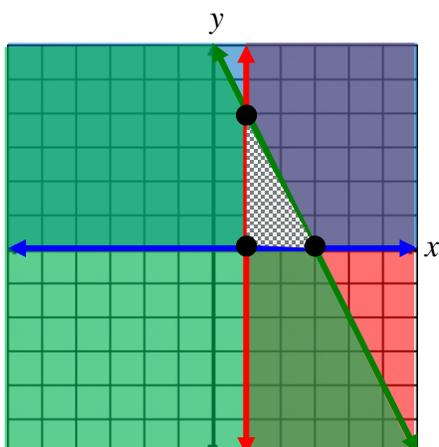


## SOLVING SYSTEMS OF LINEAR INEQUALITIES



### Unit Overview

Linear inequalities can be used to model real-world problems. For example, a linear inequality can be used to model the distance that a car with given fuel-economy ratio can be driven using no more than twenty-two gallons of gasoline. In this unit you will solve and graph linear inequalities in two variables, and then write and graph a set of constraints for a linear-programming problem.

## System of Linear Inequalities

A system of linear inequalities is a collection of linear inequalities in the same variables. The solution is any ordered pair that satisfies each of the inequalities.

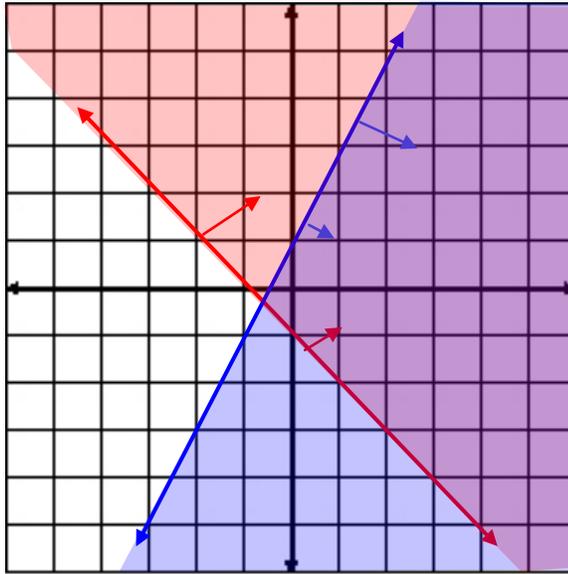
To graph a system of linear inequalities

- 1.) graph each inequality individually, decide which half-plane to shade
- 2.) after all inequalities are graphed, the solution is the intersection of all the individual solutions.

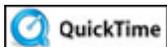
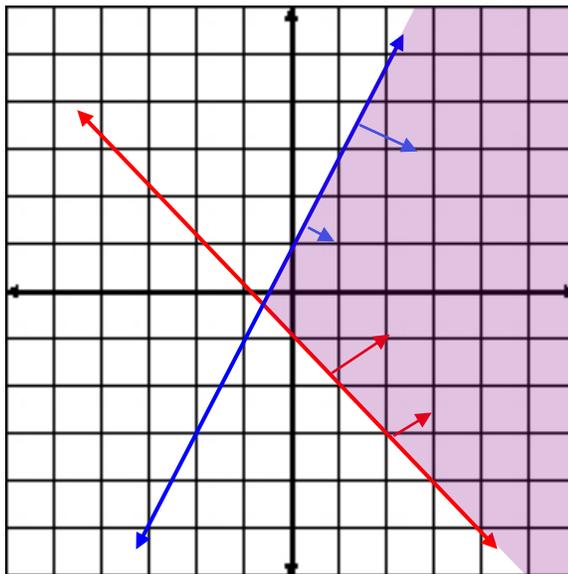
*Example #1:*  $y \geq -x - 1$   
 $y \leq 2x + 1$

The intersection of both graphs occurs in the darkened area because of the shading.

Shading occurs above the red line and to the right and below the blue line.



The solution to the system of Inequalities,  $y \geq -x - 1$  and  $y \leq 2x + 1$ , is the intersection of both graphs.



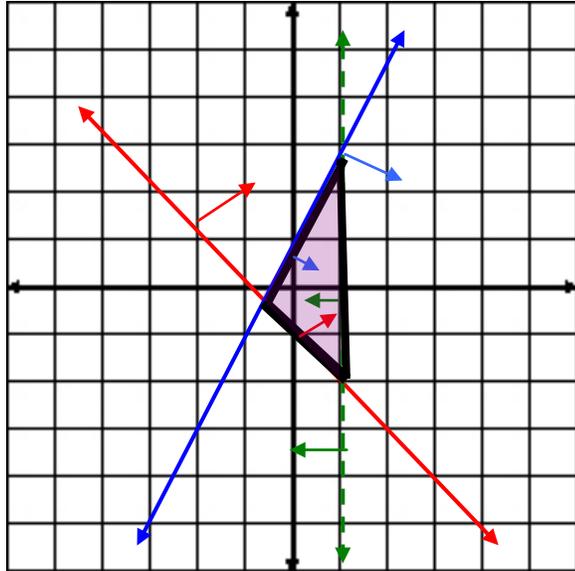
The Intersection (03:58)

Now let's add a third line,  $x < 1$ , to the system of inequalities and examine the intersection of all three lines.

*Example #2:*  $y \geq -x - 1$   
 $y \leq 2x + 1$   
 $x < 1$

The intersection of all three graphs occurs in the darkened triangular shape because of the shading.

Shading occurs above the red line, to the right and below of the blue line and to the left of the green line.



To determine a system of inequalities from a graph:

- 1.) find the equations for the boundary lines:

Determine the slope  $\frac{y_2 - y_1}{x_2 - x_1}$ , and then use the point-slope form of  $y - y_1 = m(x - x_1)$  to find the equation of the line.

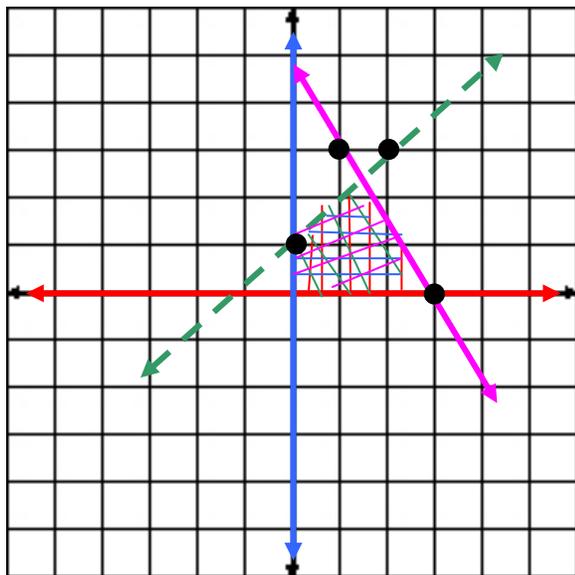
- 2.) make sure each boundary line has the appropriate inequality symbol

$\leq, \geq$  will be a solid line  
 $<, >$  will be a dashed line

*Example #3:* The blue line represents  $x \geq 0$  and the red line represents  $y \geq 0$ . To find the equations of the green and pink lines, find the slope, and then use  $y - y_1 = m(x - x_1)$  to get the equation.

The green line contains the points  $(0, 1)$  and  $(2, 3)$  so the slope is:

$$\frac{1-3}{0-2} = \frac{-2}{-2} = \frac{2}{2} = 1$$



Choose one of the given points, (2, 3), to find the equation.

$$y - 3 = 1(x - 2)$$

$$y - 3 = x - 2$$

$$y = x + 1$$

Since the green line is dashed and shaded under the line, the equation is  $y < x + 1$ .

The pink line contains the points (3, 0) and (1, 3) so the slope is

$$\frac{3-0}{1-3} = \frac{3}{-2} = \frac{-3}{2}$$

Choose one of the points, (3, 0), to find the equation.

$$y - 0 = \frac{-3}{2}(x - 3)$$

$$y = \frac{-3}{2}x + \frac{9}{2}$$

Since the pink line is solid and shaded below the equation is  $y \leq \frac{-3}{2}x + \frac{9}{2}$ .

Therefore, the systems of inequalities that make up the graph are:

$$x \geq 0$$

$$y \geq 0$$

$$y < x + 1$$

$$y \leq \frac{-3}{2}x + \frac{9}{2}$$

**Stop!** Go to Questions #1-4 about this section, then return to continue on to the next section.

# Linear Programming

**Linear programming** is a method used to find a minimum or maximum value of some quantity. Often this quantity is cost or profit and used in business applications.

**Constraints** are the inequalities used in linear programming.

**Feasible region** is the solution set.

**Objective function** is the function to be maximized or minimized.

If there is a maximum and minimum value of the objective function, it always occurs at one of the vertices of the feasible region.

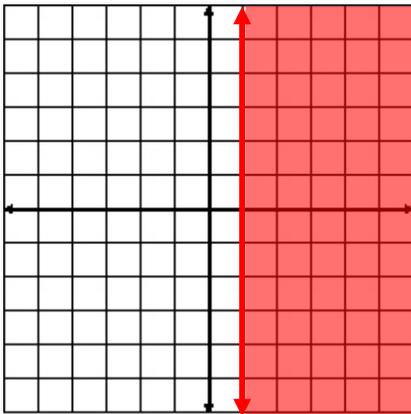
*Example #1:* Graph the following constraints. Name the coordinates of the vertices of the feasible region. Find the maximum and minimum values for the objective function  $P = 3x + y$ .

Constraints:

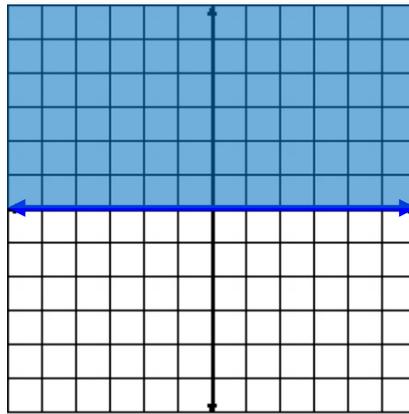
$$\begin{aligned}x &\geq 1 \\y &\geq 0 \\2x + y &\leq 6\end{aligned}$$

*Step 1:* Graph the inequalities.

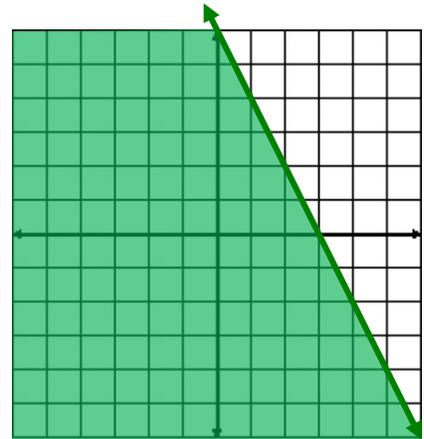
$$x \geq 1$$



$$y \geq 0$$

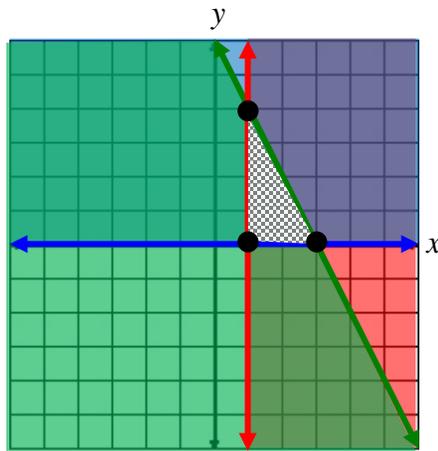


$$\begin{aligned}2x + y &\leq 6 \\y &\leq -2x + 6\end{aligned}$$

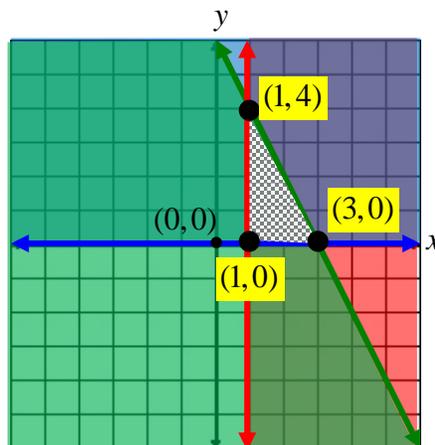


When the inequalities are all graphed in the same coordinate plane, we can determine the feasible region (solution set of points), the region that is overlapped by all three graphs.

The intersection of the boundaries are the vertices of the feasible region.



*Step 2:* Find the coordinates of the vertices of the feasible region.



The intersection of the boundaries are the vertices of the feasible region. The polygon formed is a triangle with vertices at  $(1, 4)$ ,  $(3, 0)$  and  $(1, 0)$ .

*Step 3:* Evaluate the objective function at each vertex.

$$P = 3x + y$$

$$\text{At } (1, 4): \quad P = 3(1) + 4 = 7$$

$$\text{At } (3, 0): \quad P = 3(3) + 4 = 9 \text{ (Maximum)}$$

$$\text{At } (1, 0): \quad P = 3(1) + 4 = 3 \text{ (Minimum)}$$

$P$  has a maximum value of 9 at  $(3, 0)$  and a minimum value of 3 at  $(1, 0)$ .

Let's review what we've learned thus far about linear programming. In the next few questions and/or statements, read through and fill in the blanks about linear programming. Use terms from the given word bank. After deciding which terms fit best, click to reveal the answers.

**constraints   inequalities   maximum   minimum**

**objective function   solution set   vertices**



To find the feasible region for a linear programming problem, solve the system of linear

\_\_\_\_\_ called \_\_\_\_\_.

*“Click here” to check your answer.*

**inequalities, constraints**



The points within the feasible region are the \_\_\_\_\_ of the system.

*“Click here” to check your answer.*

**solution set**



The corner points of the polygon formed are the \_\_\_\_\_ of the feasible region.

*“Click here” to check your answer.*

**vertices**



The function to be maximized or minimized is called the \_\_\_\_\_.

*“Click here” to check your answer.*

**objective function**



What significant values always occur at the vertices of the feasible region?

*“Click here” to check your answer.*

**maximum and minimum values**

To solve a real world problem using linear-programming problem:

*Step 1:* Define the variables.

*Step 2:* Write a system of linear inequalities to represent the constraints.

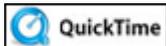
*Step 3:* Graph the system of linear inequalities to find the feasible region.

*Step 4:* Find the coordinates of the vertices of the feasible region.

*Step 5:* Write the objective function to be maximized or minimized.

*Step 6:* Evaluate the objective function at each vertex.

*Step 7:* Identify the coordinates that give the required maximum or minimum.



Linear Programming -- Business Constraints (04:36)

*Example #2:* A crafter is making two sizes of wreath arrangements. The crafter charges \$14 for the small arrangement and \$20 for the large. She wants to make at least 15 small and 10 large arrangements with a total of no more than 50 arrangements. Let  $x$  represent the number of small arrangements, and let  $y$  represent the number of large arrangements. How many of each should she make to maximize her revenue?



The question asks for the optimal combination of wreath arrangements to maximize the crafter's revenue.

$x$ : number of small arrangements

$y$ : number of large arrangements

Constraints:

$x \geq 15$       -She wants to make *at least* ( $\geq$ ) 15 small arrangements.

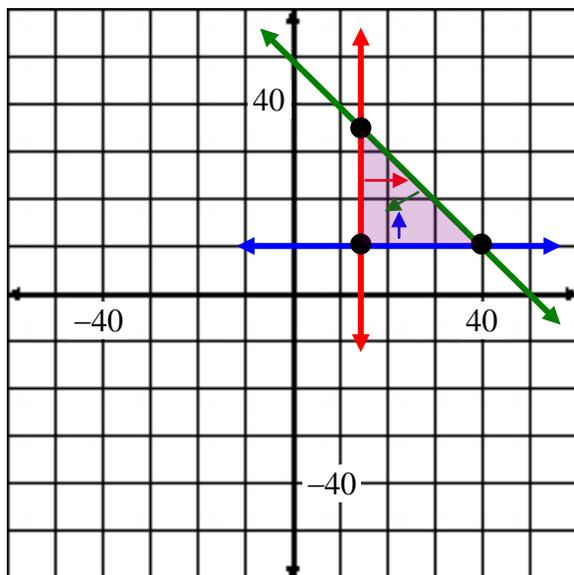
$y \geq 10$       -She wants to make *at least* ( $\geq$ ) 10 large arrangements.

$x + y \leq 50$       -She wants to make at total of *no more than* ( $\leq$ ) 50 large arrangements.

Solve  $x + y \leq 50$  for  $y$ .

$$\begin{aligned}x + y &\leq 50 \\ -x &\quad -x \\ y &\leq -x + 50\end{aligned}$$

Here is the graph displayed in the coordinate plane. Only the area that is overlapping between all three inequalities is shaded.



\*The scale of the graph is 10 units per line.

\*Notice the shading occurs to the right of the red, above the blue and below the green lines.

This will yield 3 vertices at (15, 10), (15, 35) and (40, 10).

The objective function that will represent the maximum profit is:

$$P = 14x + 20y \text{ because each small wreath costs \$14 and each large wreath costs \$20.}$$

Replace each of the vertices in for  $x$  and  $y$  to find the maximum value.

$(15, 10)$	$(15, 35)$	$(40, 10)$
$P = 14(15) + 20(10)$	$P = 14(15) + 20(35)$	$P = 14(40) + 20(10)$
$= 210 + 200$	$= 210 + 700$	$= 560 + 200$
$= 410$	$= 910$	$= 760$
	(maximum value)	

Since the maximum value is 910, the crafter should make 15 small wreaths and 35 large wreaths to gain maximum revenue.

**Stop!** Go to Questions #5-23 to complete this unit.