FUNCTIONS

This unit will introduce you to functions and function notation. You will then perform operations with functions, find the composition and inverse of functions, and determine whether the inverse of a function is a function.

Introduction to Functions

relation: a relationship between two variables such that each value of the first variable is paired with one or more values of the second variable; **a set of ordered pairs**.

Example #1: {(2, 4), (–4, 5), (2, –7), (0, 9)}

function: a relationship between two variables such that each value of the first is paired with exactly one value of the second variable; **all domain values (*x*-values) are different.**

Example #2: {(2, 4), (0, 6), (7, 4), (–9, 4)}

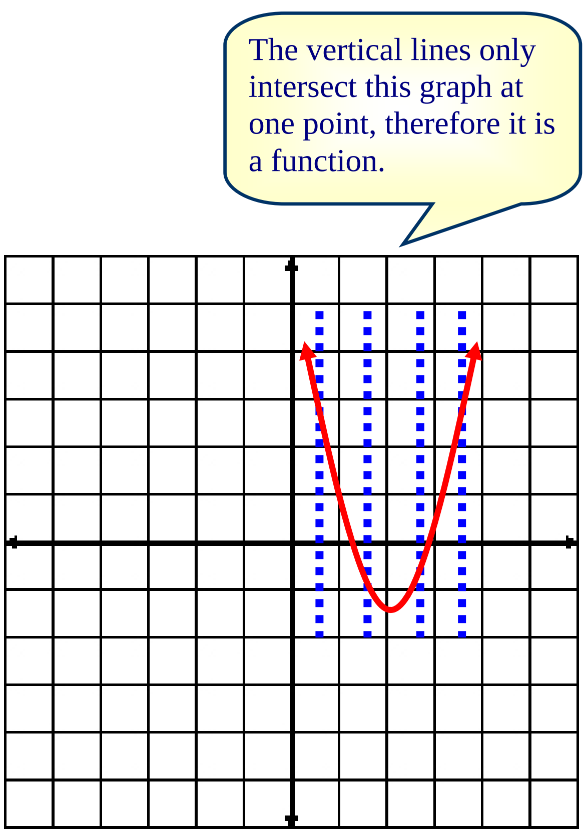
domain: the set of all possible values of the first variable (all *x*-values)

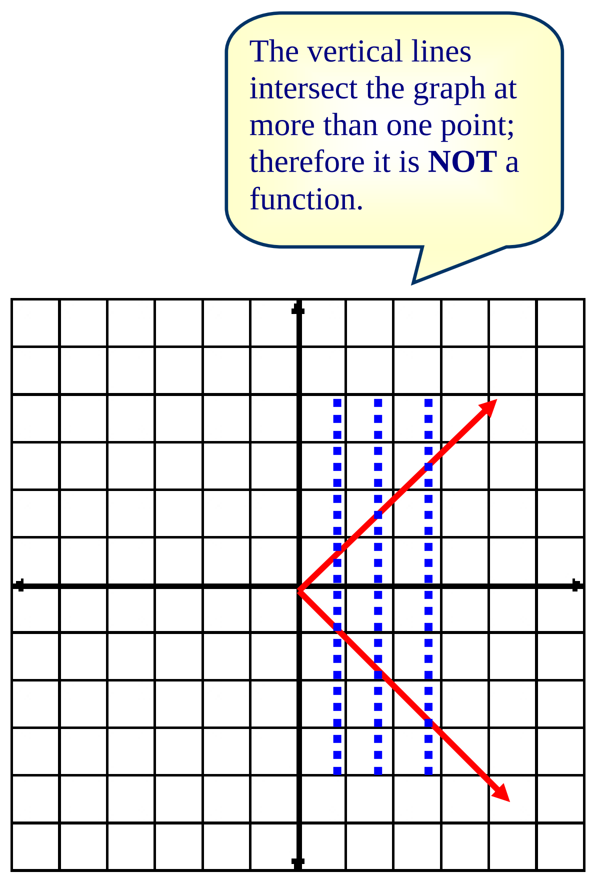
From example #2 above: domain = {2, 0, 7, –9}

range: the set of all possible values of the second variable (all *y*-values)

From example #2 above: range = {4, 6}

vertical line test: If every vertical line intersects a given graph at no more than one point, then the graph represents a function.





function notation: If there is a correspondence between values of the domain, *x*, and values of the range, *y*, that is a function; then $y = f(x)$, and (x, y) can be written as $(x, f(x))$.

$f(x)$ is read "*f* of *x*". The number represented by $f(x)$ is the value of the function *f* at *x*.

The variable ***x*** is called the **independent variable** and the variable ***y*, or *f (x)***, is called the **dependent variable**.

To evaluate a function for a specific variable, replace *x* with the given value and solve.

Example #1: Evaluate $f(x) = -1.2x^2 + 4x - 3$ for $x = 1$.

$$f(x) = -1.2x^2 + 4x - 3$$

$$f(x) = -1.2(1)^2 + 4(1) - 3$$

$$f(x) = -1.2 + 4 - 3$$

$$f(x) = -0.2$$

When $x = 1$, the value of $f(x) = -0.2$

Example #2: Evaluate $f(x) = -1.2x^2 + 4x - 3$ for $x = 5$.

$$f(x) = -1.2x^2 + 4x - 3$$

$$f(x) = -1.2(5)^2 + 4(5) - 3$$

$$f(x) = -1.2(25) + 20 - 3$$

$$f(x) = -30 + 20 - 3$$

$$f(x) = -13$$

When $x = 5$, the value of $f(x) = -13$.

Example #3: Evaluate $g(x) = 3x^2 - x + 1$ for $x = 4$.

*Note: Other letters may be used when denoting functions.

$$g(x) = 3x^2 - x + 1$$

$$g(x) = 3(4)^2 - (4) + 1$$

$$g(x) = 3(16) - 4 + 1$$

$$g(x) = 48 - 4 + 1$$

$$g(x) = 45$$

When $x = 4$, the value of $g(x) = 45$.

Operations with Functions

*functions can be combined by adding, subtracting, multiplying, and dividing.

Example #1: Let $f(x) = 4x^2 + 6x - 9$ and $g(x) = 6x^2 - x + 2$

Find *f* + *g*: $4x^2 + 6x - 9 + 6x^2 - x + 2$

$4x^2 + 6x^2 + 6x - x - 9 + 2$

$f + g(x) = 10x^2 + 5x - 7$

Find *f* – *g*: $4x^2 + 6x - 9 - (6x^2 - x + 2)$

$4x^2 + 6x - 9 - 6x^2 + x - 2$

$4x^2 - 6x^2 + 6x + x - 9 - 2$

$f - g(x) = -2x^2 + 7x - 11$

Example #2: Let $f(x) = 9x^2$ and $g(x) = 4x + 3$

Find $f \cdot g$: $9x^2(4x + 3)$

$36x^3 + 27x^2$

Example #3: Let $f(x) = 2x^2$ and $g(x) = x + 5$

Find $\frac{f}{g}$: $\frac{2x^2}{x+5}$ where $x \neq -5$ because that would make the denominator 0.

*To find restrictions on the domain, set the denominator equal to zero and solve. The result will be the restriction on the domain.

Composition of functions: when you apply a function rule on the result of another function rule, you **compose** the functions.

Let *f* and *g* be functions of *x*.

The composition of f with g is denoted by $f \circ g$ or $f(g(x))$

To find the value of a composite function:

-place the entire second function $f \circ \boldsymbol{g(x)}$ or $f(\boldsymbol{g(x)})$ into the first function in place of *x*.

Example #4: Let $f(x) = x^2 - 1$ and $g(x) = 3x$

(a) Find $f \circ g(x)$ * place $g(x)$ into $f(x)$ for x

$$f(x) = x^2 - 1$$
$$f \circ g(x) = (3x)^2 - 1$$

$$f \circ g(x) = 9x^2 - 1$$

(b) Find $g \circ f(x)$ * place $f(x)$ into $g(x)$ for x

$$g(x) = 3x$$
$$g \circ f(x) = 3(x^2 - 1)$$

$$g \circ f(x) = 3x^2 - 3$$

Another way of using compositions is to evaluate a composition for a specific value. Take a look at the example below. First you want to find the value of the specified function, and then replace that value into the other function.

Example #5: Let $f(x) = -2x^2 + 3$ and $g(x) = -4x$

Find $f(g(-1))$ * replace x in $g(x)$ with -1,
then replace that value into $f(x)$

$$g(x) = -4x$$ *determine $g(-1)$
$$g(-1) = -4(-1)$$
$$g(-1) = 4$$

$$f(g(-1)) = f(4)$$
$$f(x) = -2x^2 + 3$$ *substitute 4 for $g(-1)$ in $f(x)$
$$f(4) = -2(4)^2 + 3$$
$$= -2(16) + 3$$
$$= -32 + 3$$
$$= -29$$

Inverses of Functions

The inverse of a relation consisting of the ordered pairs (x, y) is the set of all ordered pairs (y, x). (switch the x and y)

Given the relation {(1, 2), (4, –2), (3, 2)}

- -the inverse is {(2, 1), (–2, 4), (2, 3)}
- -the domain of the inverse is {2, –2}
- -the range of the inverse is {1, 4, 3}

*the relation is a function but the inverse is not a function because the domain value 2 is paired with two range values.

To find the inverse of an equation

- -interchange the x and y
- -solve for y

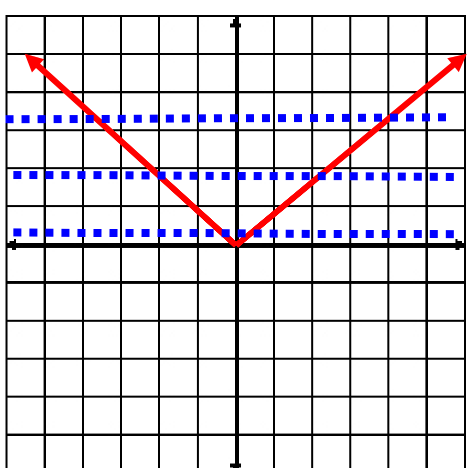
Example #1: $y = 3x - 2$

$x = 3y - 2$

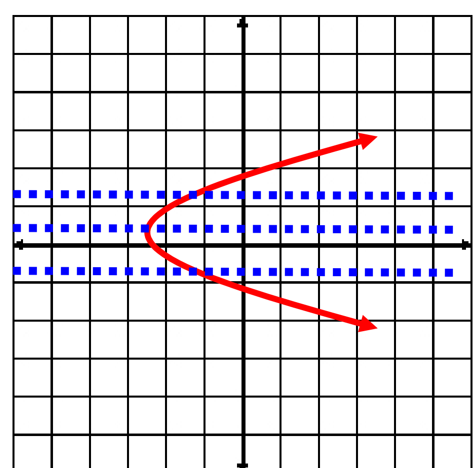
$x + 2 = 3y$

$\frac{1}{3}x + \frac{2}{3} = y$ this is the inverse of $y = 3x - 2$

The **horizontal line** test is used to determine if the inverse of a function is a function. The inverse of a function is a function, if and only if, every horizontal line intersects the graph of the given function at no more than one point.



The horizontal lines intersect the graph at **more than** 1 point. This means the inverse of the function is **not** a function.



The horizontal lines intersect the graph at only **one** point. This means the inverse of the relation is a function.

If *f* and *g* are inverse functions, then $f(g(x))$ and $g(f(x))$ will both = *x*.

Example #2: $f(x)=4x-3$ $\qquad$ $g(x)=\frac{1}{4}x+\frac{3}{4}$

$$f(g(x))=4\left(\frac{1}{4}x+\frac{3}{4}\right)-3 \qquad g(f(x))=\frac{1}{4}(4x-3)+\frac{3}{4}$$

$$=x+3-3 \qquad =x-\frac{3}{4}+\frac{3}{4}$$

$$=x \qquad =x$$

Since both compositions = *x*, they are inverses of each other.