure each boundary line has the appropriate inequality symbol

$\leq, \geq$ will be a solid line
$<, >$ will be a dashed line

Example #3: The blue line represents $x \geq 0$ and the red line represents $y \geq 0$. To find the equations of the green and pink lines, find the slope, and then use $y - y_1 = m(x - x_1)$ to get the equation.



The green line contains the points (0, 1) and (2, 3) so the slope is:

$$\frac{1-3}{0-2} = \frac{-2}{-2} = \frac{2}{2} = 1$$

Choose one of the given points, (2, 3), to find the equation.

$$y - 3 = 1(x - 2)$$
$$y - 3 = x - 2$$
$$y = x + 1$$

Since the green line is dashed and shaded under the line, the equation is $y < x + 1$.

The pink line contains the points (3, 0) and (1, 3) so the slope is

$$\frac{3-0}{1-3} = \frac{3}{-2} = \frac{-3}{2}$$

Choose one of the points, (3, 0), to find the equation.

$$y - 0 = \frac{-3}{2}(x - 3)$$

$$y = \frac{-3}{2}x + \frac{9}{2}$$

Since the pink line is solid and shaded below the equation is $y \le \frac{-3}{2} + \frac{9}{2}$.

Therefore, the systems of inequalities that make up the graph are:

$$x \ge 0$$
$$y \ge 0$$
$$y < x + 1$$
$$y \le \frac{-3}{2} + \frac{9}{2}$$

Linear Programming

Linear programming is used to find optimal solutions such as the maximum revenue for the example problem that follows.

Constraints are the inequalities used in linear programming.

Feasible region is the solution set.

Objective function is the function to be maximized or minimized.

To solve a linear-programming problem:

1.) represent the constraints as a system of linear inequalities, and graph the system to find the feasible region

2.) write the objective function to be maximized or minimized

3.) find the coordinates of the vertices of the feasible region

4.) evaluate the objective function for the coordinates of the vertices of the feasible region, and then identify the coordinates that give the required maximum or minimum

Example #1: A biologist needs at least 40 fish for her experiment. She cannot use more than 25 perch or more than 30 bass. Each perch costs $5, and each bass costs $3. How many of each fish should she use in order to minimize her cost?

x = perch y = bass

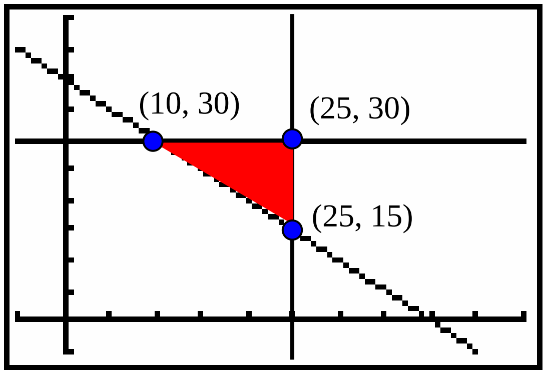
1.) constraints:

$$x + y \geq 40$$
$$x \leq 25$$
$$y \leq 30$$

2.) objective function: $5x + 3y = \text{cost}$

The objective function will represent the cost; and from it, the minimum cost can be determined.

3.) Graph



*Each mark represents 5 units both on the x and y-axis.

The shaded area represents the intersection of the three constraints. The coordinates of the three vertices are used along with the objective function to test for maximum and minimum values of the function.

4.) $5x + 3y$ = cost

(10, 30)	(25, 30)	(25, 15)
5(10) + 3(30) = cost	5(25) + 3(30) = cost	5(25) + 3(15) = cost
50 + 90 = cost	125 + 90 = cost	125 + 45 = cost
140 = cost (minimum cost)	215 = cost	170 = cost

To minimize the cost, the biologist should use 10 perch and 30 bass for the experiment.