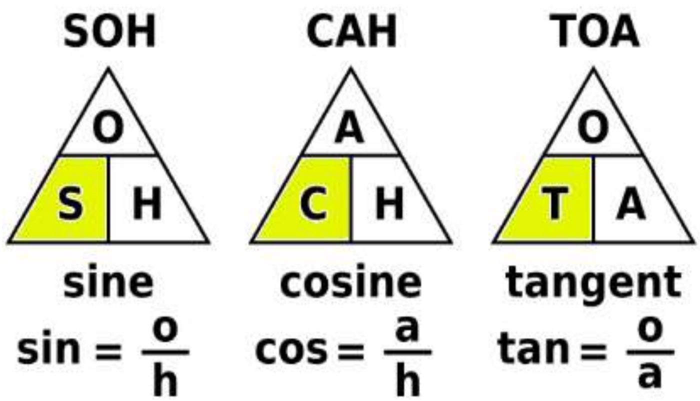
TRIGONOMETRY WITH RIGHT TRIANGLES: USING TRIGONOMETRY RATIOS TO FIND A SIDE OF A TRIANGLE



Unit Overview

In this unit, students will use Trigonometry ratios to find a missing side given one of the acute angles of a right triangle.

Key Vocabulary

Right Triangle	A triangle with a right angle
Right Angle	An angle whose measure is 90 degrees
Hypotenuse	Longest side of a right triangle
Ratio	A representation of the relative sizes of two or mor values
Adjacent	The leg that forms one side of an angle in the right triangle
Opposite	The leg/side opposite the angle is the leg that does not form a side with the angle
Θ	Greek letter that represents an unknown angle
Sine	Trigonmetric function: opposite over hypothenuse
Cosine	Trigonometric function: adjacent over hypothenuse
Tangent	Trigonometric function: opposite over adjacent

Using Trigonometry Ratios to Find a Side of a Triangle

The trigonometry ratios can be used to find many types of information, and one of their main purposes is to help solve triangles. To solve a triangle means to find the length of all the sides and the measure of all the angles. There are three main steps to help you find the side lengths of a triangle:

- ***Step 1: Choose which trigonometry ratio to use***.
  You will choose either sine, cosine, or tangent by determining which side you know and which side you are looking for.

  $$\sin A = \frac{\text{opposite}}{\text{hypotenuse}}$$

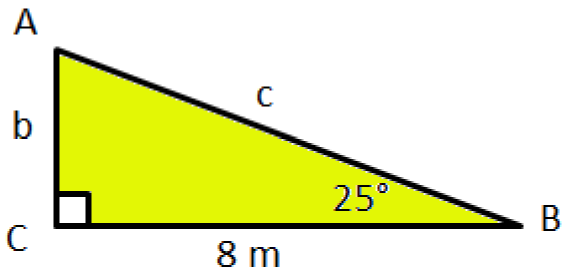
  $$\cos A = \frac{\text{adjacent}}{\text{hypotenuse}}$$

  $$\tan A = \frac{\text{opposite}}{\text{adjacent}}$$

- ***Step 2: Substitute***
  Substitute your information into the trigonometry ratio.

- ***Step 3: Solve***
  Solve the resulting equation to find the length of the side.

Example* – *Find b in the triangle below.



Step 1: Choose the correct trigonometry ratio to use.
First, we know we must look at angle B because that is the angle we know the measure. So, looking at angle B, we want to identify which sides are involved. We know one side is 8m, and that side is adjacent to angle B. The side we're looking for is opposite angle B. So we need to choose the trigonometry ratio that has opposite and adjacent. We need to use the trigonometry ratio of tangent.

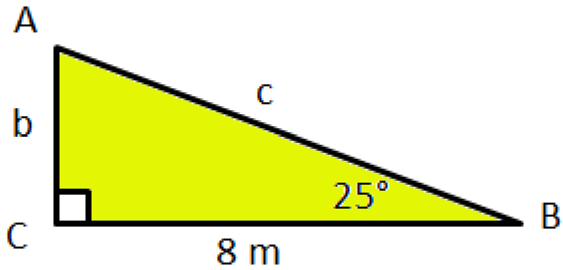
Step 2: Substitute.
Next, we write our trigonometry ratio: *tan B = opposite/adjacent*

Then, we substitute in the angle and the side we know: tan 25° = *b*/8

Step 3: Solve.
Now move the 8 to the other side by multiplying both sides by 8: 8* tan 25° = *b*
Use a calculator to find the answer. *b* = 3.7 m.

Let's Practice – Find a Side of a Triangle

1.) Identify all three step in order to ***find c*** in the triangle below.



***Step 1*: Choose the trig ratio to use.**

We're still using angle B. 8m is the **adjacent** and c is the **hypotenuse.**
The trig ratio that uses the adjacent and hypotenuse is the **cosine**.

***Step 2*: Create an equation to use.**

Write our trig ratio: $\cos B = \frac{adj}{hyp}$

***Step 3*: Substitute.**

Then, we substitute in the angle and the side we know: $\cos 25° = \frac{8}{c}$

***Step 4*: Solve.**

Since our variable is on the bottom, we can start by cross multiplying: $c \cdot \cos 25° = 8$

Then we'll divide both sides by cos 25°: $c = \frac{8}{\cos 25°}$

And use a calculator to find the answer. Well round to the nearest tenth: 8.8 m.

Summary

Watch the video below in order to complete some practice problems.

Khan Academy

Here are few reminders in order to find a side in a right-angled triangle using trigonometry.

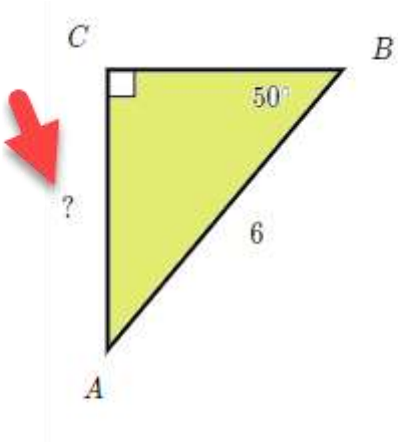
- We can find an unknown side in a right-angled triangle when we know the length, and one angle which is apart from the right angle.

- We can use trigonometric functions of sine, cosine, and tangent to find out the side.

- To find out which function we used, first we give names to the sides:
  - Adjacent: next to the angle
  - Opposite: opposite the given angle
  - Hypotenuse: longest side

- Once we know the give side and the side we are trying to find, we use “SOHCAHTOA”
  - SOH → Sine = opposite/hypotenuse
  - CAH → Cosine = adjacent/hypotenuse
  - TOA → Tangent = opposite/adjacent

Example - Given △ ABC, find AC.



Step 1: Determine which trigonometric ratio to use.

Focus on angle B since that is the angle that is given in the diagram. Note that we are given the length of the hypotenuse, and we are asked to find the length of the side opposite angle B. The trigonometric ratio that contains both of those sides is the sine.

Step 2: Create an equation using the trig ratio sine and solve for the unknown side.

Sin (B) = opposite/hypotenuse

Step 3: Substitute.

Sin (50°) = AC / 6

Step 4: Solve.

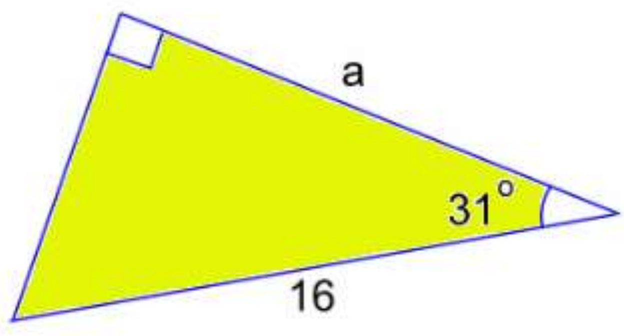
Multiply both sides by 6.

6 sin (50°) = AC

4.60 ≈ AC

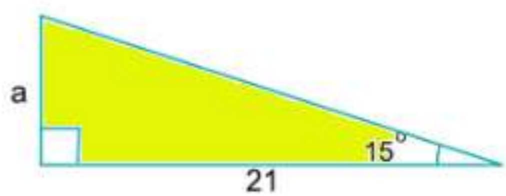
Let's Practice – Summary

2.) Determine which trigonometry ratio to use for the following triangle in order to find the missing side length of *a*.



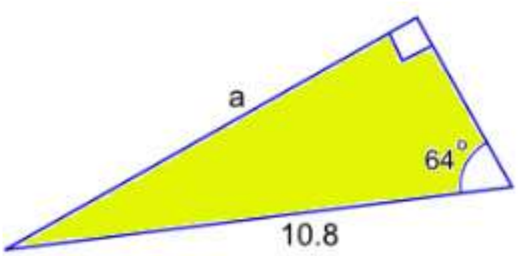
We know one angle, 31° and one side, 16, which is the hypotenuse. We want to find the side a, which is the adjacent. SohCahToa tells us that we need to use **Cosine**.

3.) Determine which trigonometry ratio to use for the following triangle in order to find the missing side length of *a*.



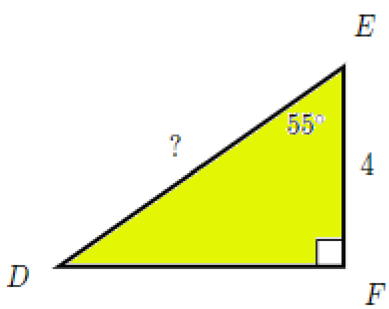
We know one angle, 15° and one side, 21, which is the adjacent. We want to find the side a, which is the opposite. SohCahToa tells us that we need to use **Tangent**.

4.) Determine which trigonometry ratio to use for the following triangle in order to find the missing side length of *a*.



This is a right-angled triangle, and we know an angle, 64°, and a side, 10.8, which is the hypotenuse. We want to find the side a, which is the opposite. SohCahToa tells us that we need to use **Sine**.

5.) Given $\triangle DEF$, find DE.



cos (E) = adjacent/hypotenuse

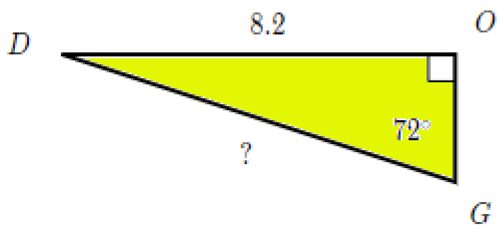
cos (55°) = 4/ED

ED*cos (55°) = 4 *multiply both sided by ED*

ED = 4/ cos (55°) *divide both sides by cos (55°)*

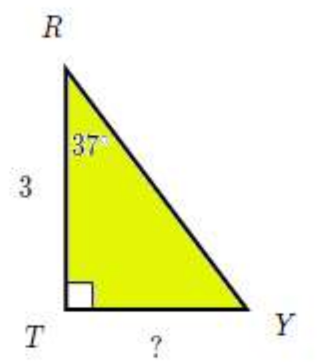
ED ≈ 6.97

6.) Given $\triangle DOG$, find DG.



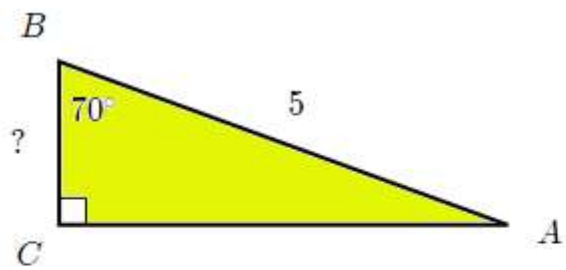
sin (G) = opposite/hypotenuse
sin (72°) = 8.2/DG
DG* sin (72°)= 8.2 *multiply both sided by DG*
DG = 8.2/ sin (72°) *divide both sides by* sin (72°)
DG ≈ 8.62

7.) Given $\triangle TRY$, find TY.



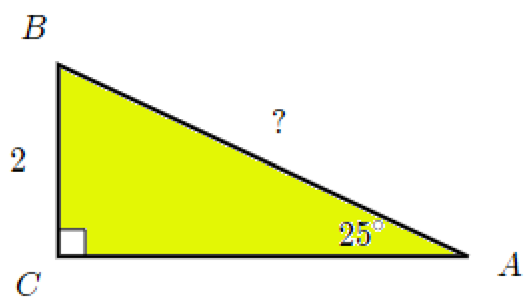
tan (R) = opposite/adjacent
tan (37°) = TY/3
3* tan (37°)= TY *multiply both sided by 3*
2.26 ≈ TY

8.) Look at the triangle below and find the side *BC*.



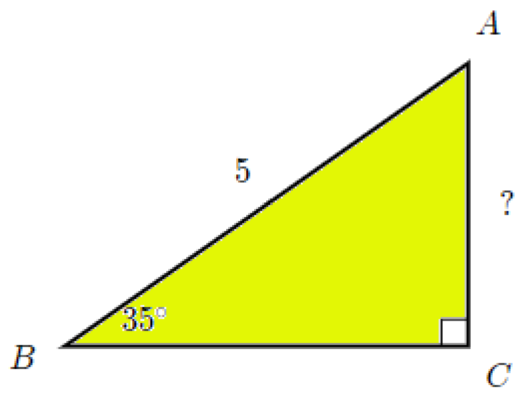
cos (B) = adjacent/hypotenuse
cos (70°) = BC/5
5* cos (70°) = BC *multiply both sided by 5*
1.71 ≈ BC

9.) Look at the triangle below and find the side *AB*.



sin (A) = opposite/hypotenuse
sin (25°) = 2/AB
AB* sin (25°) = 2 *multiply both sided by AB*
AB = 2/ sin (25°) *divide both sided by* sin (25°)
AB ≈ 4.73

10.) Look at the triangle below and find the side *AC*.



sin (B) = opposite/hypotenuse
sin (35°) = AC/5
5* sin (35°) = AC *multiply both sided by 5*
2.87 ≈ AC

Below are additional educational resources and activities for this unit.

Click on the icon to the left to practice solving for a side in right triangles.

[Practice 1: Using Trigonometry to Find Lengths]

[Practice 2 – Trigonometric Ratios]

[Practice 3 – Missing Sides (1-4)]