Course Overview

In this course, students formally define geometric figures; describe and apply the properties of similar and congruent figures; and justify conjectures involving similarity and congruence. They recognize and apply angle relationships in situations involving intersecting lines, perpendicular lines, and parallel lines; use coordinate geometry to represent and examine the properties of geometric figures including slope, midpoint, distance, parallel, and perpendicular lines; draw and construct representations of two- and three-dimensional geometric objects using a variety of tools such as straightedge, compass, and technology. Students represent and model transformations in a coordinate plane and describe results; prove or disprove conjectures and establish the validity of conjectures about geometric objects, their properties and relationships by counterexample, inductive and deductive reasoning, and critiquing arguments made by others. Students use right triangle trigonometric relationships to determine lengths and angle measures; use algebraic representations to model and solve problem situations and to describe and generalize geometric properties and relationships; connect physical, verbal, and symbolic representations of irrational numbers; calculate and explain the difference between absolute error and relative error; interpret the relationship between two variables using multiple graphical displays and statistical measures; model problems dealing with uncertainty with area models; differentiate and explain the relationship between the probability of an event and the odds of an event.

POINTS, LINES, PLANES, AND ANGLES

Unit Overview

In this course, you will learn about mathematics through lines and shapes. You will learn theory through studying theorems and postulates. You will apply theory by solving problems about lines and shapes. Look around and you will see that just about everything has a geometrical shape. Some objects have very basic shapes like rectangles and parallel lines while others are more complex like ovals and octahedrons. In essence, there is geometry behind most objects with which we are familiar. Enjoy this course and along the way learn to appreciate the beauty of geometry in your world!

In the first unit, you will examine points, lines, and planes and their connections. You will then study angles and connect various angle relationships. You will also learn how to make geometric constructions.

Points and Lines
Collinear and Coplanar
Angles
Measuring Angles
Angle Constructions
Angle Relationships

Points and Lines

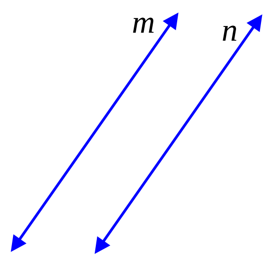
Point - A point is a location on a line. It has no dimensions but is represented by a dot.

Line - A line is a straight length that extends indefinitely into space. Lines have no width or thickness but are represented by straight edge marks.

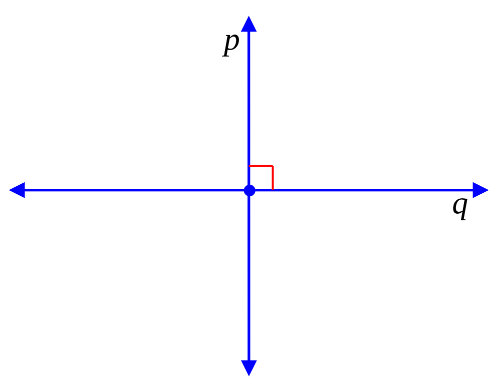
Line segment – A line segment is a part of a line usually named with its endpoints.

Intersection - Intersection is the point or line where two shapes meet. When two lines cross each other, there is one point at the place where they cross called the point of intersection. Two planes meet at and share a line of intersection.

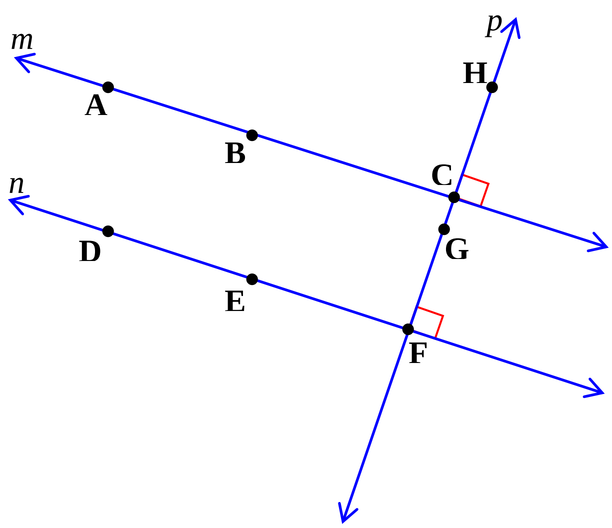
Parallel lines - Parallel lines are lines that lie in the same plane, are equidistant apart, and never meet.



Perpendicular lines - Perpendicular lines are lines that intersect and make right angles at the point of intersection. Right angles are denoted by a square shape as shown in the diagram below.



Example: Refer to the given diagram to answer the questions.



(a) Name a point.

Point A – Other points are B, C, D, E, F, G, H. Generally in this geometry course, points will be named with capital letters.

(b) Name a line.

Line *m* is one line in the diagram. It can be named another way by using any two points named on the line. Have you ever heard the expression "the shortest distance between two points is a straight line"? Well, we can name a line by using any two points on it. So line *m* can also be named as $\overleftrightarrow{AB}$, read Line AB. Other names for this line are $\overleftrightarrow{AC}, \overleftrightarrow{BC}, \overleftrightarrow{BA}, \overleftrightarrow{CA}, and\ \overleftrightarrow{CB}$.

*** When writing the name of a line on paper, you draw a mini-line above the two letters as shown in the previous paragraph. When referring to a line online, just type in "line AB" via the keyboard**.

(c) Name a line segment.

One line segment (part of a line) starts at Point A and ends at Point B. Its name is $\overline{AB}$, read segment AB. Some other segments are $\overline{BC}, \overline{GH}$, and $\overline{EF}$. There are many more segments in the diagram.

*** When writing the name of a line segment on paper, draw a mini-segment above the two letters as shown in the previous paragraph. When referring to a line segment online, just type in "segment AB" via the keyboard.**

(d) Name a point of intersection.

Point C is a point of intersection. It is the point where line *m* intersections with line *p*.

(e) Name a pair of parallel lines.

Lines *m* and *n* are parallel because they are equidistant apart. The lines may also be called $\overleftrightarrow{AB}$ and $\overleftrightarrow{DF}$. Another way to state this answer is $m \parallel n$ or $\overleftrightarrow{AB} \parallel \overleftrightarrow{DF}$.

* **When referring to parallel lines on paper, draw mini-parallel lines between the names for the lines as shown in the previous paragraph. When referring to parallel lines online, just type in "line m is parallel to line n" or "line AB is parallel to line DF".**

(f) Name a pair of perpendicular lines.

Line *m* is perpendicular to line *p* since the two lines intersect to make right angles. Another way to state this answer is $m \perp p$.

* **When referring to perpendicular lines on paper, draw mini-perpendicular lines between the names for the lines as shown in the previous paragraph. When referring to perpendicular lines online, just type in "line m is perpendicular to line n".**

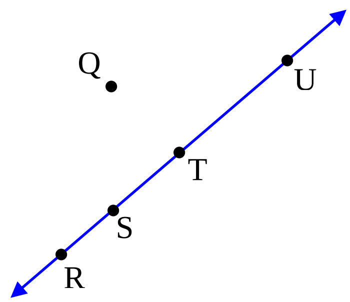
Collinear and Coplanar

Collinear points - Collinear points are points that line up in a straight line. They lie on the same line.

Plane - A plane is a flat surface. A plane extends forever in all directions. Flat tables, floors, ceilings, and walls are examples of parts of planes.

Coplanar points - Coplanar points are points that lie on the same plane.

Example 1: Refer to the diagram to answer the questions.



a) Name a line segment.

There are many line segments in the drawing. Here are a few of them:
$\overline{ST}$, $\overline{RU}$, $\overline{TR}$.

b) Name the point(s) that are collinear.

Points R, S, T, and U are collinear because they all line on the same line.

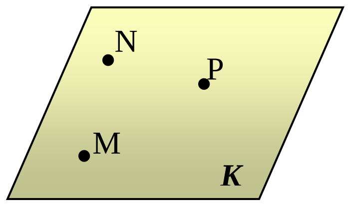
c) Name a point that is not collinear with points R and S.

Point Q is not collinear with R and S because they do not all line on the same line.

d) Name a point that is collinear with Q. Point U is collinear with Q since a straight line can be drawn through Q and U.

*Q is not collinear with **all** of the other points collectively; however, Q is collinear with any one of the points. Q is collinear with U; Q is collinear with T; Q is collinear with S; and Q is collinear with R.

Example 2: Refer to the diagram to name the plane.

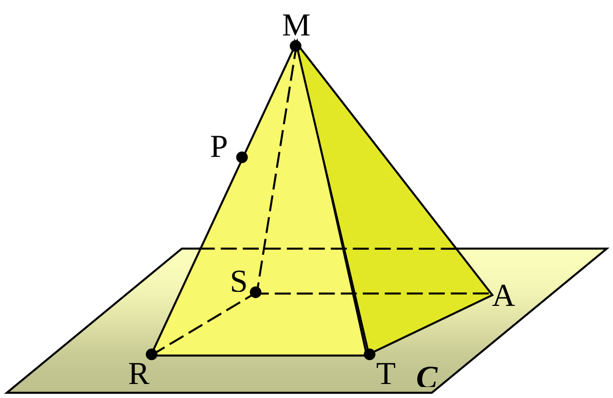


You can name the plane several ways:

K is the single letter designated as the plane's name.

Also any three letters in a plane can be used to name the plane such as plane MNP, plane MPN, plane PMN, plane PNM, plane NPM, and plane NMP.

Example 3: Refer to the 3-dimensional diagram to answer the questions.



a) Name a plane that contains point S.

plane *C* or plane *SRT*.

b) Name a plane that does not contain point S.

plane MAT or plane MTR

c) Name three coplanar points.

Points R, S, and A are coplanar because they all lie in plane *C*. Another example of coplanar points are points M, T, and A which lie in a different plane, a side of the pyramid.

d) Name a point the would not be coplanar with point A.

Point P would not be coplanar with A because they do not fall on the same planes. Point P lies in plane MRT while point A lies in planes C and MAT.

e) True or False. The bottom of the pyramid is part of plane *C*.

True. The bottom of the pyramid and the portion of plane *C* that is displayed are both parts of the same plane the extends on forever. Another name for plane C is plane RSA.

f) Name three collinear points.

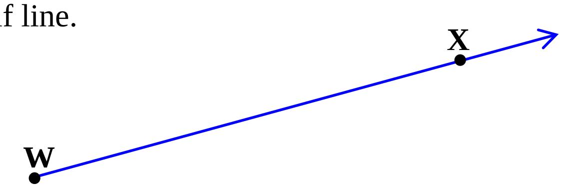
Points M, P, and R are collinear points since they all lie on the same line segment.

g) Name three non-collinear points.

Points M, S, and A are non-collinear since they do not line up in a straight line.

Angles

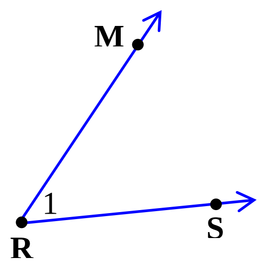
Ray – A ray is a half line.



The name of this ray is $\overrightarrow{WX}$ or Ray WX. Since the ray starts at point W, W must be the first letter of its name.

Vertex – A vertex is the point where two rays meet.

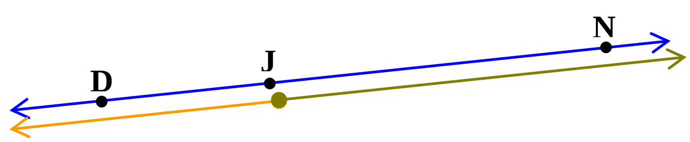
Angle – An angle is formed when two rays meet at a common point. The measure of an angle is the amount of circular rotation about a point starting with a ray and ending with a second ray.



Angle 1, also denoted as $\angle 1$, may be named $\angle MRS$ (Angle MRS). Point R is the vertex, $\overrightarrow{RS}$ and $\overrightarrow{RM}$ are the rays.

* When using three letters to name an angle, be sure to make the vertex letter the center letter of the name.

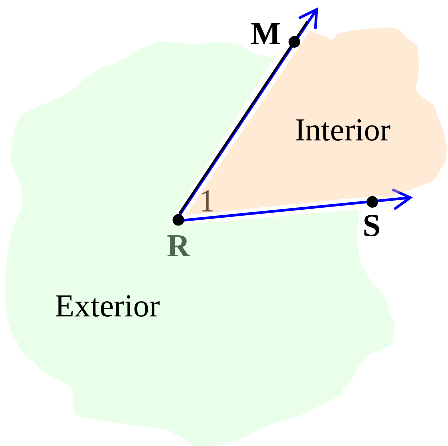
Opposite Rays – Opposite rays are two half lines that are formed at a point on a line.



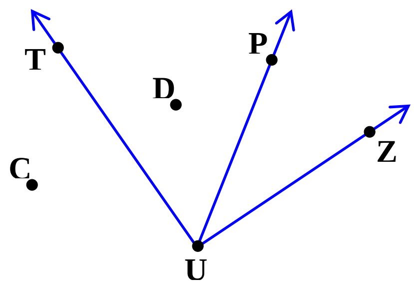
The opposite rays in this diagram are $\overrightarrow{JD}$ and $\overrightarrow{JN}$.

Interior – The interior of an angle is the area within the two rays.

Exterior – The exterior of an angle is the area outside the two rays.



Example 1: Refer to the diagram to answer the questions.



a) Name a vertex.

Point U is the vertex.

b) Name a ray.

There are three rays. They are $\overrightarrow{UT}, \overrightarrow{UP}, \overrightarrow{UZ}$.

c) Name three angles.

They are $\angle TUP, \angle PUZ, TUZ$.

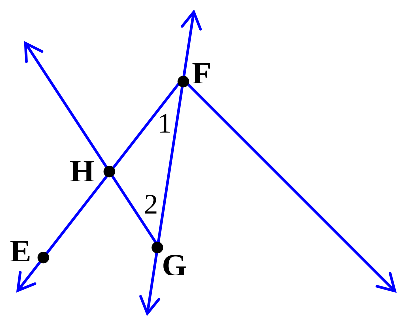
d) Name a point that lies in the interior of $\angle TUZ$.

Point D or Point P

e) Name a point that lies in the exterior of $\angle TUZ$.

Point C

Example 2: Refer to the diagram to answer the questions.



a) What are other names for $\angle 1$?

$\angle HFG$ and $\angle GFH$ (The vertex letter may be used to name an angle but should not be used when it can also be used to name other angles, so $\angle F$ would not be a good choice.)

b) What is the name of the vertex for $\angle FHG$?

Point H

c) What is the name of the vertex for $\angle 2$?

Point G

d) Name two different rays on $\overleftrightarrow{FG}$.

$\overrightarrow{FG}$ starting at Point F extending downward and $\overrightarrow{GF}$ starting at Point G and extending upward.

Measuring Angles

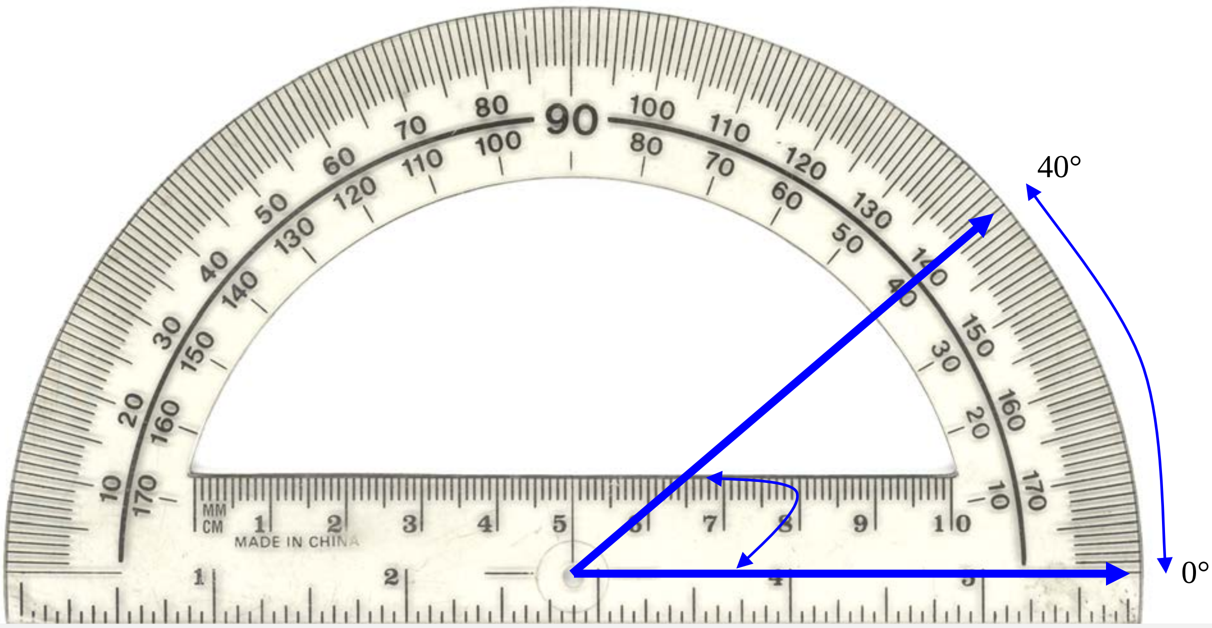
Postulate - A postulate is a mathematical statement that is accepted as true. It is based on the fundamentals of mathematical belief.

Measurement of an angle – Angles are measured in degrees.

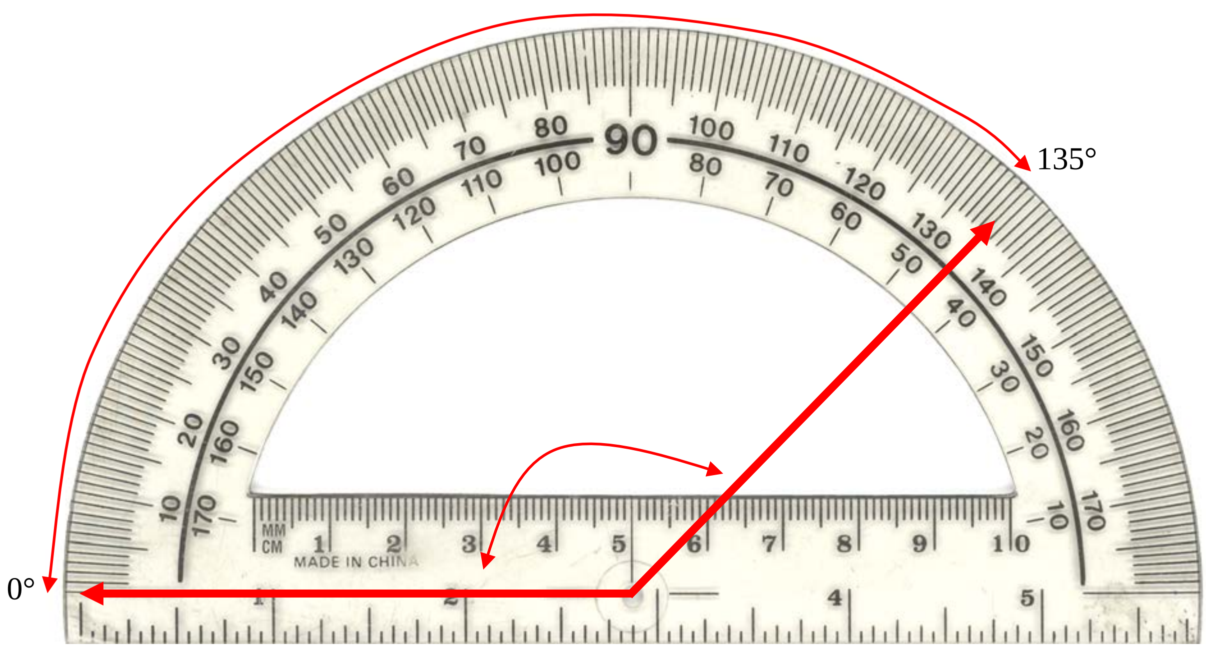
Degree – A degree is a unit of rotation around a point that may be used to measure angles. One degree is $\frac{1}{360}th$ of a rotation around a point.

Protractor – A protractor is a measurement tool used to measure angles in degrees.

Postulate 1-A Protractor Postulate	Given $\overrightarrow{AB}$ and a number r between 0 and 180, there is exactly one ray with endpoint A, extending on either side of $\overrightarrow{AB}$, such that the measure of the angle formed is r.



The starting ray, in this case, the bottom ray, is at 0°. Read the other ray. The ray is passing through both 40 and 140. You must decide which number makes sense. Think about a right angle – it measures 90°. This angle is not as open as a right angle; thus, you would read the smaller number. This angle measures 40°. Notice the numbers near the bottom ray, the lower set of numbers start at 0, then 10, 20, etc. That is the set of numbers that is used to read this angle. The starting ray starts at 0°. This is an **acute angle** because it measures more than 0° and less than 90°.



The starting ray, in this case, the bottom ray, is at 0°. Read the other ray. The ray is passing through both 45 and 135. You must decide which number makes sense. Think about a right angle – it measures 90°. This angle is open wider than a right angle; thus, you would read the larger number. This angle measures 135°. Notice the numbers near the bottom ray, the upper set of numbers start at 0, then 10, 20, etc. That is the set of numbers that is used to read this angle. The starting ray starts at 0°. This is an **obtuse angle** because it measure more than 90° and less than 180°.

Definition of Right, Acute and Obtuse Angles

$\angle A$ **is a right angle if** $m\angle A$ **is 90.**
$\angle A$ **is an acute angle if** $m\angle A$ **is less than 90.**
$\angle A$ **is an obtuse angle if** $m\angle A$ **is greater than 90 and less than 180.**

Angle Constructions

Compass – A compass is a measurement tool used to draw arcs and circles.

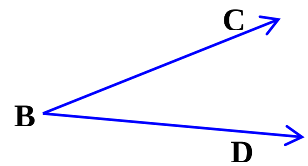
Arc – An arc is a portion of a circle.

Congruent figures – geometric figures that have the same size and shape.

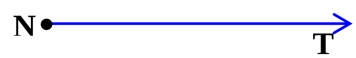
We will now look at using a compass to make some geometric constructions.

Example 1: Construct $\angle TNR$ to be equal in measure to $\angle B$.

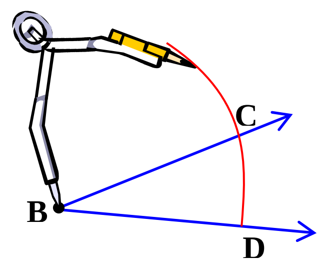
Draw $\angle B$.



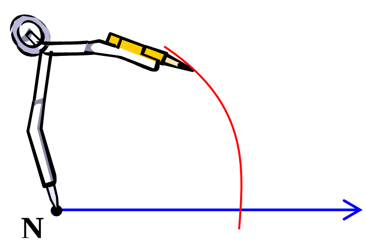
Draw $\overrightarrow{NT}$



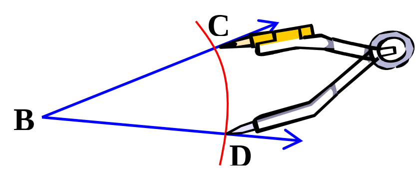
Place the metal point of the compass at vertex B and draw an arc passing through both rays of $\angle B$.



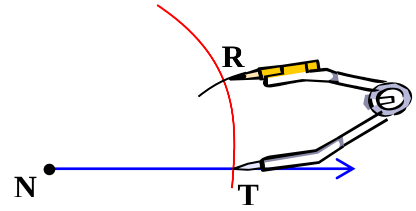
Move to $\overrightarrow{NT}$ and without changing the setting on the compass, place the metal point of the compass on point N and draw the same arc across the ray.



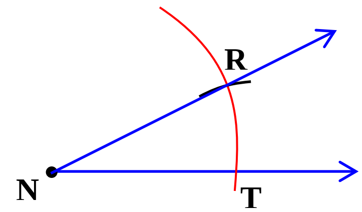
Place the metal point of the compass at point D and the pencil point at point C, where the arc crosses $\overrightarrow{BD}$. You will have to adjust the settings of the compass to do this.



Make sure the compass has not changed settings.
On $\overrightarrow{NT}$ place the metal point of the compass at the point of intersection for the ray and the arc. Use the pencil point to make a small arc that crosses the larger arc.



Draw $\overrightarrow{NR}$ starting at point N and passing through Point R, the point of intersection between the two arcs.



Angle RNT is the same size as angle CBD.

This can be represented as follows:

$m\angle RNT = m\angle CBD$

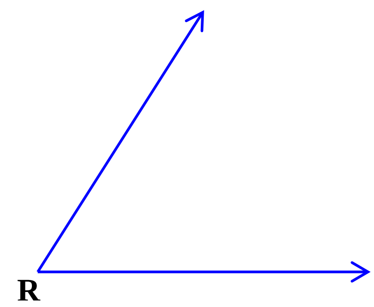
We can also say that angle RNT is congruent to angle CBD.

The symbol for congruence is $\cong$. Thus,

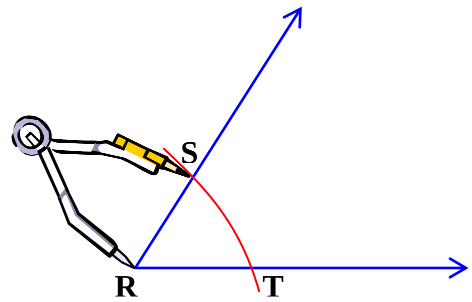
$\angle RNT \cong \angle CBD$

Example 2: Find a ray that divides the angle R into two congruent parts by constructing an angle bisector.

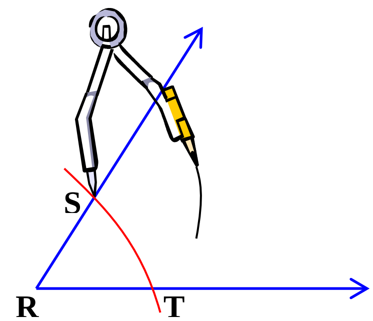
Draw $\angle R$.



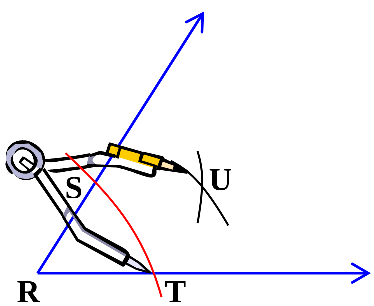
Place the metal point of the compass at point R and draw an arc through the angle rays, naming the points of intersection, S and T.



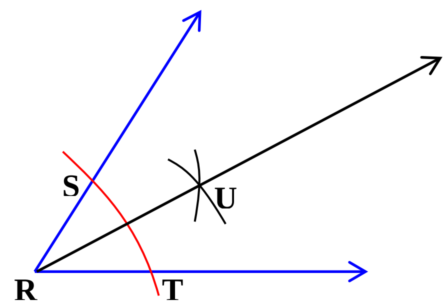
Adjust the compass settings a little wider and place the metal point of the compass at point S. Draw an arc in the interior of the angle.



Keep the compass setting the same and place the metal point of the compass at point T. Draw a second arc in the interior of the angle letting it cross the other arc.



Draw $\overrightarrow{RU}$ so that it starts at the vertex R and extends though the intersection of the two arcs, point U.



$\overrightarrow{RU}$ is the bisector of $\angle SRT$, thus $m\angle SRU = m\angle URT$ and $\angle SRU \cong \angle URT$.

Angle Relationships

Right Angle - A right angle measures 90˚.

Straight Angle – A straight angle measures 180˚.

Supplementary angles – Supplementary angles are angles that total 180˚.

Complementary angles – Complementary angles form a right angle.

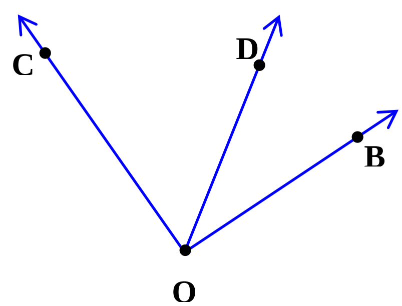
Linear pair – A linear pair is a pair of adjacent angles whose sum forms a straight angle.

Vertical angles – Vertical angles are the opposite congruent angles formed when two lines intersect.

Postulate 1-B Angle Addition

If R **is in the interior of** $\angle PQS$ **, then** $m\angle PQR + m\angle RQS = m\angle PQS$.
If $m\angle PQR + m\angle RQS = m\angle PQS$, **then** R **is in the interior of** $\angle PQS$.

Example 1: Use the diagram to answer the questions.



a) $m\angle COD + m\angle DOB = ?$

By the Angle Addition Postulate, the sum of the two angles equals $m\angle COB$.

b) If $m\angle COD$ equals 50˚ and $m\angle COB$ equals 85˚, what is $m\angle DOB$?

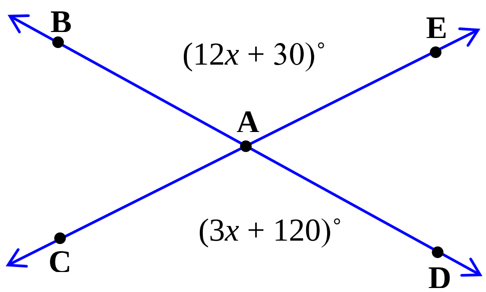
$$m\angle COD + m\angle DOB = m\angle COB$$

$$50^\circ + x = 85^\circ$$
$$x = 35^\circ$$

Vertical angles are congruent.

Example 2: Find $m\angle BAE$.

Since $\angle BAE$ and $\angle CAD$ are vertical angles, they are congruent and their measures are equal.

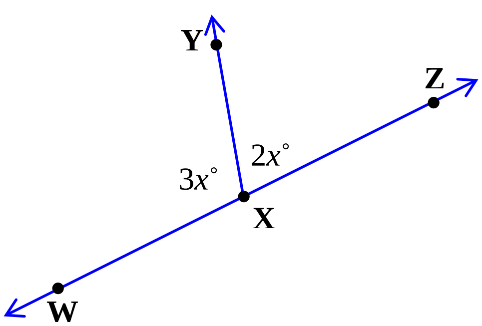


$$12x+30=3x+120$$
$$9x=90$$
$$x=10$$
$$m\angle BAE=12x+30$$
$$m\angle BAE=12(10)+30$$
$$m\angle BAE=150°$$

The sum of the measures of the angles in a linear pair is 180°.

Example 3: Find the measure of $\angle WXY$ and $\angle ZXY$.

Since $\angle WXY$ and $\angle ZXY$ are a linear pair, the sum of their measures is 180 degrees.



$$m\angle WXY+m\angle ZXY=180$$
$$3x+2x=180$$
$$5x=180$$
$$x=36$$

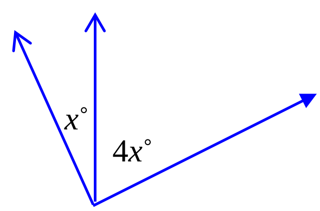
$m\angle WXY=3x$	$m\angle ZXY=2x$
$m\angle WXY=3(36)$	$m\angle ZXY=2(36)$
$m\angle WXY=108°$	$m\angle ZXY=72°$

Check: Linear pairs total $180°$. $72°+108°=180°$

The sum of the measures of complementary angles is 90°.

Example 4: A pair of angles is complementary. One of the angles is 4 times larger than the other angle. How large are each of the angles?

Draw a picture.



Let x represent the smaller angle.	x
Let $4x$ represent the larger angle.	$4x$
Complementary angles total 90.	first angle + second angle = 90
Write an equation and solve.	$x+4x=90$
	$5x=90$
	$x=18$

The first angle (x) is 18°; the second angle ($4x$) equals 4(18) which is 72°.

Check: Complementary angles total 90°. 18° + 72° = 90°