Terminal velocity

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The **terminal velocity** of an object falling towards the ground, in non-vacuum, is the speed at which the gravitational force pulling it downwards is equal and opposite to the atmospheric drag (also called air resistance) pushing it upwards. At this speed, the object ceases to accelerate downwards and falls at constant speed. An object moving downwards without power at greater than the terminal velocity (for example because it previously used power to descend, it fell from a thinner part of the atmosphere or it changed shape) will slow down until it reaches terminal velocity.

For example, the terminal velocity of a skydiver in a normal free-fall position with a closed parachute is about 195 km/h (120 Mph). This velocity is the asymptotic limiting value of the acceleration process, since the effective forces on the body more and more closely balance each other as it is approached. A speed of 50% of terminal velocity is reached after only about 3 seconds, while it takes 8 seconds to reach 90%, 15 seconds to reach 99% and so on.

Higher speeds can be attained if the skydiver pulls in his limbs (see also freeflying). In this case, the terminal velocity increases to about 320 km/h (200 Mph), which is also the maximum speed of the Peregrine Falcon diving down on its prey.

The reason an object reaches a terminal velocity is that the drag force resisting motion is directly proportional to the square of its speed. At low speeds the drag is much less than the gravitational force and so the object accelerates. As it speeds up the drag increases, until eventually it equals the weight. Drag also depends on the cross-sectional area. This is why things with a large surface area such as parachutes have a lower terminal velocity than small objects like cannon balls.

Mathematically, terminal velocity is described by the equation

$$V_t = \sqrt{\frac{2mg}{\rho A C_d}}$$

where

V_t is the terminal velocity,
m is the mass of the falling object,
g is gravitational acceleration,
C_d is the drag coefficient,
ρ is the density of the fluid the object is falling through, and
A is the object's cross-sectional area.

This equation is derived from the drag equation by setting drag equal to mg, the gravitational force on the object.

Note that the density increases with decreasing altitude, ca. 1% per 80 m (see barometric formula). Therefore, for every 160 m of falling, the "terminal" velocity decreases 1%. After reaching the local terminal velocity, while continuing the fall, speed *decreases* to change with the local terminal velocity.

Approximation

Approximating terminal velocity is much more easily done than calculating the terminal velocity because of the difficulty in finding the value of C_d. One simple small scale method is to hang an object out of a car window by a thin string. The terminal velocity of the object is the speed of the car when the object hangs at a 45° angle. This can be easily proven mathematically because it is when the atmospheric drag (in the horizontal direction) is equal to the force of gravity. It is when air resistance and gravity is the same.

Derivation

A falling object experiences two forces: gravitational force, and a large-velocity drag force. The addition of these two forces results in:

$$F = mg - qv^2$$

where

m is mass of the object
g is the acceleration due to gravity
q is $\frac{1}{2}\rho C_d A$ from the drag equation

The terminal velocity is reached when $F = 0$, so

$$mg - qv^2 = 0$$

or, solving for *v*,

$$v_t = \sqrt{\frac{mg}{q}} = \sqrt{\frac{2mg}{\rho A C_d}}$$

One can also find velocity as a function of time given gravitational and drag forces. Start, again, with $F = mg - qv^2$, and note that $F = ma$. This results in a differential equation:

$$m\frac{dv}{dt} = mg - qv^2$$

Rearange to see

$$\frac{dv}{dt} + \frac{q}{m}v^2 = g$$

The solution to this differential equation involves a hyperbolic tangent, and is

$$v(t) = \sqrt{\frac{gm}{q}}\tanh\left(\sqrt{\frac{gq}{m}}t\right)$$

Substitute *q* and the full solution is:

$$v(t) = \sqrt{\frac{2mg}{\rho A C_d}} \tanh\left(t\sqrt{\frac{g\rho C_d A}{2m}}\right)$$

We can also use this function *v*(*t*) to find the terminal velocity. As *t* approaches infinity, the tanh term approaches 1, leaving:

$$v_t = \sqrt{\frac{2mg}{\rho A C_d}}$$

See also

- Acceleration
- Drag (physics)
- Newton's Laws

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Category: Gravity

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